Finite-time Rotating Encirclement Control of Multi-agent Systems

Lipo Mo[1] Yongguang Yu[2]

Abstract This paper deals with the finite-time distributed rotating encirclement control problem of multi-agent systems, where each agent tracks one target and every target can be tracked by one agent. Firstly, finite-time encirclement tracking protocols are designed for multi-agent systems, and these protocols contain observers for the geometrical center of the targets and estimators for the maximum distance between targets and their geometrical center. Secondly, sufficient conditions for achieving finite-time encirclement tracking are obtained. Finally, simulations are provided to demonstrate the effectiveness of the presented results.

Key words Encirclement control, estimator, finite-time tracking, multi-agent systems

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1 Introduction

In the past decades, the distributed coordination control problems of multi-agent networks have received a great deal of attention. This is mainly because they have important potential applications in practical systems, such as power grids, wireless sensor networks, transportation networks and so on [1]. To achieve coordination control, we require that every agent is forced to reach consensus firstly. Therefore, consensus problem in multi-agent networks is the basic issue of the coordination control. Based on the local information, which each agent can receive from its neighbors, a mass of consensus protocols have been designed for multi-agent systems with first-order, second-order, high-order, fractional-order or mixed-order dynamics [2]−[10]. And then these results were extended to general linear multi-agent systems [11] and the time-delay multi-agent systems [12]−[15]. Of course, most of these extensions were not trivial works. In practical applications, multi-agent systems are often subjected to various disturbances and noises, to deal with these situations, robust H_∞ method and stochastic analysis method were introduced. And robust H_∞ consensus problems for first-order and high-order multi-agent systems with external disturbances were studied in [16], [17]. Some sufficient conditions were given for making all agents achieve consensus and satisfying H_∞ performance simultaneously. By using the tools of stochastic analysis, some sufficient conditions were obtained for achieving mean-square consensus for continuous or discrete-time multi-agent systems [18]−[20].

Moreover, in many real systems, it is often required that the consensus can be reached in finite-time. Compared with asymptotic convergence, finite-time convergence has higher convergence rate. Therefore, finite-time consensus problem is more appealing. In [21], finite-time consensus problems were studied and two valid distributed protocols were proposed for making the closed-loop systems achieve finite-time consensus. In [22], the finite-time consensus problem was solved by non-smooth stability theory. In [23], exponential finite-time protocols were proposed for both containment control and consensus control. For other finite-time consensus results, one can refer to [24]−[29].

In modern military applications, it is often needed to make all agents encircle some targets to attack and reconnoiter the situations of the enemy in many cases, such as ground-attack missions [30]. Therefore, it is meaningful to investigate the encirclement motion of agents in a distributed manner. However, to the best of our knowledge, few results exist on this issue to date. And the results in literatures cannot be extended directly to encirclement control, as advocated by Lin and Jia [31]. In [32], a protocol was proposed to make agents surround and track targets, and the protocol therein can make agents surround and track targets with the same formation as that of targets asymptotically. In [33]−[35], protocols were proposed for making the closed-loop system achieve encirclement asymptotically when the maximum distance between all targets and their geometry center is a fixed constant and the estimators therein are centralized. Although these studies constitute the important first step in the encircling control, three important gaps remain: 1) None of the studies considered finite-time rotating encirclement, which is more useful in military applications; 2) No results exist on the encirclement and track problem when the maximum distance between all targets and their geometry center is time variant; 3) No results exist on finite-time encirclement control for agents in manner of fully distributed protocols.

In response to the lack of research dealing specifically with the encirclement control problems, in this paper, we investigate the finite-time distributed rotating encirclement control problem of multi-agent systems, where each agent tracks one target and every target can be tracked by one agent. First, we propose a finite-time encirclement tracking protocol, which contains the estimators for the geometrical center of the targets and the estimators for the maximum distance between the target and their geometrical center. Then, we give some sufficient conditions for making the multi-agent systems achieve finite-time rotating encirclement tracking in the manner of circular formation. Finally, simulations are provided to demonstrate the effectiveness of presented results.

2 Preliminaries

2.1 Graph Theory

To solve the coordination control problems, let us introduce some basic concepts about graph theory (see [36] for details). Consider a multi-agent system with n agents. If we regard the n agents as the vertices $V = \{v_i, i =$

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$1,2,\ldots,n\}$, then the communication topology of n agents can be conveniently described by an undirected graph $G=\{V,\varepsilon\}$, where $\varepsilon\subset V\times V$ is the set of edges of the graph. $N_j=\{\ i\mid (v_i,v_j)\in\varepsilon\}$ denotes the set of labels of those agents which are neighbors of agent j $(j=1,2,\ldots,n)$. Let $A=[a_{ij}]\in\mathbb{R}^{n\times n}$ is the weighted adjacency matrix of the graph G. Then, the Laplacian of the weighted graph is defined as $L=[l_{ij}]$, where $l_{ii}=\sum_{j=1}^{n}a_{ij}$ and $l_{ij}=-a_{ij},i\neq j$, which is symmetric. A path that connects v_i and v_j is a sequence of edges in the form of $(v_{i_0},v_{i_1}),\ldots,(v_{i_{m-1}}v_{i_m})$, where $v_{i_0}=v_i,v_{i_m}=v_j$ and $(v_{i_r},v_{i_{r+1}})\in\varepsilon,0\leq r\leq m-1$. If there exists a path between any two vertices v_i and v_j $(i\neq j)$, then the graph is said to be connected. If the graph G is connected, define the distance between v_i and v_j, denoted by $d(v_i,v_j)$, as the length of the shortest path between v_i and v_j.

2.2 System Model

Consider the multi-agent system with n agents and n targets, and every agent can be described by a vertex of a graph G. The communication between agent j and i can be represented by an edge (v_j,v_i) of graph G. Each target is detected by one agent, without loss of generality, suppose target i can be detected by agent i for $i\in\mathcal{I}:=\{1,2,\ldots,n\}$. Suppose the dynamics of agents are as follows:

$$\dot{x}_i(t)=u_i(t),\ i\in\mathcal{I} \tag{1}$$

where $x_i(t)\in\mathbb{R}^2$, $u_i(t)\in\mathbb{R}^2$ are the state and the input of agent i at time t. By the polar coordinate transformation $x_i(t)=p(t)+[l_i(t)\cos(\theta_i(t)),l_i(t)\sin(\theta_i(t))]^T$, System (1) can be transferred to the following equivalent system:

$$\begin{cases}\dot{l}_i(t)=v_i(t)\\ \dot{\theta}_i(t)=\omega_i(t)\end{cases},\ i\in\mathcal{I} \tag{2}$$

where $p(t)$ represents the geometry center of targets at time t, $l_i(t),\theta_i(t)\in\mathbb{R}$ are polar radius and polar angle of the state of agent i in polar coordinate system with the ordinate origin at the geometry center of targets, and $(v_i(t),\omega_i(t)),i\in\mathcal{I}$ are new control inputs.

Combining the definitions of encirclement control [33]−[35] and finite-time stabilization [37], [38], we introduce the definition of the finite-time rotating encirclement control.

Definition 1: The protocol u_i is said to solve the finite-time rotating encirclement control problem of system (1), if there exists $T>0$, such that $\forall i\in\mathcal{I}$, the following limitations hold:

$$\begin{aligned}&\lim_{t\to T}\left[\|x_i(t)-\frac{1}{n}\sum_{k=1}^{n}r_k(t)\|\right.\\&\qquad\left.-k\times\max_{j\in I}\left\{\|r_j(t)-\frac{1}{n}\sum_{k=1}^{n}r_k(t)\|\right\}\right]=0\\&\lim_{t\to T}\left[\theta_i(t)-\theta_j(t)-\frac{2\pi(i-j)}{n}\right]=0\\&\lim_{t\to T}\left[\dot{\theta}_i(t)-\Omega(t)\right]=0\end{aligned} \tag{3}$$

where $r_i(t)\in\mathbb{R}^2$ is the state of target i, $\Omega(t)$ is the desired angular velocity of all agents, and $k>1$ is a positive parameter, which controls the radius of circle formation.

Remark 1: In the above definition, $\|x_i(t)-\frac{1}{n}\sum_{k=1}^{n}r_k(t)\|$ is the distance between agent i and their geometry center, $k\max\{\|r_i(t)-\frac{1}{n}\sum_{k=1}^{n}r_k(t)\|\}$ represents the radius of formation. The second equation implies that all of agents can evenly distribute on a circle in finite-time. The third equation implies all agents can eventually have a common angular velocity, which can be proposed according to the practical demand.

In practice, the exact position information of geometry center of targets cannot always be obtained by all agents, so we need to design an observer to calculate the geometry center of targets for agent $i\in\mathcal{I}$, the corresponding estimated value is denoted by $p_i(t)$.

$$\begin{cases}\dot{\varphi}_i(t)=\alpha\sum\limits_{j\in N_i}a_{ij}\dfrac{p_j(t)-p_i(t)}{\|p_j(t)-p_i(t)\|}\\ p_i(t)=\varphi_i(t)+r_i(t)\end{cases} \tag{4}$$

where $\varphi_i(t)\in\mathbb{R}^2$, $\varphi(0)=0$, and $\alpha>0$ is a control parameter.

Let $\rho_i(t)$ be the estimated value of the maximum distance between all targets and their the geometry center by agent i. The dynamical equation of $\rho_i(t)$ is as below:

$$\begin{cases}\dot{\rho}_i^1(t)=-k_1^1\mathrm{sign}[\rho_i^1(t)-\max\limits_{j\in N_i\cup\{i\}}(d_j(t))]\\ \dot{\rho}_i^2(t)=-k_1^2\mathrm{sign}[\rho_i^2(t)-\max\limits_{j\in N_i\cup\{i\}}(\rho_j^1(t))\}]\\ \qquad\qquad\vdots\\ \dot{\rho}_i^{N-1}(t)=-k_1^{N-1}\mathrm{sign}[\rho_i^{N-1}(t)-\max\limits_{j\in N_i\cup\{i\}}(\rho_j^{N-2}(t))]\\ \dot{\rho}_i(t)=-k_1\mathrm{sign}[\rho_i(t)-\max\limits_{j\in N_i\cup\{i\}}(\rho_j^{N-1}(t))]\end{cases} \tag{5}$$

where $\mathrm{sign}(\cdot)$ is the symbolic function, $i\in\mathcal{I}$, $k_1>0$ is the control gain, $d_j(t)=\|r_j(t)-p_j(t)\|$, $N=\max\limits_{i,j\in\mathcal{I}}\{d(v_i,v_j)\}$ and $(\rho_i^1(t),\ldots,\rho_i^{N-1}(t),\rho_i(t))^T\in\mathbb{R}^N$ is the state vector of system (5).

Remark 2: In [33] and [34], the maximum distance between all targets and the geometry center of targets by agent i is a constant, i.e., $\max_{i\in\mathcal{I}}(d_i(t))=d$, however, the maximum distance in this paper is time varying. So the problems considered in [33] and [34] are the special cases of the problem herein.

Remark 3: In [33]−[35], the estimators for the maximum distance between all targets and their geometry center are centralized. In fact, estimators therein contain the position information of all targets. But Estimator (5) herein is distributed.

In polar coordinate system, we propose the following protocol for system (2):

$$\begin{cases}v_i(t)=-k_2\mathrm{sign}(l_i(t)-k\rho_i(t)),\quad i\in\mathcal{I}\\ \omega_i(t)=-k_3\sum\limits_{j\in N_i}a_{ij}\mathrm{sign}\left(\theta_i(t)-\theta_j(t)-\dfrac{2\pi(j-i)}{n}\right)\\ \qquad\times|\theta_i(t)-\theta_j(t)-\dfrac{2\pi(j-i)}{n}|^{\kappa}+\Omega(t)\end{cases} \tag{6}$$

where $\kappa\in(0,1)$, $k_2>0,k_3>0$ are feedback gains and $\|\Omega(t)\|\leq\Omega$ is a continuous bounded function.

Assumption 1: $\forall i\in\mathcal{I}$, $\|\dot{r}_i(t)\|\leq\beta$, where $\beta>0$ is a constant.

Assumption 2: $D(t) := \max_{i\in\mathcal{I}}\{\|r_i(t) - \frac{1}{n}\sum_{k=1}^{n} r_k(t)\|\} \leq c$, where $c > 0$ is a constant.

Remark 4: In [33] and [34], it is assumed that $d_i(t) = \|r_i(t) - p_i(t)\|, i = 1, 2, \ldots, n$ are bounded, which depend on the estimators $p_i(t), i = 1, 2, \ldots, n$. This assumption is not the intrinsic property of systems. Here, we assume the maximum distance between all agents and their geometry center is bounded. In real systems, this assumption is always satisfied. So Assumption 2 is more appropriate.

Let $\widetilde{\theta}_i(t) = \theta_i(t) - \int_0^t \Omega(s)ds$, $\hat{\theta}_i(t) = \widetilde{\theta}_i(t) - 2\pi i/n$, then the closed-loop system (2)−(6) can be changed into the following form:

$$\begin{cases} \dot{l}_i(t) = -k_2\text{sign}(l_i(t) - k\rho_i(t)), \quad i \in \mathcal{I} \\ \dot{\hat{\theta}}_i(t) = -k_3 \sum\limits_{j\in N_i(t)} a_{ij}\text{sign}\left(\hat{\theta}_i(t) - \hat{\theta}_j(t)\right) \\ \qquad\qquad \times|\hat{\theta}_i(t) - \hat{\theta}_j(t)|^{\kappa}. \end{cases} \tag{7}$$

To solve the finite-time encirclement control problem of system (1), we introduce the following protocol:

$$u_i = -\gamma\text{sign}(x_i - \widehat{x}_i) \tag{8}$$

where $\widehat{x}_i(t) = p_i(t) + [l_i(t)\cos(\theta_i(t)), l_i(t)\sin(\theta_i(t))]^T$, $\gamma > 0$ is the feedback gain.

Remark 5: The protocols proposed in [33]−[35] contain derivative, which cannot be realized in physical systems because derivative terms may amplify noises in applications. The protocol (8) overcomes this weakness.

3 Finite-time Stability Analysis

To analyze the finite-time rotating encirclement control problem of multi-agent systems, we first introduce the following lemmas.

Lemma 1 [37]: Consider system $\dot{x}_i(t) = \sum_{j\in N_i(t)} a_{ij}\text{sign}(x_j - x_i)|x_j - x_i|^{\kappa}$, $i = 1, 2, \ldots, n$. If the graph G is connected, then the system is finite-time stable.

Lemma 2: Consider system (4). Take $\alpha > \beta(n-1)$, and suppose Assumption 1 holds, then there exists $T_1 > 0$ such that $\lim_{t\to T_1}\left[p_i(t) - \frac{1}{n}\sum_{k=1}^{n} r_k(t)\right] = 0$, $\forall i \in \mathcal{I}$, i.e., for each agent, the estimated value of the geometry center of targets can eventually converge to its accurate value in finite-time.

The proof of this lemma is provided in Appendix A.

Remark 6: This lemma shows that the estimated value of each agent for the geometrical center of the targets can converge to the precise value. In addition, it is easy to see that $T_1 \leq \frac{2V^{\frac{1}{2}}(0)}{\sqrt{\frac{2}{n}}\left(\frac{\alpha}{2} - \frac{\beta(n-1)}{2}\right)n^2}$ by the Lyapunov finite-time stability theorem.

Lemma 3: Consider system (5). Suppose Assumption 1 holds. If $k_1^j > 2\beta$, $j = 1, 2, \ldots, N-1$ and $k_1 > 2\beta$, then system (5) is finite-time stable.

The proof of this lemma is provided in Appendix B.

Remark 7: In practical applications, instead of (5) we can represent maximum estimated velocity by the equation

$$\dot{\rho}_i(t) = -k_1\text{sign}[\rho_i(t) - \sum_{k=1}^{N-1}\max_{j\in N_i}(\rho_i^k(t))1_{\{\rho_i^k(t)\neq\max\limits_{j\in N_i}(\rho_j^{k-1}(t));\ \rho_i^{k-1}(t)=\max\limits_{j\in N_i}(\rho_j^{k-2}(t))\}}]$$

where $\rho_i^0(t) = \max\limits_{j\in N_i}(d_j(t))$ and $1_{\{\cdot\}}$ is the indicative function.

In fact, this equation is equivalent to the first equation when $t < T_2^1$, the second equation when $T_2^1 \leq t < T_2^2$, and so on.

Lemma 4: Consider system (2) with protocol (6). Suppose Assumption 1 holds. If $k_1^j > 2\beta$, $j = 1, 2, \ldots, N-1$, $k_1 > 2\beta$ and $k_2 > 2k\beta$, then the closed-loop system (2)−(6) is finite-time stable.

The proof of this lemma is provided in Appendix C.

Next, let us state the main result of this paper.

Theorem 1: Consider system (1). Suppose graph G is connected and Assumptions 1 and 2 hold. Take $\alpha > (n-1)\beta$, $k_1^j > 2\beta$, $j = 1, 2, \ldots, N-1$, $k_1 > 2\beta$, $k_2 > 2k\beta$ and $\gamma > \beta + 2k_2 + 2kc\Omega$. Then, control protocol (8) can solve the finite-time encirclement control problem of system (1), i.e., the closed-loop system can achieve finite-time circle formation, and the task of distributed rotating encirclement tracking moving targets is implemented.

Proof: Choose the following Lyapunov function:

$$V_1 = |x_i - \widehat{x}_i|. \tag{9}$$

Compute the derivative of V_1 along closed-loop system (1)−(9), we have

$$\begin{aligned}\dot{V}_1 &= \text{sign}(x_i - \widehat{x}_i)(\dot{x}_i - \dot{\widehat{x}}_i) \\ &= -\gamma - \text{sign}(x_i - \widehat{x}_i)\dot{\widehat{x}}_i \leq -\gamma + \|\dot{\widehat{x}}_i\|.\end{aligned}$$

According to Lemma 2, there exists $T_1 > 0$, such that $p_i(t) = \frac{1}{n}\sum_{j=1}^{n} p_j(t)$ for $t > T_1$. So $\dot{p}_i = \frac{1}{n}\sum_{j=1}^{n}\dot{p}_j(t) = \frac{1}{n}\sum_{j=1}^{n}\dot{r}_j(t)$. Therefore, $\|\dot{p}_i(t)\| = \|\frac{1}{n}\sum_{j=1}^{n}\dot{r}_j(t)\| \leq \beta$. And according to Lemma 3, there exists $T_2 > T_1$, such that $\rho_i(t) = \max_{j\in\mathcal{I}}(d_j(t))$ when $t \geq T_2$. Moreover, since $\dot{l}_i(t) = -k_2\text{sign}(l_i(t) - k\rho_i(t))$, we have $\|\dot{l}_i(t)\| \leq k_2$. According to Lemma 3, there exists $T_4 > T_2$ such that $l_i(t) = k\rho_i(t) = kD(t)$ when $t > T_4$. So we have $\|l_i(t)\| \leq kc$. On the other hand, from Lemma 1, it is easy to see that $\widehat{\theta}_i(t)$ can uniformly and ultimately converge in finite-time, i.e., there exists $T_3 > 0$, $\lim_{t\to T_3}(\hat{\theta}_i(t) - \hat{\theta}_j(t)) = 0$, So $\lim_{t\to T_3}(\widetilde{\theta}_i(t) - \widetilde{\theta}_j(t) - 2(i-j)\pi/n) = 0$, thus $\lim_{t\to T_3}(\theta_i(t) - \theta_j(t) - 2(i-j)\pi/n) = 0$, i.e., agents are evenly distributed across circle formation in manner of rotation in finite-time. It is easy to get $\dot{\widetilde{\theta}}_i(t) = 0$ and $\dot{\theta}_i(t) = \Omega(t)$, thus $\|\dot{\theta}_i(t)\| = \|\Omega(t)\| \leq \Omega$ when $t > T_3$. So we have $\|l_i(t)\dot{\widetilde{\theta}}_i(t)\| \leq kc\Omega$ when $t \geq \max\{T_2, T_3\}$. Therefore, if $t > \max\{T_3, T_4\}$, then $\|\dot{\widehat{x}}_i(t)\| \leq \beta + 2k_2 + 2kc\Omega$. Because $\gamma > \beta + 2k_2 + 2kc\Omega$, the closed-loop system is finite-time stable. Having noted Lemma 3 and Lemma 4, it is easy to see that the closed-loop multi-agent system can achieve circle formation in finite-time, and the task of distributed rotating encirclement tracking moving targets is completed. ■

Remark 8: Theorem 1 shows that the protocols proposed here can force agents not only achieve encirclement tracking in the manner of distribute formation, but also rotate around targets.

Remark 9: In contrast to the asymptotic rotating control [31] and encirclement control [33]−[35], finite-time control provides faster responses, and has better disturbance rejection even than exponential convergence. Just because of this, the finite-time control has been intensively inves-

tigated in [38]. The proposed protocols in this paper can drive all agents evenly distribute and rotate around the targets in finite-time.

Remark 10: Although the discontinuous control has strong anti-jamming capability, while, the chattering phenomenon is inevitable in practice, how to eliminate or weaken the chattering phenomenon is the common problem for discontinuous dynamical systems (such as [33]−[35]), which is still an open problem [39]. To eliminate chattering phenomenon, we can use the following continuous protocol to replace protocol (8):

$$u_i = -\gamma \text{sign}(x_i - \widehat{x}_i)|x_i - \widehat{x}_i|^{\sigma_1} + \dot{\widehat{x}}_i$$

where $0 < \sigma_1 < 1$. However, this protocol contains the derivative term, which might amplify noises. For (4), we can replace it with the following continuous protocol:

$$\begin{cases} \dot{\varphi}_i(t) = \alpha \sum\limits_{j\in N_i} a_{ij} \dfrac{p_j(t) - p_i(t)}{\|p_j(t) - p_i(t)\| + \sigma_2} \\ p_i(t) = \varphi_i(t) + r_i(t) \end{cases}$$

where $\sigma_2 > 0$ is a small enough scalar. This protocol can make the system avoid chattering phenomenon, but cannot force the system to converge to equilibrium in finite-time. For other protocols, we can do the similar change to eliminate chattering phenomenon. In practical applications, we can adjust the protocol to seek some kind of balance based on actual needs. In addition, as stated in [39], the chattering phenomenon can be weakened by adjusting the system parameters. Therefore, the results of this paper are still valuable. For the future work, we will study deeply the chattering problem of discontinuous multi-agent systems.

4 Simulations

In this section, numerical simulations are given to illustrate the theoretical results obtained in the previous sections. Fig. 1 shows a graph with $n = 5$ nodes.

Suppose the dynamics of targets are

$$\begin{aligned}
\dot{r}_{11}(t) &= -(r_{12}(t) - 1) - (r_{11}(t) - 1) \\
&\quad \times[(r_{11}(t) - 1)^2 + (r_{12}(t) - 1)^2 - 0.01] \\
\dot{r}_{12}(t) &= (r_{11}(t) - 1) - (r_{12}(t) - 1) \\
&\quad \times[(r_{11}(t) - 1)^2 + (r_{12}(t) - 1)^2 - 0.01] \\
\dot{r}_{21}(t) &= -(r_{22}(t) - 1) - r_{21}(t) \\
&\quad \times[(r_{21}(t))^2 + (r_{22}(t) - 1)^2 - 0.01] \\
\dot{r}_{22}(t) &= r_{21}(t) - (r_{22}(t) - 1) \\
&\quad \times[(r_{21}(t))^2 + (r_{22}(t) - 1)^2 - 0.01] \qquad (10) \\
\dot{r}_{31}(t) &= -(r_{32}(t) - 1) - (r_{31}(t) - 2) \\
&\quad \times[(r_{31}(t) - 2)^2 + (r_{32}(t) - 1)^2 - 0.01] \\
\dot{r}_{32}(t) &= (r_{31}(t) - 2) - (r_{32}(t) - 1) \\
&\quad \times[(r_{31}(t) - 2)^2 + (r_{32}(t) - 1)^2 - 0.01] \qquad (11) \\
\dot{r}_{41}(t) &= -(r_{42}(t) - 2) - (r_{41}(t) - 1) \\
&\quad \times[(r_{41}(t) - 1)^2 + (r_{42}(t) - 2)^2 - 0.01] \\
\dot{r}_{42}(t) &= (r_{41}(t) - 1) - (r_{42}(t) - 2) \\
&\quad \times[(r_{41}(t) - 1)^2 + (r_{42}(t) - 2)^2 - 0.01] \qquad (12) \\
\dot{r}_{51}(t) &= -r_{52}(t) - (r_{51}(t) - 1) \\
&\quad \times[(r_{51}(t) - 1)^2 + (r_{52}(t))^2 - 0.01] \\
\dot{r}_{52}(t) &= (r_{51}(t) - 1) - r_{52}(t) \\
&\quad \times[(r_{51}(t) - 1)^2 + (r_{52}(t))^2 - 0.01]. \qquad (13)
\end{aligned}$$

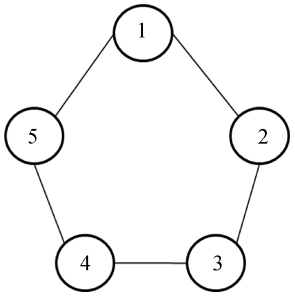

Fig. 1. Communication topology.

And the initial conditions of system (10)−(14) are $(1,2),(2,3),(3,4),(3,2),(1,1)$. Take $\Omega(t) = 1, \beta = 0.2, \alpha = 0.81, k_1 = k_1^i = 0.5, i = 1,2,3,4,5, k_2 = 0.0.8, \gamma = 6.6$. It is easy to see the conditions of Theorem 1 are satisfied.

Fig. 2 shows the movement paths of targets. Fig. 3 shows the estimates of targets' geometry center and the real path of targets' geometry center, which is marked by black line. Fig. 4 shows the estimates of polar radius. From Figs. 3 and 4, it is easy to see that these estimators are finite-timeconvergent. Fig. 5 shows the paths of agents and targets of multi-agent systems, which implies that the closed-loop multi-agent system can achieve circle formation in finite-time, and the task of distributed rotating encirclement tracking moving targets is completed.

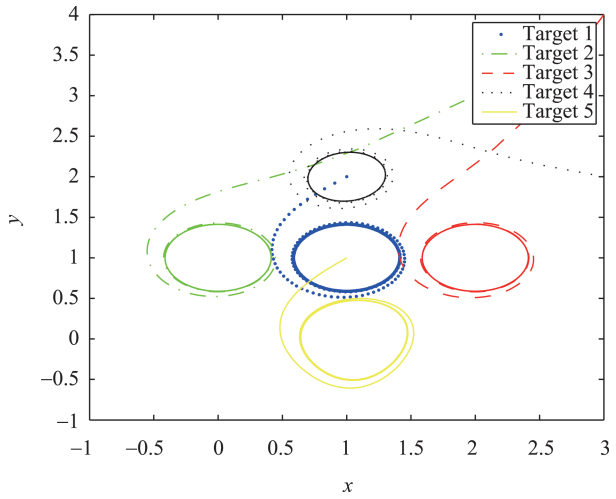

Fig. 2. States of targets.

5 Conclusion

In this paper, the focus was mainly on the problem of how to implement encircling and tracking of moving targets for agents. We designed some protocols, under which, the task of rotating encirclement and tracking moving targets for agents was completed. Based on the neighbor information of each agent, some estimators were proposed to estimate geometry center of targets, the maximum distance between all targets and their geometry center. Sufficient conditions were obtained for making agents encircle and track targets in circle formation, and the radius of formation can be changed with targets simultaneously. By the theory of

finite-time Lyapunov stability, it is proved that geometry center position of targets can be gotten in finite-time by agents and circle formation can be formed in finite-time. Finally, simulation results are provided to demonstrate the effectiveness of presented results. The future work will focus on the encirclement control of the time-delay systems with external disturbances.

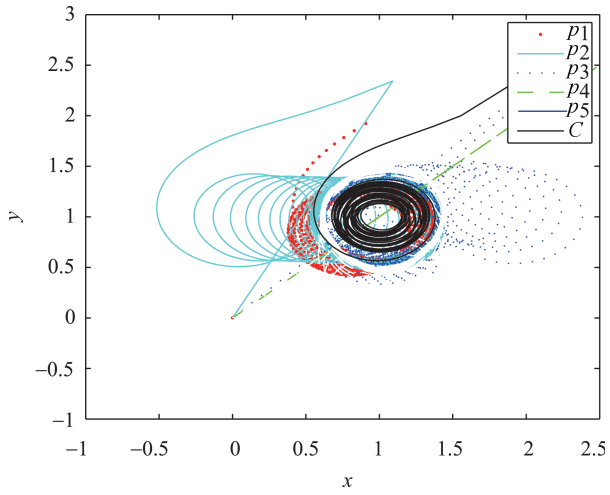

Fig. 3. Geometry center.

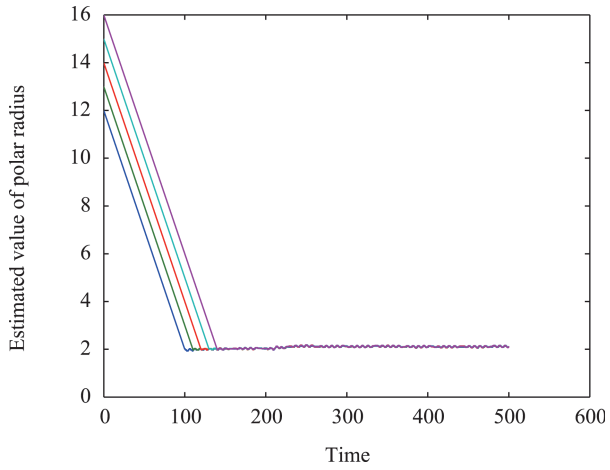

Fig. 4. The estimates of polar radius.

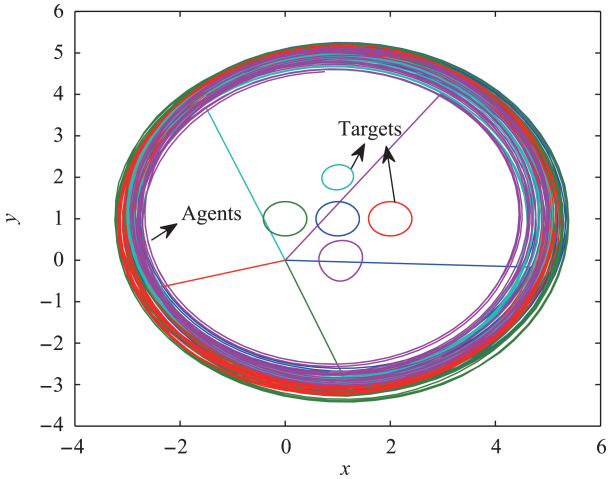

Fig. 5. Encircling motion of agents and targets.

Appendix

A. Proof of Lemma 2

Let $\overline{p}(t) = \left[p_1^T(t), p_2^T(t), \ldots, p_n^T(t)\right]^T$, we define the following Lyapunov function:

$$V_1(t) = \frac{1}{2}\left[\overline{p}(t) - \frac{1}{n}\mathbf{1}_n \otimes \left[\ \mathbf{1}_n^T \otimes E_2\overline{p}(t)\ \right]\right]^T \times \left[\overline{p}(t) - \frac{1}{n}\mathbf{1}_n \otimes \left[\ \mathbf{1}_n^T \otimes E_2\overline{p}(t)\ \right]\right] \quad (14)$$

where E_2 is the identity matrix with order two.

Computing the derivative of $V_1(t)$ along system (4), yields,

$$\begin{aligned}\dot{V}_1(t) &= \sum_{i=1}^{n}\left(\left[p_i(t) - \frac{1}{n}\sum_{k=1}^{n}p_k(t)\right]^T \times \left[\dot{p}_i(t) - \frac{1}{n}\sum_{k=1}^{n}\dot{p}_k(t)\right]\right)\\ &= \sum_{i=1}^{n}\left\{\left[p_i(t) - \frac{1}{n}\sum_{k=1}^{n}p_k(t)\right]^T \times\left[\alpha\sum_{j\in N_i}a_{ij}\frac{p_j(t)-p_i(t)}{\|p_j(t)-p_i(t)\|} + \dot{r}_i(t) - \frac{1}{n}\sum_{k=1}^{n}\dot{p}_k(t)\right]\right\}.\end{aligned}$$

Since

$$\begin{aligned}&\alpha\sum_{i=1}^{n}\left\{\left[p_i(t) - \frac{1}{n}\sum_{k=1}^{n}p_k(t)\right]^T\sum_{j\in N_i}a_{ij} \times\frac{p_j(t)-p_i(t)}{\|p_j(t)-p_i(t)\|}\right\}\\ =\ &\frac{\alpha}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\left\{a_{ij}\left[p_i(t) - \frac{1}{n}\sum_{k=1}^{n}p_k(t)\right]^T \times\frac{p_j(t)-p_i(t)}{\|p_j(t)-p_i(t)\|}\right\} - \frac{\alpha}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\left\{a_{ij}[p_j(t) - \frac{1}{n}\sum_{k=1}^{n}p_k(t)]^T\frac{p_j(t)-p_i(t)}{\|p_j(t)-p_i(t)\|}\right\}\\ =\ &\frac{\alpha}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\left\{a_{ij}[p_i(t)-p_j(t)]^T\frac{p_j(t)-p_i(t)}{\|p_j(t)-p_i(t)\|}\right\}\\ \le\ &-\frac{\alpha}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}a_{ij}\|p_i(t)-p_j(t)\|.\end{aligned}$$

From Assumption 1, we can get

$$\begin{aligned}&\sum_{i=1}^{n}\left\{(p_i(t) - \frac{1}{n}\sum_{k=1}^{n}p_k(t))^T\dot{r}_i(t)\right\}\\ \le\ &\beta\sum_{i=1}^{n}\|(p_i(t) - \frac{1}{n}\sum_{k=1}^{n}p_k(t))\|\\ =\ &\frac{\beta}{n}\sum_{i=1}^{n}\|np_i(t) - \sum_{k=1}^{n}p_k(t)\|\\ \le\ &\beta\sum_{j=1,j\neq i}^{n}\max_{i=1,2\ldots,n}(\|p_i(t)-p_j(t)\|)\\ \le\ &(n-1)\beta\max_{i,j=1,2\ldots,n}\|p_i(t)-p_j(t)\|\\ \le\ &\frac{\beta(n-1)}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}a_{ij}\|p_i(t))-p_j(t)\|\end{aligned}$$

and

$$\begin{aligned}&\sum_{i=1}^{n}\left\{\left[p_i(t)-\frac{1}{n}\sum_{k=1}^{n}p_k(t)\right]^T\left[-\frac{1}{n}\sum_{k=1}^{n}\dot{p}_k(t)\right]\right\}\\ =\;&\left[\sum_{i=1}^{n}p_i(t)-\sum_{k=1}^{n}p_k(t)\right]\left[-\frac{1}{n}\sum_{k=1}^{n}\dot{p}_k(t)\right]=0.\end{aligned}$$

So we have

$$\begin{aligned}\dot{V}_1(t) &\le -\frac{\alpha}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}a_{ij}\|p_i(t)-p_j(t)\|\\ &\quad+\frac{\beta(n-1)}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}a_{ij}\|p_i(t)-p_j(t)\|\\ &\le \left(\frac{\beta(n-1)}{2}-\frac{\alpha}{2}\right)n^2 s(t)\end{aligned}$$

where $s(t)=\max_{i,j\in I}\|p_i(t)-p_j(t)\|$. Since $\alpha>\beta(n-1)$, we have $\dot{V}_1(t)\le 0$. And $\left\|p_i(t)-\frac{1}{n}\sum_{k=1}^{n}p_k(t)\right\|\le\frac{1}{n}\|p_i(t)-p_j(t)\|\le s(t)$, which yields $V_1(t)\le ns(t)^2/2$. Hence

$$\begin{aligned}&\dot{V}_1(t)+\sqrt{\frac{2}{n}}\left(\frac{\alpha}{2}-\frac{\beta(n-1)}{2}\right)n^2V_1(t)^{\frac{1}{2}}\\ &\le\left(\frac{\beta(n-1)}{2}-\frac{\alpha}{2}\right)n^2s(t)+\left(\frac{\alpha}{2}-\frac{\beta(n-1)}{2}\right)n^2s(t)=0.\end{aligned}$$

By the Lyapunov finite-time stability theorem [38], there exists $T_1>0$ such that $\lim_{t\to T_1}\left[p_i(t)-\frac{1}{n}\sum_{k=1}^{n}r_k(t)\right]=0$, which yields $\lim_{t\to T_1}\left[p_i(t)-p_j(t)\right]=0$.

B. Proof of Lemma 3

Define the following candidate Lyapunov function:

$$V_2(t)=|\varepsilon_i| \tag{15}$$

where $\varepsilon_i=\rho_i(t)-\max_{j\in N_i}\{d_j(t)\}$. Then, the derivative of $V_2(t)$ along system (5) is

$$\begin{aligned}\dot{V}_2(t) &= \mathrm{sign}(\varepsilon_i)\dot{\varepsilon}_i\\ &= \mathrm{sign}(\varepsilon_i)[-k_1\mathrm{sign}(\rho_i(t)-\max_{j\in N_i\cup\{i\}}(d_j(t))\\ &\quad-\max_{j\in N_i\cup\{i\}}(\dot{d}_j(t))].\end{aligned}$$

Let $d(t):=\max_{j\in N_i\cup\{i\}}\{d_j(t)\}$. From Lemma 2, there exists $T_1>0$ such that $p_j(t)-\frac{1}{n}\sum_{k=1}^{n}r_k(t)=0$ for all $t>T_1$, which yields $p_j(t)=\frac{1}{n}\sum_{k=1}^{n}r_k(t)$. Hence, when $t>T_1$,

$$\begin{aligned}d(t) &= \max_{j\in N_i\cup\{i\}}\left\{\left\|r_j(t)-\frac{1}{n}\sum_{k=1}^{n}r_k(t)\right\|\right\}\\ &= \max_{j\in N_i\cup\{i\}}\left\{\left\|\frac{n-1}{n}r_j(t)-\frac{1}{n}\sum_{k=1,k\ne j}^{n}r_k(t)\right\|\right\}.\end{aligned} \tag{16}$$

Having noted Assumption 1, it is easy to see that

$$\begin{aligned}\dot{d}(t) &= \left|\max_{j\in N_i\cup\{i\}}(\dot{d}_j(t))\right|\\ &= \left|\max_{j\in N_i\cup\{i\}}\left\{\left\|\dot{r}_j(t)-\frac{1}{n}\sum_{k=1}^{n}\dot{r}_k(t)\right\|\right\}\right|\\ &= \left|\max_{j\in N_i\cup\{i\}}\left\{\left\|\frac{n-1}{n}\dot{r}_j(t)-\frac{1}{n}\sum_{k=1,k\ne j}^{n}\dot{r}_k(t)\right\|\right\}\right|\\ &\le \max_{j\in N_i\cup\{i\}}\left\{\left\|\frac{n-1}{n}\dot{r}_j(t)-\frac{1}{n}\sum_{k=1,k\ne j}^{n}\dot{r}_k(t)\right\|\right\}\\ &\le 2\beta.\end{aligned}$$

Therefore,

$$\dot{V}_2(t)\le\mathrm{sign}(\varepsilon_i)\left[-k_1\mathrm{sign}(\varepsilon_i)\right]+2\beta=-k_1\left|\mathrm{sign}(\varepsilon_i)\right|+2\beta. \tag{17}$$

From $k_1^1>2\beta$, it is easy to prove that the first equation of system (5) is finite-time stable, i.e., there exists $T_2^1>0$, such that $\rho_i^1(t)=\max_{j\in N_i\cup\{i\}}(d_j(t))$ for all $t\ge T_2^1$.

Similarly, we can prove that the other equations in system (5) are also finite-time stable, i.e., there exist $T_2^1<T_2^2<\cdots<T_2^{N-1}<T_2$, such that $\rho_i^k(t)=\max_{j\in N_i\cup\{i\}}(\rho_i^{k-1}(t))$ for all $t\ge T_2^k$, $k=2,\ldots,N-1$, and $\rho_i(t)=\max_{j\in N_i\cup\{i\}}(\rho_i^{N-1}(t))$ for all $t\ge T_2$. From the definition of N, it is not hard to see that $\rho_i(t)=\max_{j\in\mathcal{I}}(d_j(t))$ when $t\ge T_2$.

C. Proof of Lemma 4

Choose the candidate Lyapunov function as follows:

$$V_3(t)=|e_i| \tag{18}$$

where $e_i(t)=l_i(t)-k\rho_i(t)$.

Computing the derivative of $V_3(t)$ along system (7), we have

$$\begin{aligned}\dot{V}_3(t) &= \mathrm{sign}(e_i(t))\dot{e}_i\\ &= \mathrm{sign}(e_i(t))\left[-k_2\mathrm{sign}(l_i(t)-k\rho_i(t))-k\dot{\rho}_i(t)\right]\\ &\le -k_2\left|\mathrm{sign}(e_i(t))\right|-\mathrm{sign}(e_i(t)k\dot{\rho}_i(t).\end{aligned}$$

By Lemma 3, there exists $T_2>0$, such that when $t>T_2$, $\rho_i(t)=\max_{j\in\mathcal{I}}\{d_j(t)\}$, so $|\dot{\rho}_i(t)|=\left|\dot{d}(t)\right|\le 2\beta$, thus,

$$\dot{V}_3(t)\le -k_2\left|\mathrm{sign}(e_i(t))\right|+2k\beta. \tag{19}$$

It is easy to see that the closed-loop system is finite-time stable when $k_2>2k\beta$.

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