Optimal State Estimation for Discrete-time Systems with Random Observation Delays

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Abstract This paper investigates the linear minimum mean square error state estimation for discrete-time systems with Markov jump delays. In order to solve the optimal estimation problem, the single Markov delayed measurement is rewritten as an equivalent measurement with multiple constant delays, then a delay-free Markov jump linear system is obtained via state augmentation. The estimator is derived on the basis of the geometric arguments in the Hilbert space, and a recursive equation of the filter is obtained by solving the Riccati equations. It is shown that the proposed state estimator is exponentially stable under standard assumptions.

Key words Linear estimation, discrete-time systems, Markov jump delays, Riccati equation

In many signal estimation problems, it is usually assumed that the observation packets are received either immediately or with a constant and known delay[1−2]. However, in certain new engineering applications, the observation packets need to be transmitted through a communication network which may induce considerable and randomly varying transmission delays. Examples are offered by complex networked sensor systems containing a large number of low power sensors, as well as complex dynamical processes like advanced aircraft, spacecraft, and manufacturing process, where time division multiplexed networks are employed for exchange of information between spatially distributed plant components. All these new engineering applications motivate the development of a new chapter of control and estimation theory that deals with networked control systems, in which control and communication issues are combined together while all the delays and limitation of the communication channels between sensors, actuators, and controllers are taken into account.

State estimation for delayed systems was considered in a number of papers[3−4]. The situation where the observations were transmitted to the estimator via communication channels with irregular transmission time was investigated in [5−6]. The case of randomly delayed measurements was systematically studied by Ray (see [7−8], and the literature therein) under the assumption that the sensor delays did not exceed the sampling rate and form a stationary sequence of mutually independent random quantities with a priori known statistics. When the delay process is itself modeled as a finite-state Markov chain, state estimation algorithms for a resulting hidden Markov model were presented in [9], while an H_∞ filtering was presented in [10]. However, in [9−10], they only considered the case where the delay steps were known on-line via time-stamped data. The case where the transition probabilities were uncertain was not studied.

In this paper, we study the linear minimum mean square error state estimation for discrete-time systems subject to random communication delays, where the delay process is modeled as a finite-state Markov chain. Different from [9−10], the Markov delay considered in this paper is not known in advance. In order to solve the optimal estimation problem, the single Markov delayed measurement is firstly written as an equivalent measurement with multiple constant delays, and the estimation problem is then reformulated as an optimal linear filter of a class of Markov jump linear systems via state augmentation[11−15]. A finite-dimensional recursive filter is obtained in terms of Riccati difference equations. We also show that the estimator is stable under standard assumptions.

The rest of this paper is organized as follows. Section 1 introduces the system under consideration. The minimum variance state estimator is presented in Section 2, and its stability is studied in Section 3, which requires most of our efforts. In Section 4, a numerical example is presented to illustrate the main results. Finally, some conclusions are drawn in Section 5.

Notations. The notation in this paper is fairly standard. $\mathbf{R}^n$ denotes the n-dimensional Euclidean space. $\mathbf{R}^{m\times n}$ denotes the norm bounded linear space of all $m\times n$ matrices. For $L\in\mathbf{R}^{n\times n}$, L^{T} indicates the transpose of L, and $L\geq 0$ ($L>0$) means that the symmetric matrix L is positive semi-definite (positive definite). For a collection of N matrices $D_1,\cdots,D_N$, with $D_j\in\mathbf{R}^{m\times n}$, $\mathrm{diag}\{D_j\}\in\mathbf{R}^{Nm\times Nn}$ represents the diagonal matrix formed by D_j in the diagonal, and zero elsewhere. In addition, $L\otimes K\in\mathbf{R}^{mr\times ns}$ stands for the Kronecker product for any $L\in\mathbf{R}^{m\times n}$ and $K\in\mathbf{R}^{r\times s}$, and $\mathrm{E}[\cdot]$ represents the operator expected value.

1 Problem statement

Consider the following discrete-time linear system

$$\boldsymbol{x}(k+1) = A\boldsymbol{x}(k)+C\boldsymbol{w}(k) \quad (1)$$

$$\boldsymbol{y}(k) = H\boldsymbol{x}(k-d(k))+\boldsymbol{v}(k) \quad (2)$$

where $\boldsymbol{x}(k)\in\mathbf{R}^n$ is the state, $\boldsymbol{w}(k)\in\mathbf{R}^r$ is a process disturbance, $\boldsymbol{y}(k)\in\mathbf{R}^m$ is a sensor measurement, and $\boldsymbol{v}(k)\in\mathbf{R}^m$ is a noise. Here, $d(k)$ represents the random time delay. We make the following assumptions for the above system.

Assumption 1. $\boldsymbol{w}(k)$ and $\boldsymbol{v}(k)$ are null mean second-order, independent, wide-sense, stationary sequences with covariance matrices Q_w and Q_v, respectively.

Assumption 2. The initial state $\boldsymbol{x}(0)$ is also a second-order, independent, wide-sense, stationary sequence with null mean and covariance matrix $\mathrm{E}[\boldsymbol{x}(0)\boldsymbol{x}^{\mathrm{T}}(0)]=V$.

Assumption 3. $d(k)$ is a discrete-time Markov chain with finite state space $\{0,1,\cdots,d\}$, and transition probability matrix $\Lambda=[\lambda_{ij}], i,j=0,1,\cdots,d$. We set $\pi_i(k)=P(d(k)=i)$ and denote $\boldsymbol{\pi}(k)=[\pi_0(k),\pi_1(k),\cdots,\pi_d(k)]^{\mathrm{T}}$, which satisfies the difference equation $\boldsymbol{\pi}(k)=\Lambda^{\mathrm{T}}\boldsymbol{\pi}(k-1)$.

Assumption 4. $\boldsymbol{x}(0)$ and $\{d(k)\}$ are independent of $\{\boldsymbol{w}(k)\}$ and $\{\boldsymbol{v}(k)\}$.

Remark 1. All matrices A, C, H, and Λ could be functions of k and the argument has been dropped here for simplicity in Notations.

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It is clear from Assumptions 1 and 2 that $\boldsymbol{x}(k)$ and $\boldsymbol{y}(k)$ are sequences of second-order random vectors, and we shall denote by $\mathcal{F}_k$ the σ-field generated by the random vectors and variables $\{\boldsymbol{x}(s), \boldsymbol{y}(s), d(s); s = 0, \cdots, k\}$. From (2) and Assumption 2, we get that $\mathrm{E}(\boldsymbol{y}(k)) = 0$. Define $\mathcal{L}\{\boldsymbol{y}(s)|_{0\le s\le k}\}$ the linear space spanned by the observation $\boldsymbol{y}(s), s = 0, \cdots, k$. For any $\boldsymbol{r}, \boldsymbol{\eta} \in \mathcal{L}\{\boldsymbol{y}(s)|_{0\le s\le k}\}$, the inner product $\langle .; . \rangle$ in $\mathcal{L}\{\boldsymbol{y}(s)|_{0\le s\le k}\}$ is given by $\langle \boldsymbol{r}; \boldsymbol{\eta} \rangle = \mathrm{E}(\boldsymbol{r}\boldsymbol{\eta}^{\mathrm{T}})$, and therefore $\boldsymbol{r}$ and $\boldsymbol{\eta}$ are orthogonal if $\langle \boldsymbol{r}; \boldsymbol{\eta} \rangle = 0$. In addition, for any second random vector $\boldsymbol{r}(k)$, we shall denote $\hat{\boldsymbol{r}}(k|k)$ as the estimator of $\boldsymbol{r}(k)$ given $\mathcal{L}\{\boldsymbol{y}(s)|_{0\le s\le k}\}$. From the geometric arguments as in [16], the linear mean square filter of $\hat{\boldsymbol{r}}(k|k)$ satisfies the following properties: $\hat{\boldsymbol{r}}(k|k) \in \mathcal{L}\{\boldsymbol{y}(s)|_{0\le s\le k}\}$, and $\boldsymbol{r}(k) - \hat{\boldsymbol{r}}(k|k)$ is orthogonal to $\mathcal{L}\{\boldsymbol{y}(s)|_{0\le s\le k}\}$.

Then, the optimal estimation problem can be stated as: Given the observation $\{\boldsymbol{y}(s)|_{0\le s\le k}\}$, find a linear mean square error estimator $\hat{\boldsymbol{x}}(k|k)$ of $\boldsymbol{x}(k)$.

2 Construction of the state estimator

In this section, we shall present a solution to the optimal estimator via the state augmentation and the projection theory in the Hilbert space.

2.1 Preliminaries

Let $\phi_i(k)$ represent an indictor of the Markov process $d(k)$, which is defined as follows

$$\phi_i(k) = \begin{cases} 1, & \text{if } d(k) = i \\ 0, & \text{otherwise} \end{cases}$$

Then, at time k, the observation equation (2) can be reformulated as

$$\boldsymbol{y}(k) = \sum_{i=0}^{d} \phi_i(k) H \boldsymbol{x}(k-i) + \boldsymbol{v}(k) \tag{3}$$

For a given time k, delay $d(k)$ can only take one value from set $\{0, 1, \cdots, d\}$, and thus, only one number of set $\{\phi_0(k), \phi_1(k), \cdots, \phi_d(k)\}$ is 1 and the rest of the members should be zero. Hence, $\phi_i(k)$ satisfies

$$\phi_i(k) \times \phi_j(k) = 0, \quad i \neq j \tag{4}$$

At the sampling time k, if we augment the state-variable as

$$\bar{\boldsymbol{x}}(k) = [\ \boldsymbol{x}^{\mathrm{T}}(k) \quad \boldsymbol{x}^{\mathrm{T}}(k-1) \quad \cdots \quad \boldsymbol{x}^{\mathrm{T}}(k-d)\ ]^{\mathrm{T}}$$

then the discrete-time system (1) and (2) can be written as

$$\bar{\boldsymbol{x}}(k+1) = \begin{bmatrix} A & 0 & \cdots & 0 & 0 \\ I & 0 & \cdots & 0 & 0 \\ 0 & I & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & I & 0 \end{bmatrix} \bar{\boldsymbol{x}}(k) + \begin{bmatrix} C \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix} \boldsymbol{w}(k) \tag{5}$$

$$\boldsymbol{y}(k) = [\ \phi_0(k)H \quad \phi_1(k)H \quad \cdots \quad \phi_d(k)H\ ]\bar{\boldsymbol{x}}(k) + \boldsymbol{v}(k) \tag{6}$$

As discussed in [1], the correct initialization of the augmented state and its covariance are given by

$$\bar{\boldsymbol{x}}(0) \triangleq [\ \overbrace{0, \cdots, 0}^{(d+1)\text{ blocks}}\ ]^{\mathrm{T}}$$

$$\bar{V} \triangleq \mathrm{E}[\bar{\boldsymbol{x}}(0)\bar{\boldsymbol{x}}^{\mathrm{T}}(0)] = \mathrm{diag}\{V, \overbrace{0, \cdots, 0}^{d\text{ blocks}}\}$$

Denote

$$\boldsymbol{\theta}(k) = [\phi_0(k)\ \phi_1(k)\ \cdots\ \phi_d(k)]^{\mathrm{T}} \tag{7}$$

Since delay $d(k)$ can be chosen to be only one value among $\{0, 1 \cdots, d\}$ at each time k, it is implied that only one value is 1 in set $\{\phi_0(k), \phi_1(k), \cdots, \phi_d(k)\}$ and the others are zeros. So, it is not difficult to show that $\boldsymbol{\theta}(k)$ is a homogeneous Markov chain with the state space

$$S = \{\boldsymbol{e}_0,\ \boldsymbol{e}_1,\ \cdots,\ \boldsymbol{e}_d\}$$

where $\boldsymbol{e}_i$ is an $(d+1) \times 1$-dimensional vector with all its components null except the $(i+1)$-th one is equal to one. It is obvious that $P(\boldsymbol{\theta}(k+1) = \boldsymbol{e}_j | \boldsymbol{\theta}(k) = \boldsymbol{e}_i) = P(d(k+1) = j | d(k) = i) = \lambda_{ij}$ and $P(\boldsymbol{\theta}(k) = \boldsymbol{e}_i) = P(d(k) = i) = \pi_i(k)$. Thus, we can conclude that $\boldsymbol{\theta}(k)$ has the same transition probability matrix as $d(k)$, as well as the same initial distribution. By the definition of $\boldsymbol{\theta}(k)$, we describe system (5) and (6) as follows

$$\bar{\boldsymbol{x}}(k+1) = \bar{A}\bar{\boldsymbol{x}}(k) + \bar{C}\boldsymbol{w}(k) \tag{8}$$

$$\boldsymbol{y}(k) = \bar{H}_{\theta(k)}\bar{\boldsymbol{x}}(k) + \boldsymbol{v}(k) \tag{9}$$

where

$$\bar{A} = \begin{bmatrix} A & 0 & \cdots & 0 & 0 \\ I & 0 & \cdots & 0 & 0 \\ 0 & I & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & I & 0 \end{bmatrix}, \quad \bar{C} = \begin{bmatrix} C \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}$$

$$\bar{H}_{\theta(k)} = [\ \phi_0(k)H \quad \phi_1(k)H \quad \cdots \quad \phi_d(k)H\ ]$$

It can be seen that the obtained system representation is a delay-free jump linear system with one mode modeled by a homogeneous Markov chain. However, at time k, the random communication delay $d(k)$ is unknown in advance, and thus, the new jumping parameter $\boldsymbol{\theta}(k)$ representing the same jumping properties as $d(k)$ cannot be determined at the same time. In this paper, we shall estimate $\bar{\boldsymbol{x}}(k)$ and $\boldsymbol{\theta}(k)$ synchronously by introducing the new stochastic variables

$$\boldsymbol{z}_i(k) = \bar{\boldsymbol{x}}(k) 1_{\{\boldsymbol{\theta}(k) = \boldsymbol{e}_i\}}, \quad i = 0, 1 \cdots, d \tag{10}$$

$$\boldsymbol{z}(k) = [\boldsymbol{z}_0^{\mathrm{T}}(k)\ \boldsymbol{z}_1^{\mathrm{T}}(k)\ \cdots\ \boldsymbol{z}_d^{\mathrm{T}}(k)]^{\mathrm{T}} \tag{11}$$

where $1_{\{.\}}$ stands for the Dirac measure, that is, for any $\boldsymbol{\omega} \in S$,

$$1_{\{\boldsymbol{\theta}(k) = \boldsymbol{e}_i\}}(\boldsymbol{\omega}) = \begin{cases} 1, & \text{if } \boldsymbol{\theta}(k)(\boldsymbol{\omega}) = \boldsymbol{e}_i \\ 0, & \text{if } \boldsymbol{\theta}(k)(\boldsymbol{\omega}) \neq \boldsymbol{e}_i \end{cases}$$

From the definition of $\boldsymbol{z}_i(k)$, we have clearly that

$$\bar{\boldsymbol{x}}(k) = \sum_{i=0}^{d} \boldsymbol{z}_i(k)$$

Thus, the state estimator $\hat{\bar{\boldsymbol{x}}}(k|k)$ can be obtained via the estimator of $\boldsymbol{z}_i(k)$.

In light of (11), the observation equation can be written equivalently as

$$\boldsymbol{y}(k) = \bar{\mathcal{H}}\boldsymbol{z}(k) + \boldsymbol{v}(k) \tag{12}$$

where

$$\bar{\mathcal{H}} = [H_0\ H_1\ \cdots\ H_d] \in \mathbf{R}^{m \times n(d+1)^2}$$

$$H_i = [0\ \cdots H \cdots\ 0] \in \mathbf{R}^{m \times n(d+1)}$$

$$i = 0, 1, \cdots, d$$

that is, H_i has all elements being zeros except for the $(i+1)$-th block being H. To derive the optimal filter, we first define the innovation sequence as

$$\boldsymbol{\varepsilon}(k) = \boldsymbol{y}(k) - \hat{\boldsymbol{y}}(k|k-1) \tag{13}$$

where $\hat{\boldsymbol{y}}(k|k-1)$ is the projection of $\boldsymbol{y}(k)$ onto the linear space of

$$\mathcal{L}\{\boldsymbol{y}(s)|_{0\le s\le k-1}\} \tag{14}$$

Then, from (12), we get

$$\boldsymbol{\varepsilon}(k) = \bar{\mathcal{H}}\tilde{\boldsymbol{z}}(k|k-1) + \boldsymbol{v}(k) \tag{15}$$

where $\tilde{\boldsymbol{z}}(k|k-1) = \boldsymbol{z}(k) - \hat{\boldsymbol{z}}(k|k-1)$ with $\hat{\boldsymbol{z}}(k|k-1)$ is the projection of $\boldsymbol{z}(k)$ onto the linear space of (14). It is well known that the elements of the sequence $\{\boldsymbol{\varepsilon}(s)|_{0\le s\le k-1}\}$ are mutually uncorrelated and spans the same linear space of (14). Thus, the estimation problem can be transformed into the one given the innovation sequences generated by (15).

2.2 Optimal estimator

In order to calculate the covariance matrices of the estimator errors and thus compute the optimal estimator, we first define the following matrices associated with the second moment of the above variables. Given the time instant k, we define

$$\begin{aligned}
Z(k) &= \mathrm{E}[\boldsymbol{z}(k)\boldsymbol{z}^{\mathrm{T}}(k)] \in \mathbf{R}^{n(d+1)^2\times n(d+1)^2}\\
Z_i(k) &= \mathrm{E}[\boldsymbol{z}_i(k)\boldsymbol{z}_i^{\mathrm{T}}(k)] \in \mathbf{R}^{n(d+1)\times n(d+1)}\\
\hat{Z}(k|l) &= \mathrm{E}[\hat{\boldsymbol{z}}(k|l)\hat{\boldsymbol{z}}^{\mathrm{T}}(k|l)] \in \mathbf{R}^{n(d+1)^2\times n(d+1)^2}\\
P(k|l) &= \mathrm{E}[\tilde{\boldsymbol{z}}(k|l)\tilde{\boldsymbol{z}}^{\mathrm{T}}(k|l)] \in \mathbf{R}^{n(d+1)^2\times n(d+1)^2}
\end{aligned}$$

Then, from the definitions of $Z_i(k)$ and $Z(k)$, the following lemma is obtained.

Lemma 1. For any given time k, the covariance matrix of $\boldsymbol{z}_i(k)$ satisfies the following difference equation

$$Z_j(k+1) = \sum_{i=0}^{d}\lambda_{ij}\bar{A}Z_i(k)\bar{A}^{\mathrm{T}} + \sum_{i=0}^{d}\pi_i(k)\lambda_{ij}\bar{C}Q_w\bar{C}^{\mathrm{T}} \tag{16}$$

where the initial value $Z_i(0) = \pi_i(0)\bar{V}, i = 0, 1, \cdots, d$.

Proof. For any given time k, we have from (8) that

$$\begin{aligned}
Z_j(k+1) &= \mathrm{E}[\boldsymbol{z}_j(k+1)\boldsymbol{z}_j^{\mathrm{T}}(k+1)] =\\
&\sum_{i=0}^{d}\lambda_{ij}\bar{A}Z_i(k)\bar{A}^{\mathrm{T}} + \sum_{i=0}^{d}\pi_i(k)\lambda_{ij}\bar{C}Q_w\bar{C}^{\mathrm{T}}
\end{aligned}$$

so (16) is hold. Recalling that $\mathrm{E}[\bar{\boldsymbol{x}}(0)\bar{\boldsymbol{x}}^{\mathrm{T}}(0)] = \bar{V}$, we get the initial covariance matrix $Z_i(0) = \pi_i(0)\bar{V}$. □

In the following, we shall present the Riccati difference equation for $P(k|k-1)$.

Theorem 1. For a given time instant k, the covariance matrix of the estimation error $\tilde{\boldsymbol{z}}(k|k-1)$ can be obtained by solving the following Riccati equation

$$\begin{aligned}
P(k+1|k) &= \bar{\mathcal{A}}P(k|k-1)\bar{\mathcal{A}}^{\mathrm{T}} + \mathcal{B}(Q(k)) +\\
&\quad \mathrm{diag}\{\sum_{i=1}^{d}\lambda_{ij}\pi_i(k)\bar{C}Q_w\bar{C}^{\mathrm{T}}\} -\\
&\quad \bar{\mathcal{A}}P(k|k-1)\bar{\mathcal{H}}^{\mathrm{T}}(\bar{\mathcal{H}}P(k|k-1)\bar{\mathcal{H}}^{\mathrm{T}} +\\
&\quad Q_v)^{-1}\bar{\mathcal{H}}P(k|k-1)\bar{\mathcal{A}}^{\mathrm{T}}\\
P(0|-1) &= \mathrm{diag}\{\pi_0(0)\bar{V}, \cdots, \pi_d(0)\bar{V}\}
\end{aligned} \tag{17}$$

where $\bar{\mathcal{A}} = \Lambda^{\mathrm{T}}\otimes\bar{A}$ and

$$\begin{aligned}
\mathcal{B}(Q(k)) &= \mathrm{diag}\left\{\sum_{i=0}^{d}\lambda_{ij}\bar{A}Z_i(k)\bar{A}^{\mathrm{T}}\right\} -\\
&\quad \bar{\mathcal{A}}\mathrm{diag}\{Z_j(k)\}\bar{\mathcal{A}}^{\mathrm{T}}
\end{aligned} \tag{18}$$

Proof. Considering the geometric argument as in [16], the estimator $\hat{\boldsymbol{z}}(k|k-1)$ satisfies the following equations

$$\hat{\boldsymbol{z}}(k|k-1) = \mathrm{E}[\boldsymbol{z}(k)((\boldsymbol{y})^{k-1})^{\mathrm{T}}]\mathrm{cov}((\boldsymbol{y})^{k-1})^{-1}\times((\boldsymbol{y})^{k-1}) \tag{19}$$

$$\begin{aligned}
\hat{\boldsymbol{z}}(k|k) &= \hat{\boldsymbol{z}}(k|k-1) + \mathrm{E}[\boldsymbol{z}(k)\boldsymbol{\varepsilon}^{\mathrm{T}}(k)]\mathrm{E}[\boldsymbol{\varepsilon}(k)\boldsymbol{\varepsilon}^{\mathrm{T}}(k)]^{-1}\times\\
&\quad (\boldsymbol{y}(k) - \hat{\boldsymbol{y}}(k|k-1))
\end{aligned} \tag{20}$$

where $(\boldsymbol{y})^{k-1} = (\boldsymbol{y}^{\mathrm{T}}(0), \cdots, \boldsymbol{y}^{\mathrm{T}}(k-1))^{\mathrm{T}}$.

It has been shown that

$$\boldsymbol{y}(k) = \bar{\mathcal{H}}\boldsymbol{z}(k) + \boldsymbol{v}(k) \tag{21}$$

and the corresponding innovation sequence can be given as

$$\boldsymbol{\varepsilon}(k) = \bar{\mathcal{H}}\tilde{\boldsymbol{z}}(k|k-1) + \boldsymbol{v}(k) \tag{22}$$

From Assumption 4, $\boldsymbol{v}(k)$ is independent of $\{\boldsymbol{\theta}(k), (\boldsymbol{y})^{k-1}\}$. Then, we have

$$\begin{aligned}
&\langle\boldsymbol{\beta}^{\mathrm{T}}\boldsymbol{v}(s); \boldsymbol{\alpha}^{\mathrm{T}}(\boldsymbol{y})^{s-1}\rangle = \mathrm{E}[\boldsymbol{v}^{\mathrm{T}}(s)\boldsymbol{\beta}\boldsymbol{\alpha}^{\mathrm{T}}(\boldsymbol{y})^{s-1}] =\\
&\boldsymbol{\beta}\boldsymbol{\alpha}^{\mathrm{T}}\mathrm{E}(\boldsymbol{v}^{\mathrm{T}}(s))\mathrm{E}((\boldsymbol{y})^{s-1}) = 0
\end{aligned} \tag{23}$$

which shows that $\boldsymbol{v}(k)$ is orthogonal to $\mathcal{L}\{\boldsymbol{y}(s)|_{0\le s\le k-1}\}$. Similar reasoning indicates the orthogonality between $\tilde{\boldsymbol{z}}(k|k-1)$ and $\boldsymbol{v}(k)$. Recalling that $\hat{\boldsymbol{z}}(k|k-1) \in \mathcal{L}\{\boldsymbol{y}(s)|_{0\le s\le k-1}\}$ and $\tilde{\boldsymbol{z}}(k|k-1)$ is orthogonal to $\mathcal{L}\{\boldsymbol{y}(s)|_{0\le s\le k-1}\}$, we obtain that $\hat{\boldsymbol{z}}(k|k-1)$ is orthogonal to $\tilde{\boldsymbol{z}}(k|k-1)$. Then, from (21) and (22), we get

$$\mathrm{E}[\boldsymbol{\varepsilon}(k)\boldsymbol{\varepsilon}^{\mathrm{T}}(k)] = \bar{\mathcal{H}}P(k|k-1)\bar{\mathcal{H}}^{\mathrm{T}} + Q_v \tag{24}$$

$$\mathrm{E}[\boldsymbol{z}(k)\boldsymbol{\varepsilon}^{\mathrm{T}}(k)] = P(k|k-1)\bar{\mathcal{H}}^{\mathrm{T}} \tag{25}$$

Recalling that $\mathrm{E}[\boldsymbol{y}(k)] = 0$, we have from (19) that

$$\begin{aligned}
&\hat{\boldsymbol{z}}_j(k|k-1) =\\
&\quad \mathrm{E}[\boldsymbol{z}_j(k)((\boldsymbol{y})^{k-1})^{\mathrm{T}}]\mathrm{cov}((\boldsymbol{y})^{k-1})^{-1}((\boldsymbol{y})^{k-1}) =\\
&\quad \sum_{i=0}^{d}\lambda_{ij}\bar{A}\hat{\boldsymbol{z}}_i(k-1|k-1)
\end{aligned} \tag{26}$$

where it has been readily shown that $\mathrm{cov}((\boldsymbol{y})^{k-1}) > 0$. The initial state $\hat{\boldsymbol{z}}(0|-1) = \mathrm{E}[\boldsymbol{z}(0)] = 0$. Owning to (26), the following relationship between $\hat{\boldsymbol{z}}(k|k-1)$ and $\hat{\boldsymbol{z}}(k-1|k-1)$ is obtained

$$\hat{\boldsymbol{z}}(k|k-1) = \bar{\mathcal{A}}\hat{\boldsymbol{z}}(k-1|k-1) \tag{27}$$

where $\bar{\mathcal{A}} = \Lambda^{\mathrm{T}}\otimes\bar{A}$.

In view of (20), (24), and (25), we obtain the recursive formulation of the linear mean square filter as follows

$$\hat{\boldsymbol{z}}(k|k) = \hat{\boldsymbol{z}}(k|k-1) + K(k)(\boldsymbol{y}(k) - \bar{\mathcal{H}}\hat{\boldsymbol{z}}(k|k-1)) \tag{28}$$

where

$$K(k) = P(k|k-1)\bar{\mathcal{H}}^{\mathrm{T}}(\bar{\mathcal{H}}P(k|k-1)\bar{\mathcal{H}}^{\mathrm{T}} + Q_v)^{-1}$$

From (28), and after writing that $\boldsymbol{y}(k) - \bar{\mathcal{H}}\hat{\boldsymbol{z}}(k|k-1) = \bar{\mathcal{H}}\tilde{\boldsymbol{z}}(k|k-1) + \boldsymbol{v}(k)$, $\hat{\boldsymbol{z}}(k|k-1), \tilde{\boldsymbol{z}}(k|k-1)$, and $\boldsymbol{v}(k)$ are

orthogonal among themselves, we get

$$\begin{aligned}&\mathrm{E}[\hat{\boldsymbol{z}}(k|k)\hat{\boldsymbol{z}}^{\mathrm{T}}(k|k)] = \\ &\hat{Z}(k|k-1)+K(k)[\bar{\mathcal{H}}P(k|k-1)\bar{\mathcal{H}}^{\mathrm{T}}+Q_v]K^{\mathrm{T}}(k) = \\ &\hat{Z}(k|k-1)+K(k)\bar{\mathcal{H}}P(k|k-1)\end{aligned} \tag{29}$$

In light of (27), we have

$$\begin{aligned}\hat{Z}(k+1|k) &= \bar{\mathcal{A}}\hat{Z}(k|k)\bar{\mathcal{A}}^{\mathrm{T}} \\ \hat{Z}(0|-1) &= \mathrm{E}(\hat{\boldsymbol{z}}(0|-1))\mathrm{E}(\hat{\boldsymbol{z}}^{\mathrm{T}}(0|-1)) = 0\end{aligned} \tag{30}$$

Meanwhile, in view of Lemma 1, (16) can be reformulated as

$$\begin{aligned}Z(k+1) &= \bar{\mathcal{A}}Z(k)\bar{\mathcal{A}}^{\mathrm{T}}+\mathcal{B}(Q(k))+ \\ &\qquad \mathrm{diag}\left\{\sum_{i=0}^{d}\pi_i(k)\lambda_{ij}\bar{C}Q_w\bar{C}^{\mathrm{T}}\right\} \\ \mathcal{B}(Q(k)) &= \mathrm{diag}\left\{\sum_{i=0}^{d}\lambda_{ij}\bar{\mathcal{A}}Z_i(k)\bar{\mathcal{A}}^{\mathrm{T}}\right\}-\bar{\mathcal{A}}Z(k)\bar{\mathcal{A}}^{\mathrm{T}} \\ Z(0) &= \mathrm{diag}\{\pi_0(0)\bar{V},\cdots,\pi_d(0)\bar{V}\}\end{aligned} \tag{31}$$

Considering (29) $\sim$ (31) and noting that $P(k|k-1) = Z(k)-\hat{Z}(k|k-1)$, we get

$$\begin{aligned}&P(k+1|k) = \bar{\mathcal{A}}P(k|k-1)\bar{\mathcal{A}}^{\mathrm{T}}+\mathcal{B}(Q(k))+ \\ &\mathrm{diag}\{\sum_{i=0}^{d}\pi_i(k)\lambda_{ij}\bar{C}Q_w\bar{C}^{\mathrm{T}}\}-\bar{\mathcal{A}}K(k)\bar{\mathcal{H}}P(k|k-1)\bar{\mathcal{A}}^{\mathrm{T}}\end{aligned} \tag{32}$$

Then, (17) is immediately obtained from (32) with $P(0|-1) = Z(0)-\hat{Z}(0|-1) = \mathrm{diag}\{\pi_0(0)\bar{V},\cdots,\pi_d(0)\bar{V}\}$. □

Note that for the case with no jumps ($d = 0$), we would have $\mathcal{B}(Q(k)) = 0$, and therefore, (17) would reduce to the standard recursive Riccati equation for the Kalman filter. In the following, we shall give an analytical solution to the optimal filtering by applying the geometric arguments.

Corollary 1. Considering (1), (2), and given time $k > d$, the optimal filter $\hat{\boldsymbol{x}}(k|k)$ is given by

$$\hat{\boldsymbol{x}}(k|k) = [I\ 0\ \cdots\ 0]\hat{\bar{\boldsymbol{x}}}(k|k) \tag{33}$$

$$\hat{\bar{\boldsymbol{x}}}(k|k) = \sum_{i=0}^{d}\hat{\boldsymbol{z}}_i(k|k) \tag{34}$$

where $\hat{\boldsymbol{z}}_i(k|k)$ is the $(i+1)$-th element of $\hat{\boldsymbol{z}}(k|k)$, and $\hat{\boldsymbol{z}}(k|k)$ satisfies the following recursive equation

$$\begin{aligned}\hat{\boldsymbol{z}}(k|k) &= \hat{\boldsymbol{z}}(k|k-1)+P(k|k-1)\bar{\mathcal{H}}^{\mathrm{T}}\times \\ &\qquad (\bar{\mathcal{H}}P(k|k-1)\bar{\mathcal{H}}^{\mathrm{T}}+Q_v)^{-1}\times \\ &\qquad (\boldsymbol{y}(k)-\bar{\mathcal{H}}\hat{\boldsymbol{z}}(k|k-1))\end{aligned} \tag{35}$$

$$\hat{\boldsymbol{z}}(k|k-1) = \bar{\mathcal{A}}\hat{\boldsymbol{z}}(k-1|k-1),\hat{\boldsymbol{z}}(0|-1) = 0 \tag{36}$$

where $P(k|k-1)$ can be computed by (17).

Proof. The proof of this theorem is apparent and omitted here. □

In Corollary 1, we consider the case that the initial value $\mathrm{E}[\boldsymbol{x}(0)] = \boldsymbol{0}$. While $\mathrm{E}[\boldsymbol{x}(0)] = \boldsymbol{\mu} \neq \boldsymbol{0}$, it can be shown that equations (33) $\sim$ (36) are also satisfied. On the other hand, the obtained estimation algorithm can be applied to a broader class of systems which involve Markov jumping parameters and jumping delays simultaneously.

3 Stability of the state estimator

Stability of the estimator is essential when estimation is performed over a long period of time. As is well known, the observation delays are the source of potential instability. In this section, we show that the proposed estimator is stable provided that the following assumptions hold.

Assumption 5. System (1) and (2) is mean square stable (MSS) according to the definition in [11].

Assumption 6. The Markov chain $d(k)$ is ergodic.

It follows from Assumption 6 that $\lim_{k\to\infty}P(\boldsymbol{\theta}(k) = \boldsymbol{e}_i)$ exists and it is independent of $\boldsymbol{\theta}(0)$. We define

$$\pi_i = \lim_{k\to\infty}P(\boldsymbol{\theta}(k) = \boldsymbol{e}_i) = \lim_{k\to\infty}\pi_i(k)$$

With regard to Assumptions 5 and 6, and Proposition 8 in [11], we see that $Q(k)\to Q$ as $k\to\infty$, where $Q = (Z_0,\cdots,Z_d)$ is the unique solution of

$$Z_j = \sum_{i=0}^{d}\lambda_{ij}(\bar{\mathcal{A}}Z_i\bar{\mathcal{A}}^{\mathrm{T}}+\pi_i\bar{C}Q_w\bar{C}^{\mathrm{T}}),\quad j = 0,\cdots,d \tag{37}$$

Now, we introduce an operator for any positive semi-definite matrix L as follows

$$T(L) = \bar{\mathcal{A}}L\bar{\mathcal{H}}^{\mathrm{T}}(\bar{\mathcal{H}}L\bar{\mathcal{H}}^{\mathrm{T}}+Q_v)^{-1}$$

and let $\inf_{l\geq k}\pi_i(l) > 0$ hold for all $i = 0,1,\cdots,d$ (since $\pi_i(k)\to\pi_i > 0$ as $k\to\infty$, we have that this number exists). Defining $\alpha_i(k) = \inf_{l\geq k}\pi_i(l+k)$, then we get

$$\begin{aligned}&\pi_i(k+k)\geq\alpha_i(k)\geq\alpha_i(k-1) \\ &k = 1,2,\cdots,\ i = 0,1,\cdots,d\end{aligned} \tag{38}$$

and $\alpha_i(k)\to\pi_i(k\to\infty)$ exponentially fast.

In the following, we define

$$\begin{aligned}R(k+1) &= \bar{\mathcal{A}}R(k)\bar{\mathcal{A}}^{\mathrm{T}}+\mathcal{B}(\bar{Q}(k))+ \\ &\qquad \mathrm{diag}\left\{\sum_{i=0}^{d}\alpha_i(k)\lambda_{ij}\bar{C}Q_w\bar{C}^{\mathrm{T}}\right\}- \\ &\qquad \bar{\mathcal{A}}R(k)\bar{\mathcal{H}}^{\mathrm{T}}(\bar{\mathcal{H}}R(k)\bar{\mathcal{H}}^{\mathrm{T}}+Q_v)^{-1}\bar{\mathcal{H}}R(k)\bar{\mathcal{A}}^{\mathrm{T}}\end{aligned}$$

where $R(0) = 0$ and $\bar{Q}(k) = (\bar{Z}_0(k),\cdots,\bar{Z}_d(k))$ with

$$\begin{aligned}\bar{Z}_j(k+1) &= \sum_{i=0}^{d}\lambda_{ij}(\bar{\mathcal{A}}\bar{Z}_i(k)\bar{\mathcal{A}}^{\mathrm{T}}+\alpha_i(k)\bar{C}Q_w\bar{C}^{\mathrm{T}}) \\ \bar{Z}_j(0) &= 0,\ j = 0,1,\cdots,d\end{aligned}$$

The rest of this section is devoted to proving the stability of the covariance matrix of the estimation error. In doing so, the following fact will play a key role.

Lemma 2. For each $k = 0,1,\cdots$, we have

$$0\leq R(k)\leq R(k+1)\leq P(k+1+k|k+k) \tag{39}$$

Proof. It follows from the same arguments as in Lemma 3 in [13]. □

Theorem 2. Suppose that Assumptions 1, 5, and 6 hold. Consider the algebraic Riccati equation given by

$$\begin{aligned}P &= \bar{\mathcal{A}}P\bar{\mathcal{A}}^{\mathrm{T}}+\mathcal{B}(Q)+\mathrm{diag}\left\{\sum_{i=0}^{d}\lambda_{ij}\pi_i\bar{C}Q_w\bar{C}^{\mathrm{T}}\right\}- \\ &\qquad \bar{\mathcal{A}}P\bar{\mathcal{H}}^{\mathrm{T}}(\bar{\mathcal{H}}P\bar{\mathcal{H}}^{\mathrm{T}}+Q_v)^{-1}\bar{\mathcal{H}}P\bar{\mathcal{A}}^{\mathrm{T}}\end{aligned} \tag{40}$$

satisfies (37), where $Q = (Z_0,\cdots,Z_d)$. Then, there exists a unique nonnegative definite solution P to (40). Moreover, $\bar{\mathcal{A}}-T(P)\bar{\mathcal{H}}$ is a stable matrix, and for any $Q(0) =$

$(Z_0(0), \cdots, Z_d(0))$ with $Z_i(0) \geq 0, i = 0, 1, \cdots, d$, and $P(0|-1) \geq 0$, we have that $P(k+1|k)$ given by (16) and (17) satisfies $P(k+1|k) \to P, k \to \infty$.

Proof. From MSS of (1) and Proposition 5 in [11], we have that $r_\sigma(\bar{\mathcal{A}}) < 1$. According to the standard results for algebraic Riccati equations, there exists a unique positive semi-definite solution P to (40) and moreover $r_\sigma(\bar{\mathcal{A}} - T(P)\bar{\mathcal{H}}) < 1$. Furthermore, P satisfies

$$P = (\bar{\mathcal{A}} - T(P)\bar{\mathcal{H}})P(\bar{\mathcal{A}} - T(P)\bar{\mathcal{H}})^{\mathrm{T}} + \mathcal{B}(Q) + \mathrm{diag}\{\sum_{i=0}^{d} \lambda_{ij}\pi_i \bar{C} Q_w \bar{C}^{\mathrm{T}}\} + T(P)Q_v T^{\mathrm{T}}(P) \quad (41)$$

Define $N(0) = P(0|-1)$ and

$$N(k+1) = (\bar{\mathcal{A}} - T(P)\bar{\mathcal{H}})N(k)(\bar{\mathcal{A}} - T(P)\bar{\mathcal{H}})^{\mathrm{T}} + \mathcal{B}(Q(k)) + \mathrm{diag}\left\{\sum_{i=0}^{d} \lambda_{ij}\pi_i(k) \bar{C} Q_w \bar{C}^{\mathrm{T}}\right\} + T(P)Q_v T^{\mathrm{T}}(P) \quad (42)$$

Let us show by induction on k that $N(k) \geq P(k|k-1)$. Since

$$\begin{aligned} P(k+1|k) = {} & \mathcal{B}(Q(k)) + \\ & \mathrm{diag}\{\sum_{i=0}^{d} \pi_i(k)\lambda_{ij} \bar{C} Q_w \bar{C}^{\mathrm{T}}\} + T(P)Q_v T^{\mathrm{T}}(P) + \\ & (\bar{\mathcal{A}} - T(P)\bar{\mathcal{H}})P(k|k-1)(\bar{\mathcal{A}} - T(P)\bar{\mathcal{H}})^{\mathrm{T}} - \\ & (T(k) - T(P))(\bar{\mathcal{H}}P(k|k-1)\bar{\mathcal{H}}^{\mathrm{T}} + Q_v) \times \\ & (T(k) - T(P))^{\mathrm{T}} \end{aligned} \quad (43)$$

we have from (42) and (43) that

$$\begin{aligned} & (N(k+1) - P(k+1|k)) = \\ & (\bar{\mathcal{A}} - T(P)\bar{\mathcal{H}})(N(k) - P(k|k-1))(\bar{\mathcal{A}} - T(P)\bar{\mathcal{H}})^{\mathrm{T}} + \\ & (T(k) - T(P))(\bar{\mathcal{H}}P(k|k-1)\bar{\mathcal{H}}^{\mathrm{T}} + Q_v) \times \\ & (T(k) - T(P))^{\mathrm{T}} \end{aligned} \quad (44)$$

By definition, $N(0) = P(0|-1)$. Suppose that $N(k) \geq P(k|k-1)$. From (44), we have that $N(k+1) \geq P(k+1|k)$. Therefore, we have shown by induction that $N(k) \geq P(k|k-1)$ for all $k = 0, 1, \cdots$. From Assumptions 5 and 6, we have that $Q(k) \to Q$ as $k \to \infty$ and $\mathrm{diag}\{\sum_{i=0}^{d} \lambda_{ij}\pi_i(k) \bar{C} Q_w \bar{C}^{\mathrm{T}}\} \to \mathrm{diag}\{\sum_{i=0}^{d} \lambda_{ij}\pi_i \bar{C} Q_w \bar{C}^{\mathrm{T}}\}$ as $k \to \infty$ exponentially fast. From $r_\sigma(\bar{\mathcal{A}} - T(P)\bar{\mathcal{H}}) < 1$ and Proposition 2 in [11], we get that $N(k) \to N$ as $k \to \infty$, where N satisfies

$$N = (\bar{\mathcal{A}} - T(P)\bar{\mathcal{H}})N(\bar{\mathcal{A}} - T(P)\bar{\mathcal{H}})^{\mathrm{T}} + \mathcal{B}(Q) + \mathrm{diag}\{\sum_{i=0}^{d} \lambda_{ij}\pi_i \bar{C} Q_w \bar{C}^{\mathrm{T}}\} + T(P)Q_v T^{\mathrm{T}}(P) \quad (45)$$

and N is the unique solution to (45). Recalling that P satisfies (41), we have that P is also a solution to (45) and from uniqueness, $P = N$.

Therefore,

$$P(k|k-1) \leq N(k) \quad (43)$$

and $N(k) \to P$ as $k \to \infty$. From (46), it follows that $0 \leq R(k) \leq R(k+1) \leq N(k+1+k)$, and thus, we can conclude that $R(k) \to R$ whenever $k \to \infty$ for some $R \geq 0$. Moreover, from the fact that $\alpha_i(k) \to \pi_i$ and $\bar{Q}(k) \to Q$ as $k \to \infty$, we have that R satisfies (41). From uniqueness of positive semi-definite solution to (41), we can conclude that $R = P$. From (46) and (39), $R(k) \leq P(k+k|k-1+k) \leq N(k+k)$. Since $N(k) \to P$ and $R(k) \to P$ as $k \to \infty$, we get that $P(k|k-1) \to P$ as $k \to \infty$. □

With Theorem 2, we obtain sufficient conditions for the convergence of the error covariance matrix to a stationary value for the linear minimum mean square error (LMMSE) estimation. It is shown that if the system is MSS and the Markov delay is ergodic, then the error covariance matrix will converge to the unique nonnegative definite solution of an algebraic Riccati equation associated with the problem. Moreover, the filter equation with the stationary gain will be stable.

4 Numerical example

In this section, we present a numerical example to illustrate the previous theoretical results. Consider a dynamic system described in (1) and (2) with the following specifications:

$$A = \begin{bmatrix} 0.9 & 0 \\ 0 & 0.5 \end{bmatrix}, \quad \boldsymbol{C} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}, \quad \boldsymbol{H} = [0.15 \;\; 0.3]$$

and we denote that $\boldsymbol{x}(k) = [x_1^{\mathrm{T}}(k),\ x_2^{\mathrm{T}}(k)]^{\mathrm{T}}$. For the convenience of illustration of the proposed results and without loss of generality, we assume that the Markovian delay $d(k)$ is restricted to the set $\{0, d\}$ with transition probabilities:

$$\begin{aligned} p_{00} &= P(d(k+1) = 0 \mid d(k) = 0) = 0.85 \\ p_{0d} &= P(d(k+1) = d \mid d(k) = 0) = 0.15 \\ p_{d0} &= P(d(k+1) = 0 \mid d(k) = d) = 0.7 \\ p_{dd} &= P(d(k+1) = d \mid d(k) = d) = 0.3 \end{aligned}$$

The initial conditions are as follows: $x_i(0) \sim N(0, 1)$, $p_0(0) = P(d(0) = 0) = 0.5$, $p_d(0) = P(d(0) = d) = 0.5$. In the actual system, we use $\boldsymbol{x}(0) = 1$ for all the simulation trials to generate $\boldsymbol{z}_0(k)$ and $\boldsymbol{z}_d(k)$. Furthermore, $\{\boldsymbol{w}(k)\}$ and $\{\boldsymbol{v}(k)\}$ are mutually independent noise sequences. In this example, we consider three kinds of distributions for the noises: normal (with $\mu = 0$ and $\sigma = 1$), uniform (with parameters $a = -1$ and $b = 1$), and T-distribution (with parameters $v = 20$ and $\delta = 0$). The state estimation performances were evaluated by averaging over 50 Monte Carlo simulations from $k = 0$ to 200.

The simulation results are obtained as follows. Figs. 1 and 2 represent the real states and their estimators subject to different noises and random delay $d(k) \in \{0, 5\}$ based on the given path. Fig. 3 shows the square root of the mean square errors (RMS) subject to different delays. It can be seen from the simulation results that the obtained linear estimator for systems with Markovian time delay are tracking well to the real state value and the proposed method is efficient to different noises and delays, that is, the estimation scheme proposed in this paper produces good performance.

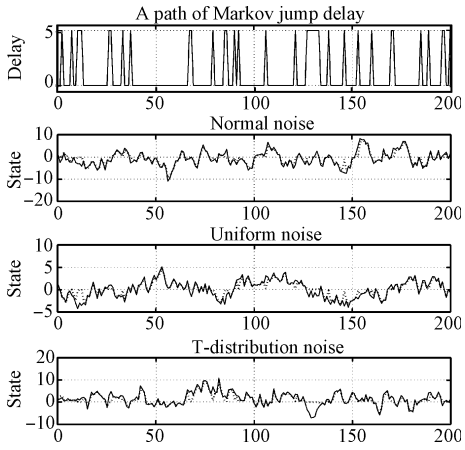

Fig. 1 Trajectories of state x_1: real state value (solid line) and its estimations (dashed lines) subject to different noises and Markovian delay $d(k) \in \{0, 5\}$ based on the given path

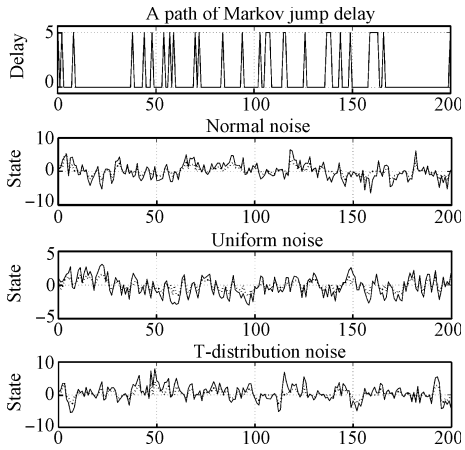

Fig. 2 Trajectories of state x_2: real state value (solid line) and its estimations (dashed lines) subject to different noises and Markovian delay $d(k) \in \{0, 5\}$ based on the given path

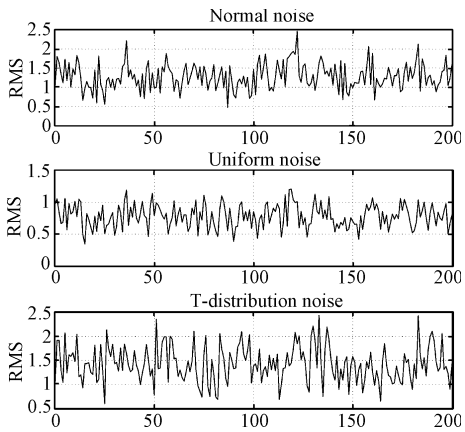

Fig. 3 RMS state estimation errors for system with different noises and Markovian delay $d(k) \in \{0, 5\}$

5 Conclusion

This paper has addressed the state estimation problem in an environment where state measurements are sent over a communication network. The random delay introduced by the network is modeled as a finite-state Markov chain. Complete analytical solution has been obtained by solving the recursive Riccati equations. It has been shown that the proposed state estimator is exponentially stable under standard assumptions.

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