Robust Exponential Stability of Discrete Time Perturbed Impulsive Switched Systems[1)]

ZONG Guang-Deng WU Yu-Qiang

(*Research Institute of Automation, Qufu Normal University, Qufu* 273165)

(E-mail: zonggdeng@yahoo.com.cn)

Abstract The robust exponential stability of a class of discrete time impulsive switched systems with structure perturbations is studied. Based on the average dwell time concept and by dividing the total activation time into the time with stable subsystems and the time with unstable subsystems, it is shown that if the average dwell time and the activation time ratio are properly large, the given switched system is robustly exponentially stable with a desired stability degree. Compared with the traditional Lyapunov methods, our layout is more clear and easy to carry out. Simulation results validate the correctness and effectiveness of the proposed algorithm.

Key words Switched systems, robust exponential stability, structure perturbations, average dwell time

1 Introduction

A switched system is an important class of hybrid systems consisting of a family of continuous-time or discrete-time subsystems and a rule that orchestrates the switching among them. Switched systems are different from the common continuous-time or discrete-time systems, because they have some special properties. It is very important how to choose the rule. Recently, the study on the switched systems analysis and switching control has attracted more and more attentions[1~12]. Its main result lies in: 1) many systems encountered in practice exhibit switching phenomena between several subsystems due to the inherent multi-model or various environmental factors[3,4]; 2) the methods of intelligent control design are based on the idea of switching between different controllers[3,5]; 3) switching controllers can achieve a better performance than the traditional feedback controllers, *e.g.*, Narendra and Balakrishnan improved the system performance by using a set of switched adaptive controllers[6].

In this paper, under the assumption that the subsystems include stable and unstable subsystems, we study the robust exponential stability of a class of discrete time impulsive switched systems with structure perturbations. [7] considered the exponential stability of a class of continuous time switched systems that only contained stable subsystems, and proved that the switched system is exponentially stable when the dwell time of the switching signal (the time between every two consequent switchings) is large enough. [8] proposed the concept of the average dwell time, and proved that the switched system is still exponentially stable when the average dwell time is large enough. [9] studied the switched systems containing stable and unstable subsystems. The total activation time of the switched system was divided into the activation time of the stable subsystems and unstable subsystems. The authors proved that the switched system is exponentially stable when the average dwell time and the total activation time period ratio between the stable subsystems and unstable subsystems are both large enough. Similar to [9], [11] considered the exponential stability of a class of discrete time switched systems.

However, there are few results on the robustness of the uncertain switched systems. Meanwhile, for the practical systems, the impulsive effect often occurs when switching exists. Therefore, in this note, we consider the robust exponential stability of the impulsive switched systems with structure perturbations. With the knowledge of the average dwell time and the modular matrix, we prove that the given switched system is robustly exponentially stable with the desired stability degree under the proposed switching signal, if the average dwell time and the total activation time period ratio between the stable subsystems and unstable subsystems are both properly large. Finally, the numerical simulations validate the algorithm in this paper.

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2 Problem formulation and preliminaries

Consider the following discrete time perturbed switched system

$$x(k+1) = [A_{\sigma(k)} + \Delta A_{\sigma(k)}]x(k), \quad x(0) = x_0 \tag{1}$$

where $x(k) \in R^n$ is the state, and $\sigma \in I_N = \{1, \cdots, N\}$ is the switching signal which is a piecewise constant function. When $\sigma(k) = i$, $i \in I_N$, system (1) switches to the i-th subsystem. $\Delta A_{\sigma(k)}$ describes the structure perturbation of the system mode.

Definition 1. $|G|_m = [|g_{ij}|]$ is the modular matrix of $G = [g_{ij}]$, *i.e.*, the elements of $|G|_m$ are the absolute values of the corresponding elements of matrix G.

Assumption 1. Suppose that $\Delta A_{\sigma(k)}$ satisfies

$$|\Delta A_i|_m \leqslant \Phi(i), \quad 1 \leqslant i \leqslant N \tag{2}$$

where $\Phi(i)(1 \leqslant i \leqslant N)$ is a nonnegative matrix.

In order to make the whole switched system more practical, we assume that the sampling rates for all the subsystems are with the same constant T_0. For simplicity, let "k" and "$k_2 - k_1$" denote time "kT_0" and time interval "$(k_2 - k_1)T_0$", respectively, where $k, k_1, k_2 (k > 0, k_2 > k_1)$ are arbitrary nonnegative integrals. Without loss of generality, we assume that all the system matrices $A(i)(1 \leqslant i \leqslant N)$ include both stable and unstable matrices.

Usually, there are three switching modes that exist for the practical systems: 1) the system switches automatically from mode i to mode j after some duration time in mode i[10]; 2) the system will switch from mode i to mode j, if the state of mode i is in a switching conditional set $S_{ij}(x)$, which is of the form[10]

$$S_{ij}(x) = \{x | h_{ij} x = 0, h_{ij} \in R^n\} \tag{3}$$

3) in discrete event dynamical systems, the system will switch from mode i to mode j when some event happens.

Let k_i denote the i-th switching time, and at time k_i, some impulsive switchings will occur as follows

$$x(k_i^+) = [D(\sigma(k_i),\ \sigma(k_{i+1})) + \Delta D(\sigma(k_i),\ \sigma(k_{i+1}))]\ x(k_i) \tag{4}$$

where $D(\cdot, \cdot)$ is the reset map. $D(\cdot, \cdot)$ and $\Delta D(\cdot, \cdot)$ reflect the influence of the switching to the system state. $\Delta D(\cdot, \cdot)$ represents the structured perturbations of the reset map satisfying

$$|\Delta D(i,j)|_m \leqslant \Gamma(i,j),\ 1 \leqslant i,j \leqslant N \tag{5}$$

where $\Gamma(i,j)(1 \leqslant i,j \leqslant N)$ is a nonnegative matrix.

Our aim is to find an appropriate switching signal $\sigma(k)$ such that the given discrete time perturbed switched system (1)~(5) is exponentially stable.

For any matrixes $A = [a_{ij}] \in R^{n \times n}$ and $B = [b_{ij}] \in R^{n \times n}$, let A^{T}, $\lambda(A)$ and $\|A\|$ denote respectively the transpose matrix, eigenvalue and the norm. Define $\|A\| = \sqrt{\lambda_{\max}(A^{\mathrm{T}}A)}$ for simplicity, where $\lambda_{\max}(A^{\mathrm{T}}A)$ is the maximum eigenvalue of the matrix $A^{\mathrm{T}}A$. It is known from [10] that

$$\|A\| \leqslant \||A|_m\|, \quad A + B \leqslant |A + B|_m \leqslant |A|_m + |B|_m \tag{6}$$

where $A \geqslant B$ if and only if $a_{ij} \geqslant b_{ij}$.

Definition 2[11]**.** For a certain switching signal $\sigma(k)$, the given switched system is said to be globally exponentially stable with stability degree λ, if there exist a constant $c > 0$ and $0 < \lambda < 1$ such that $x(k)$ satisfies $\|x(k)\| \leqslant c\lambda^k \|x_0\| (k \geqslant 0)$.

Let us introduce the concept of average dwell time to the discrete time switched system[11]. For any switching signal $\sigma(\cdot)$, denote $N_\sigma(0,k)$ as the switching times of $\sigma(\cdot)$ in the time interval $[0,k)$. Given constants $N_0(N_0 \geqslant 0)$ and $\tau_a(\tau_a > 0)$, denote $S_a[\tau_a, N_0]$ as all the switching signals satisfying

$$N_\sigma(0,k) \leqslant N_0 + k/\tau_a \tag{7}$$

where τ_a is called the average dwell time, and N_0 the chatter bound[8].

Since the system matrixes $A(i)(1 \leqslant i \leqslant N)$ contain both stable and unstable matrices, we assume

$$\lambda_1 = \max_i\{\|A_i\| | A_i(1 \leqslant i \leqslant N) \text{ is stable}\},\ \lambda_2 = \max_i\{\|A_i\| | A_i(1 \leqslant i \leqslant N) \text{ is unstable}\} \tag{8}$$

It easily follows that $0 \leqslant \lambda_1 < 1$ and $\lambda_2 \geqslant 1$.

3 Main results

Case 1. Stability analysis of the switched system (1)~(5) when $\Delta A_{\sigma(k)} = 0$ and $\Delta D(\cdot,\cdot) = 0$.

Let $0 = k_0 < k_1 < k_2 < \cdots$ denote the discontinuous points (or switching points) of $\sigma(\cdot)$, and denote $\sigma(k_{j-1}) = p_j (j = 1, 2, \cdots)$. For the reset map $D(i, j)(1 \leqslant i, j \leqslant N)$, it is assumed that $\max\limits_{1\leqslant i,j\leqslant N} \|D(i,j)\| \leqslant \alpha$, where $\alpha > 0$ is a constant. Let $K^-(0,k)(or\ K^+(0,k))$ be the total activation time of the stable subsystems (or unstable subsystems) in $[0,k)$ for any switching signal $\sigma(\cdot)$. Assuming that $k_i \leqslant k < k_{i+1}$, then we have

$$\begin{aligned} &\sigma(l) = \sigma(l_{j-1}) = p_j,\ x(l+1) = A_{p_j}x(l),\ x(k_j^+) = D(p_j, p_{j+1})x(k_j) \\ &x(k_j) = A_{p_j}^{(k_j-k_{j-1})}x(k_{j-1}^+),\ l \in [k_{j-1}, k_j),\ (1 \leqslant j \leqslant i+1) \end{aligned} \tag{9}$$

Since $k_i \leqslant k < k_{i+1}$, *i.e.*, $k \in [k_i, k_{i+1})$, we derive

$$\begin{aligned} x(k) =& A_{p_{i+1}}^{(k-k_i)}D(p_i,p_{i+1})x(k_i^+) = A_{p_{i+1}}^{(k-k_i)}D(p_i,p_{i+1}) \cdot A_{p_i}^{(k_i-k_{i-1})}x(k_{i-1}^+) = \cdots = \\ & A_{p_{i+1}}^{(k-k_i)} \cdot \prod_{j=1}^{i} D(p_j,p_{j+1})A_{p_j}^{(k_j-k_{j-1})}x(0) \end{aligned} \tag{10}$$

Based on the assumptions and the analysis we obtain

$$\|x(k)\| \leqslant \alpha^i \prod_{j=1}^{i+1} \|A_{p_j}^{(k_j-k_{j-1})}\| \|x(0)\| \leqslant \alpha^i \lambda_1^{K^-(0,k)}\lambda_2^{K^+(0,k)}\|x(0)\| = \alpha^{N_\sigma(0,k)}\lambda_1^{K^-(0,k)}\lambda_2^{K^+(0,k)}\|x(0)\| \tag{11}$$

where $N_{\sigma(0,k)}$ denote all the switching times of the switching signal $\sigma(\cdot)$ in the time interval $[0,k)$.

In order to guarantee the exponential stability of the switched system (1)~(5), we design the switching signal $\sigma(k)$ and present the main theorem as follows.

SL1: Design the switching signal $\sigma(k)(\sigma(k) \in S_a[\tau_a, N_0])$ such that

$$\frac{K^-(0,k)}{K^+(0,k)} \geqslant \frac{\ln\lambda_2 - \ln((1-\varepsilon_0)\lambda_0)}{\ln((1-\varepsilon_0)\lambda_0) - \ln\lambda_1} \tag{12}$$

$$\tau_a \geqslant \tau_d,\ N_0 \leqslant N_d,\ N_d = \ln c_1/\ln\alpha,\ \tau_d = \ln\alpha/(\ln\lambda_0 - \ln((1-\varepsilon_0)\lambda_0)) \tag{13}$$

where $k \geqslant 0, \lambda_0 \in (\lambda_1, 1), 0 < \varepsilon_0 < 1$ is a small constant and satisfies $(1-\varepsilon_0)\lambda_0 > \lambda_1$, and $c_1 > 0$ is a constant.

Theorem 1. When $\Delta A_{\sigma(k)} = 0$ and $\Delta D(\cdot,\cdot) = 0$, the given switched system is globally exponentially stable with the stability degree λ_0 under the switching signal (SL1).

Proof. From (12), we have

$$\lambda_1^{K^-(0,k)}\lambda_2^{K^+(0,k)} \leqslant ((1-\varepsilon_0)\lambda_0)^k \tag{14}$$

(11) and (14) imply that

$$\|x(k)\| \leqslant \alpha^{N_\sigma(0,k)}\lambda_1^{K^-(0,k)}\lambda_2^{K^+(0,k)}\|x(0)\| \leqslant \alpha^{N_\sigma(0,k)}((1-\varepsilon_0)\lambda_0)^k\|x(0)\| \tag{15}$$

From (13), we derive

$$N_\sigma(0,k)\ln\alpha + k\ln((1-\varepsilon_0)\lambda_0) = N_0\ln\alpha + [\ln\alpha/\tau_a + \ln((1-\varepsilon_0)\lambda_0)]k \leqslant \ln c_1 + k\ln\lambda_0 \tag{16}$$

Therefore, from (11)~(16), we obtain

$$\|x(k)\| \leqslant \alpha^{N_\sigma(0,k)}\lambda_1^{K^-(0,k)}\lambda_2^{K^+(0,k)}\|x(0)\| \leqslant c_1\lambda_0^k\|x(0)\| \tag{17}$$

Then according to Definition 2, we can see that the given switched system is globally exponentially stable with stability degree λ_0 under the switching law (SL1) when $\Delta A_{\sigma(k)}=0$ and $\Delta D(\cdot,\cdot)=0$. □

Remark 1. Similar to the way of the division of activation time in this paper, [11] considered the exponential stability of a class of discrete time switched systems without perturbation and impulsive effect. Its results are equivalent to the special case of Theorem 1 with $D(i,j)\equiv I$, $(1\leqslant i,j\leqslant N)$, that is, the trajectory of the system is continuous at the switching instants. However, our results not only contain the case of the trajectory being continuous but also contain the case of trajectory being discontinuous. Therefore, our results can be considered as an extension to [11] in some sense.

Remark 2. As shown in [9,11], a simple choice of the ratio condition (12) is

$$K^-(0,k)=2[\ln\lambda_2-\ln((1-\varepsilon_0)\lambda_0)]K_0,\ K^+(0,k)=[\ln((1-\varepsilon_0)\lambda_0)-\ln\lambda_1]K_0 \tag{18}$$

where $K_0>0$ is a time unit, which satisfies condition (13), and $0<\varepsilon_0<1$ is a sufficiently small constant.

Case 2. The stability analysis of the switched system (1)∼(5) when $\Delta A_{\sigma(k)}\neq 0$ and $\Delta D(\cdot,\cdot)\neq 0$.

Assumption 2. $\max\limits_{1\leqslant i\leqslant N}\|\varPhi(i)\|=\omega$, where $\omega>0$ is a constant and $\lambda_1+\omega<1$;

Assumption 3. $\max\limits_{1\leqslant i,j\leqslant N}[\|D(i,j)\|+\|\varGamma(i,j)\|]\leqslant\bar\alpha$, where $\bar\alpha>0$ is a constant.

SL2: Design the switching signal $\sigma(k)(\sigma(k)\in S_a[\tau_a,N_0])$ such that

$$\frac{\bar K^-(0,k)}{\bar K^+(0,k)}\geqslant\frac{\ln(\lambda_2+\omega)-\ln((1-\varepsilon_1)\bar\lambda_0)}{\ln((1-\varepsilon_1)\bar\lambda_0)-\ln(\lambda_1+\omega)} \tag{19}$$

$$\tau_a\geqslant\bar\tau_d,\ N_0\leqslant\bar N_d,\bar\tau_d=\ln\bar\alpha/(\ln\bar\lambda_0-\ln((1-\varepsilon_1)\bar\lambda_0)),\ \bar N_d=\ln c_2/\ln\bar\alpha \tag{20}$$

where $k\geqslant 0$, $\bar\lambda_0\in(\lambda_1,1)$, $0<\varepsilon_1<1$ is a very small constant and satisfies $(1-\varepsilon_1)\bar\lambda_0>\lambda_1+\omega$. $\bar K^-(0,k)(\bar K^+(0,k))$ denotes the total activation time of all the stable(unstable) subsystems respectively in the interval $[0,k)$.

Theorem 2. If Assumption 1, 2 and 3 are satisfied, the switched system (1)∼(5) is globally exponentially stable with the stability degree $\bar\lambda_0$ under the switching law (SL2).

Proof. Let $0=k_0<k_1<k_2<\cdots$ be the switching points of $\sigma(\cdot)$, and $\sigma(k_{j-1})=p_j, j=1,2,\cdots$. Assume that $k_i\leqslant k<k_{i+1}$. Similar to the above analysis, for any $l\in[k_{j-1},k_j)$, $(1\leqslant j\leqslant i+1)$ we have

$$\begin{aligned}&\sigma(l)=\sigma(l_{j-1})=p_j,\ x(l+1)=[A_{p_j}+\Delta A_{p_j}]x(l)\\&x(k_j^+)=[D(p_j,p_{j+1})+\Delta D(p_j,p_{j+1})]x(k_j)\\&x(k_j)=(A_{p_j}+\Delta A_{p_j})^{(k_j-k_{j-1})}x(k_{j-1}^+),\quad j=1,2,\cdots,i+1\end{aligned} \tag{21}$$

From (21) we get

$$x(k)=(A_{p_{i+1}}+\Delta A_{p_{i+1}})^{(k-k_i)}\prod_{j=1}^{i}[D(p_j,p_{j+1})+\Delta D(p_j,p_{j+1})](A_{p_j}+\Delta A_{p_j})^{(k_j-k_{j-1})}x(0) \tag{22}$$

Therefore,

$$\|x(k)\|\leqslant\|A_{p_{i+1}}+\Delta A_{p_{i+1}}\|^{(k-k_i)}\prod_{j=1}^{i}\|D(p_j,p_{j+1})+\Delta D(p_j,p_{j+1})\|\|A_{p_j}+\Delta A_{p_j}\|^{(k_j-k_{j-1})}\|x(0)\| \tag{23}$$

From (2), (5), (6), A1) and A2), we derive

$$\|A_{p_{j+1}}+\Delta A_{p_{j+1}}\|\leqslant\|A_{p_{j+1}}\|+\||\Delta A_{p_{j+1}}|_m\|\leqslant\|A_{p_{j+1}}\|+\max_{1\leqslant l\leqslant N}\|\varPhi(l)\|=\|A_{p_{j+1}}\|+\omega \tag{24}$$

$$\|D(p_j,p_{j+1})+\Delta D(p_j,p_{j+1})\|\leqslant\max_{1\leqslant i,j\leqslant N}[\|D(i,j)\|+\|\varGamma(i,j)\|]\leqslant\bar\alpha \tag{25}$$

Substituting (24), (25) into (23) results in

$$\|x(k)\|\leqslant\bar\alpha^i\prod_{j=1}^{i+1}(\|A_{p_{j+1}}\|+\omega)^{(k_j-k_{j-1})}\|x(0)\|\leqslant\bar\alpha^{N_\sigma(0,k)}(\lambda_1+\omega)^{\bar K^-(0,k)}(\lambda_2+\omega)^{\bar K^+(0,k)}\|x(0)\| \tag{26}$$

where $N_\sigma(0,k)$ denotes the switching times of the switching signal $\sigma(\cdot)$ in the interval $[0,k)$.

From (19) we obtain

$$(\lambda_1+\omega)^{\bar K^-(0,k)}(\lambda_2+\omega)^{\bar K^+(0,k)} \leqslant ((1-\varepsilon_1)\bar\lambda_0)^k \tag{27}$$

Considering that $\tau_a \geqslant \bar\tau_d$, $N_0 \leqslant \bar N_d$, we get

$$N_\sigma(0,k)\ln\bar\alpha + k\ln((1-\varepsilon_1)\bar\lambda_0) = N_0\ln\bar\alpha + [\ln\bar\alpha/\tau_a + \ln((1-\varepsilon_1)\bar\lambda_0)]k \leqslant \ln c_2 + k\ln\bar\lambda_0 \tag{28}$$

Therefore,

$$\begin{aligned}\|x(k)\| \leqslant \bar\alpha^{N_\sigma(0,k)}(\lambda_1+\omega)^{\bar K^-(0,k)}(\lambda_2+\omega)^{\bar K^+(0,k)}\|x(0)\| \leqslant \bar\alpha^{N_\sigma(0,k)}((1-\varepsilon_1)\bar\lambda_0)^k\|x(0)\| \leqslant \\ \exp[N_\sigma(0,k)\ln\bar\alpha + k\ln((1-\varepsilon_1)\bar\lambda_0)]\|x(0)\| \leqslant \exp[\ln c_2 + k\ln\bar\lambda_0]\|x(0)\| = c_2\bar\lambda_0^k\|x(0)\|\end{aligned} \tag{29}$$

According to Definition 2, we can see that the switched system (1)~(5) is globally exponentially stable with stability degree $\bar\lambda_0$. □

Remark 3. In [10], a class of continuous time impulsive switched systems with structure perturbations similar to (1)~(5) was considered, and the switching strategies were proposed which guaranteed the locally exponential stability of the switched system by the method of "loop analysis and matrix measure". Compared to the results in this paper, the judging condition of [10] is complex and hard to verify. The algorithm in this paper can guarantee the globally exponential stability of the given discrete time impulsive switched system. Moreover, it can be seen from Theorem 2 that compared to the traditional Lyapunov designing method[1,15], the division method of the motivation time of the switched system is more practically useful.

Remark 4. According to the above analysis, we can choose

$$\bar K^-(0,k) = 2[\ln(\lambda_2+\omega) - \ln((1-\varepsilon_1)\bar\lambda_0)]\bar K_0,\quad \bar K^+(0,k) = [\ln((1-\varepsilon_1)\bar\lambda_0) - \ln(\lambda_1+\omega)]\bar K_0 \tag{30}$$

where $\bar K_0 > 0$ is a time unit satisfying the condition in (20), and $0 < \varepsilon_1 < 1$ is a sufficiently small constant.

4 Simulation

Consider the following uncertain discrete time switched system

$$x(k+1) = \lfloor A_{\sigma(k)} + \Delta A_{\sigma(k)}\rfloor x(k),\ x(0) = x_0 \tag{31}$$

where

$$A_1 = \begin{bmatrix} -0.6 & 0.1 \\ 0 & 1.2 \end{bmatrix},\ A_2 = \begin{bmatrix} 0.2 & -0.1 \\ 0 & -0.3 \end{bmatrix}$$

$$\Delta A_1 = \begin{bmatrix} 0.1\sin(x_1(k)) & 0 \\ 0 & 0.1\cos(x_2(k)) \end{bmatrix},\ \Delta A_2 = \begin{bmatrix} -0.1\sin(x_1(k)) & 0 \\ 0 & -0.1\cos(x_2(k)) \end{bmatrix}$$

It is easy to know that A_1 is unstable and A_2 is stable. After some manipulations, we get $\lambda_1 = 0.2$, $\lambda_2 = 1.2$ and $|\Delta A_1|_m \leqslant \Phi(1) = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$, $|\Delta A_2|_m \leqslant \Phi(2) = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$, $\max\limits_{1\leqslant i\leqslant N}\|\Phi(i)\| = \omega = 0.1$.

Assume that at the switching time k_i, the system trajectory will jump as follows

$$x(k_i^+) = [D(\sigma(k_i),\ \sigma(k_{i+1})) + \Delta D(\sigma(k_i),\ \sigma(k_{i+1}))]x(k_i) \tag{32}$$

where

$$D(1,2) = \begin{bmatrix} \exp(0.4)-0.1 & 0.1 \\ 0.1 & \exp(0.4)-0.1 \end{bmatrix},\ D(2,1) = \begin{bmatrix} \exp(-0.5)-0.1 & 0.1 \\ 0.1 & \exp(-0.5)-0.1 \end{bmatrix}$$

$$\Delta D(1,2) = \begin{bmatrix} -0.1\sin(x_1(k)) & 0 \\ 0 & -0.1\cos(x_2(k)) \end{bmatrix},\ \Delta D(2,1) = \begin{bmatrix} 0.1\sin(x_1(k)) & 0 \\ 0 & 0.1\cos(x_2(k)) \end{bmatrix}$$

By some manipulations, we get

$$|\Delta D(1,2)|_m \leqslant \Gamma(1,2) = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},\ |\Delta D(2,1)|_m \leqslant \Gamma(2,1) = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$$

$$D(1,2)+\Gamma(1,2)=\begin{bmatrix}\exp(0.4) & 0.1\\ 0.1 & \exp(0.4)\end{bmatrix},\ D(2,1)+\Gamma(2,1)=\begin{bmatrix}\exp(-0.5) & 0.1\\ 0.1 & \exp(-0.5)\end{bmatrix}$$

$$\|D(1,2)+\Gamma(1,2)\|\leqslant 1.5918,\ \|D(2,1)+\Gamma(2,1)\|\leqslant 0.7065,\ \bar{\alpha}=1.5918$$

Taking $\bar{\lambda}_0=0.6\in(\lambda_1,1)$, $c_2=1$, and $\varepsilon_1=0.2$, $(1-\varepsilon_1)\bar{\lambda}_0=0.48>\lambda_1+\omega=0.3$, we derive

$$\bar{\tau}_d=\ln 1.9518/(\ln 0.6-\ln(0.48))=2.9972\approx 3,\ \bar{N}_d=\ln c_2/\ln\bar{\alpha}=0$$

$$\frac{\bar{K}^-(0,k)}{\bar{K}^+(0,k)}\geqslant\frac{\ln(\lambda_2+\omega)-\ln((1-\varepsilon_1)\bar{\lambda}_0)}{\ln((1-\varepsilon_1)\bar{\lambda}_0)-\ln(\lambda_1+\omega)}=\frac{\ln 1.3-\ln 0.48}{\ln 0.48-\ln 0.3}=2.1198\approx 2.2$$

Choose $\bar{K}_0=6.5>0$; then we have $\bar{K}^-(0,k)\approx 13$, $\bar{K}^+(0,k)\approx 3$. In $S_a[\tau_a,N_0)$, we choose the following quasi-periodical switching strategy

$$A(1)\to A(2)\mapsto A(1)\to A(2)\mapsto\cdots$$

and activate the first and second subsystems in 3 seconds and 13 seconds respectively. The system trajectories $x_1(k)$ and $x_2(k)$ are given in Fig. 1 and Fig. 2 during two quasi-periods. From Fig. 1 and Fig. 2, we see that the system states jump because of impulsive effect at the switching points. The state trajectory of the whole switched system converges with an exponential rate under the proposed switching law.

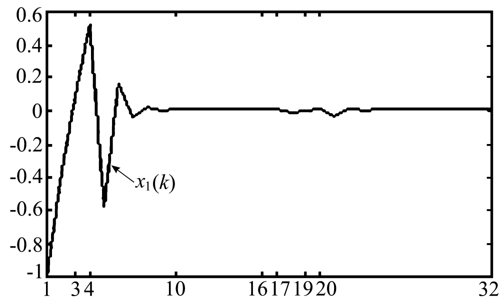

Fig. 1 The state trajectory $x_1(k)$ of the switched system

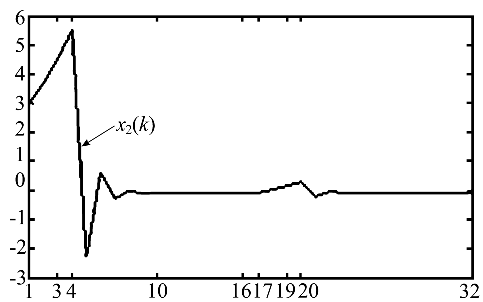

Fig. 2 The state trajectory $x_2(k)$ of the switched system

5 Conclusion

This paper considers the robust exponential stability of a class of uncertain discrete time switched systems. The given switched system contains stable and unstable subsystems. With the application of the concept of average dwell time and by dividing the activation time of the whole system into two parts, we have proved that the switched system is globally exponentially stable and has the desired stability degree if the average dwell time of the switched system and the total activation time period ratio between the stable subsystems and unstable subsystems are both properly large. Compared with the traditional Lyapunov function design method, the method in this paper is easily understood and carried out. Therefore we provide a way of research on the robust control of the switched systems.

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ZONG Guang-Deng Received his Ph. D. degree from Southeast University, Nanjing, P. R. China in 2005. He currently works as a lecturer in the Institute of Automation at Qufu Normal University. His research interests include hybrid dynamical systems and switching control.

WU Yu-Qiang Received his Ph. D. degree from Southeast University, Nanjing, P. R. China in 1994. He is now a full professor in the Institute of Automation at Qufu Normal University. His research interests include variable structure control, switching control, process control, and network control.