Adaptive Dynamic Feedback Tracking Control for a Robot-camera System with Unknown Parameters

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Abstract The tracking problem of nonholonomic mobile robots with uncertainties is investigated in this paper. An uncertain model of the nonholonomic kinematic system is presented based on the visual feedback and the state and input transformations for a kind of mobile robots in chained form with uncertainties. Two transformations are exploited based on the idea of backstepping and the structure of tracking error system. Then, both an adaptive control law and a dynamic feedback robust controller are designed to track the desired trajectory by using Lyapunov direct method and the extended Barbalat Lemma. The asymptotic convergence of a closed-loop error system is proved rigorously. Finally, simulation results demonstrate the effectiveness of the proposed strategies.

Key words Adaptive control, chained system, feedback, mobile robot, tracking control

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The control of nonholonomic systems has received a great deal of attention over the past twenty years[1−2]. In [3] and some references therein, it is shown that many systems with nonholonomic constraints can be transformed, either locally or globally, to chained systems by using coordinate and state-feedback transformations. Several new control strategies were developed around the important nonholonomic chained models[4−6]. A wheeled mobile robot (WMR) is one of the well-known systems with nonholonomic constraints[7]. In the control of nonholonomic WMR, it is usually assumed that the states are available using sensor measurements. But in practice, there exist uncertainties, such as uncalibrated parameters in the kinematic models, mechanical limitations, noise and so on. In recent ten years, the study of nonholonomic systems with uncertainties has received considerable attention. Many strategies have been investigated to stabilize the uncertain nonholonomic systems[8−11]. Adaptive strategies were often used to control the dynamic nonholonomic systems with modeling or parametric uncertainties[12−13].

The tracking control is a complicated problem due to coupled and nonlinear system dynamics[12−16]. In [12], adaptive force tracking controllers were proposed which not only ensure the entire state of the system to asymptotically converge to the desired trajectory but also ensure the constraint force to asymptotically converge to the desired force. In [13], Wang et al. proposed a robust adaptive tracking controller which not only can guarantee robustness to parametric and dynamics uncertainties but also can reject any bounded, immeasurable disturbances entering the system. Based on Lyapunov′s direct method and backstepping technique in [6] and [15], the time-varying global adaptive controllers were presented which simultaneously solved both tracking and stabilization for mobile robots with unknown kinematic and dynamic parameters.

Visual feedback is an important approach to improve the control performance for robots and manipulators since it mimics the human sense of vision and allows for operating on the basis of noncontact measurement and unstructured environment. Since the late 1980s, tremendous effort has been made to visual servoing[16−21] and vision-based manipulations. In order to develop an adaptive tracking controller for a mobile robot that compensated for the parametric uncertainty in the camera and the mobile robot dynamics, the feedback from an uncalibrated, fixed (ceiling-mounted) camera was used in [16]. In [18], a visual servo tracking controller was developed for a monocular camera system mounted on an underactuated WMR subject to nonholonomic motion constraints. In [19], a new controller for controlling a number of feature points on a robot manipulator was presented to track desired trajectories specified on the image plane of a fixed camera. Recently, [20] presented a dynamic feedback tracking controller for the nonholonomic WMR of unicycle type with unknown camera parameters. In [11], a series of new chained models of nonholonomic mobile robots with uncalibrated visual parameters were shown. In [21], the trajectory tracking control problem of another kind of uncertain dynamic nonholonomic mobile robot (called type $(1, 1)$ robot) was addressed where a new adaptive torque tracking controller was presented for tracking error model. For type $(1, 2)$ robot, which has two steering wheels and one castor wheel with unknown visual parameters, a new and simple robust stabilization controller[11, 22] was designed for a particular case. However, the corresponding tracking problem has not been discussed. Comparing with [16], in this paper, we design an adaptive dynamic feedback controller to compensate for the unknown camera parameter. Based on Lyapunov direct method and the idea of back-stepping technique, two transformations are chosen. The controllers can make the mobile robot tracking the desired trajectory in the image space and work-space.

The paper is organized as follows. Section 1 addresses robot-camera system configuration. In Section 2, an uncertain chained form model is presented, and the tracking problems are proposed. Section 3 addresses the designs of adaptive and dynamic feedback tracking controllers for

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the uncertain kinematic error system and gives the rigorous proof of the asymptotical convergence of the closed-loop error system. Then, the tracking problems in work-space of the mobile robot are presented. In Section 4, simulation results are provided to illustrate the effectiveness of the proposed control strategy. Finally, the major contributions of the paper are summarized in Section 5.

1 Robot-camera system

In this section, we will address robot-camera system configuration.

In Fig. 1, a robot-camera system is shown. It is assumed that a pinhole camera is fixed to the ceiling, the type $(1,2)$ mobile robot is under the camera. The movement of the mobile robot can be measured by using a fixed camera. It is assumed that the camera plane runs parallel to the mobile robot plane, and the camera can capture images throughout the entire robot workspace. In the robot-camera system, three coordinate frames exist, namely the inertial frame X-Y-Z, the camera frame i-j-k and the image frame i_1-o_1-j_1. Assume that the i-j plane of the camera frame is parallel to the plane of the image coordinate plane. The direction of corresponding coordinate axis is identical. But the coordinate of the original point of the camera frame with respect to the image frame is defined by (O_{c1}, O_{c2}). $C(c_x, c_y)$ is the crossing point between the optical axis of the camera and X-Y plane.

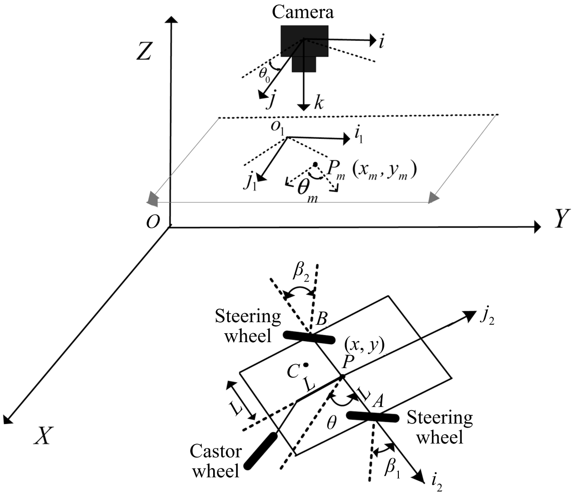

Fig. 1 Wheeled mobile robots with monocular camera

1.1 Robot kinematic system

In Fig. 1, the type $(1,2)$ mobile robot[7] is in the X-Y plane which has two steering wheels (conventional centered orientable wheels) and one castor wheel (conventional off-centered orientable wheel). P is the mid-distance point between the centers of these two steering wheels, with i_2 aligned along the line joining their centers. A and B are the center points of two steering wheels respectively. L is the distance between point P and point A (or point P and point B). It is also the distance between point P and the joint point of the castor wheel. θ denotes the angle between i_2 axis and X axis, β_1 and β_2 denote the angles between the orientation of the plane of steering wheels and i_2 axis respectively. Assume that the geometric center point and the mass center point of the robot are the same. The nonholonomic constraints are described[7] by

$$(\cos\beta_1, \sin\beta_1, L\sin\beta_1)G(\theta)\dot{q} = 0$$
$$(-\cos\beta_2, -\sin\beta_2, L\sin\beta_2)G(\theta)\dot{q} = 0$$

where $q = (x, y, \theta)^{\rm T}$, and

$$G(\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Then the nonholonomic kinematic system[7] can be given by

$$\begin{cases} \dot{x} = -Lv_1[\sin\beta_1\sin(\theta+\beta_2) + \sin\beta_2\sin(\theta+\beta_1)] \\ \dot{y} = Lv_1[\sin\beta_1\cos(\theta+\beta_2) + \sin\beta_2\cos(\theta+\beta_1)] \\ \dot{\theta} = v_1\sin(\beta_2-\beta_1) \\ \dot{\beta}_1 = v_2 \\ \dot{\beta}_2 = v_3 \end{cases} \tag{1}$$

where v_1 is the velocity of the robot, v_2 and v_3 are the angular velocities of two steering wheels respectively.

For system (1), if $\sin(\beta_2 - \beta_1) = 0$, then $\beta_1 = \beta_2 = 0$ or $\beta_2 = \beta_1 + k\pi$ $(k = \pm1, \pm2, \cdots)$. Consider the two nonholonomic constraints along the wheel plane given by [7]. One obtains that the type $(1,2)$ mobile robot is stopped (i.e., $v_1 = 0$) or this robot is vestigial to type $(2,0)$ robot. The tracking and stabilization problems are discussed in many papers such as [20−21].

For system (1), if $\sin(\beta_2-\beta_1) \neq 0$, choose the state-input transformation[3] as

$$\begin{cases} z_0 = \theta \\ z_1 = x\cos\theta + y\sin\theta \\ z_2 = -x\sin\theta + y\cos\theta - 2L\dfrac{\sin\beta_1\sin\beta_2}{\sin(\beta_2-\beta_1)} \\ z_3 = x\sin\theta - y\cos\theta \\ z_4 = x\cos\theta + y\sin\theta - L\dfrac{\sin(\beta_1+\beta_2)}{\sin(\beta_2-\beta_1)} \\ \sigma_0 = v_1\sin(\beta_2-\beta_1) \\ \sigma_1 = -x_4v_1\sin(\beta_2-\beta_1) - \dfrac{2Lv_2\sin^2\beta_2}{\sin^2(\beta_2-\beta_1)} + \dfrac{2Lv_3\sin^2\beta_1}{\sin^2(\beta_2-\beta_1)} \\ \sigma_2 = x_2v_1\sin(\beta_2-\beta_1) - \dfrac{Lv_2\sin(2\beta_2)}{\sin^2(\beta_2-\beta_1)} + \dfrac{Lv_3\sin(2\beta_1)}{\sin^2(\beta_2-\beta_1)} \end{cases} \tag{2}$$

We obtain the following chained system

$$\begin{cases} \dot{z}_0 = \sigma_0 \\ \dot{z}_1 = z_2\sigma_0 \\ \dot{z}_2 = \sigma_1 \\ \dot{z}_3 = z_4\sigma_0 \\ \dot{z}_4 = \sigma_2 \end{cases} \tag{3}$$

System (3) is called canonical chained form system[8]. Generally, (x, y) in (1) needs to be measured for feedback. The encoders can be used to do it. However, over-shoot and low precision are their main drawbacks. Camera is a convenient sensor to implement non-contact and unstructured measurement. The data from a camera can be used for the robot tracking problem.

1.2 Camera system model

In Fig. 1, the coordinate of the mass center P is (x, y) for the robot with respect to X-Y plane. Suppose that P_m

(x_m, y_m) is the coordinate of (x, y) relative to the image frame. Pinhole camera model yields

$$\begin{bmatrix} x_m \\ y_m \end{bmatrix} = \begin{bmatrix} \alpha_1 & 0 \\ 0 & \alpha_2 \end{bmatrix} H(\theta_0)\left[\begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} c_x \\ c_y \end{bmatrix}\right] + \begin{bmatrix} O_{c1} \\ O_{c2} \end{bmatrix} \tag{4}$$

where α_1 and α_2 are positive constants and dependent on the depth information, focal length, scale factors[16] defined as follows

$$\alpha_1 = \rho_1 \frac{f}{z}, \quad \alpha_2 = \rho_2 \frac{f}{z}$$

where $z \in \mathbf{R}^1$ represents the constant height of the camera optical center with respect to the task-space plane, $f \in \mathbf{R}^1$ is a constant representing the camera′s focal length, the positive constants denoted by $\rho_1,\ \rho_2 \in \mathbf{R}^1$, represent the cameras constant scale factors (in pixels/m) along their respective Cartesian directions, respectively[16]. And

$$H(\theta_0) = \begin{bmatrix} \cos\theta_0 & \sin\theta_0 \\ -\sin\theta_0 & \cos\theta_0 \end{bmatrix}$$

where θ_0 denotes the angle between j axis and X axis which represents the constant, anticlockwise rotation angle of the camera coordinate system with respect to the task-space coordinate system.

Therefore, the kinematic system in the image frame can be rewritten as

$$\begin{bmatrix} \dot{x}_m \\ \dot{y}_m \end{bmatrix} = \begin{bmatrix} \alpha_1 & 0 \\ 0 & \alpha_2 \end{bmatrix} \begin{bmatrix} \cos\theta_0 & \sin\theta_0 \\ -\sin\theta_0 & \cos\theta_0 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} \tag{5}$$

In this paper, it is assumed that (x, y) in (1) is measured by using a camera with uncalibrated visual parameters shown in Fig. 1. The pose of the mobile robot in the workspace is (x, y, θ). the pose of the robot in the image plane is (x_m, y_m, θ_m). Then, by using the state and input transformations in Section 2, a kinematic model with unknown visual parameters will be deduced in the following section.

Remark 1. The first formula $H(\theta_0)$ on this page is a rotation matrix which is different from that denoted by $R(\theta_0)$ in [16]. In our paper, $H(\theta_0)$ is a matrix of anticlockwise rotation, but $R(\theta_0)$ in [16] is a matrix of clockwise rotation.

2 Problem formulation

In this section, we will present an uncertain chained system by using (5) and using the state and input transformations for type $(1, 2)$ mobile robot with unknown visual parameters. Then, we will propose the tracking problem for the uncertain chained system and type $(1, 2)$ mobile robot.

For system (1), suppose $\sin(\beta_2 - \beta_1) \neq 0$, and consider (5). We have[22]

$$\begin{bmatrix} \dot{x}_m \\ \dot{y}_m \end{bmatrix} = \begin{bmatrix} -\alpha_1 L v_1 (\sin\beta_1 s_{\Delta 2} + \sin\beta_2 s_{\Delta 1}) \\ \alpha_2 L v_1 (\sin\beta_1 c_{\Delta 2} + \sin\beta_2 c_{\Delta 1}) \end{bmatrix} \tag{6}$$

where

$$s_{\Delta i} = \sin(\theta - \theta_0 + \beta_i),\ c_{\Delta i} = \cos(\theta - \theta_0 + \beta_i), \quad i = 1, 2$$

Considering kinematic system (1) in the robot workspace, we have

$$\tan\theta = \frac{\dot{y}}{\dot{x}} = -\frac{\sin\beta_1\cos(\theta + \beta_2) + \sin\beta_2\cos(\theta + \beta_1)}{\sin\beta_1\sin(\theta + \beta_2) + \sin\beta_2\sin(\theta + \beta_1)}$$

Then,

$$\tan(\theta - \theta_0) = -\frac{\sin\beta_1 c_{\Delta 2} + \sin\beta_2 c_{\Delta 1}}{\sin\beta_1 s_{\Delta 2} + \sin\beta_2 s_{\Delta 1}}$$

Now, considering (6) in the image space, we have

$$\tan\theta_m = \frac{\dot{y}_m}{\dot{x}_m} = -\frac{\alpha_2}{\alpha_1}\frac{\sin\beta_1 c_{\Delta 2} + \sin\beta_2 c_{\Delta 1}}{\sin\beta_1 s_{\Delta 2} + \sin\beta_2 s_{\Delta 1}}$$

Hence, we obtain the following relationships

$$\tan\theta_m = \frac{\alpha_2}{\alpha_1}\tan(\theta - \theta_0)$$

$$\sec^2\theta_m = \frac{\alpha_1^2\cos^2(\theta - \theta_0) + \alpha_2^2\sin(\theta - \theta_0)}{\alpha_1^2\cos^2(\theta - \theta_0)} \tag{7}$$

After taking the time derivative of (7), we have that

$$(\sec^2\theta_m)\dot{\theta}_m = \left[\frac{\alpha_2}{\alpha_1}\sec^2(\theta - \theta_0)\right]\dot{\theta}$$

Therefore, we obtain

$$\dot{\theta} = \left[\frac{\alpha_1}{\alpha_2}\cos^2(\theta - \theta_0) + \frac{\alpha_2}{\alpha_1}\sin^2(\theta - \theta_0)\right]\dot{\theta}_m$$

If $\alpha_1 = \alpha_2 = \alpha$, we have $\dot{\theta}_m = \dot{\theta}$ and $\theta_m = \theta - \theta_0 + k\pi$ ($k = 0, \pm1, \pm2, \cdots$). Then, the nonholonomic kinematic system with uncalibrated parameters in the image-plane can be described by the following system

$$\begin{bmatrix} \dot{x}_m \\ \dot{y}_m \\ \dot{\theta} \\ \dot{\beta}_1 \\ \dot{\beta}_2 \end{bmatrix} = \begin{bmatrix} -\alpha L v_1 (\sin\beta_1 s_{\Delta 2} + \sin\beta_2 s_{\Delta 1}) \\ \alpha L v_1 (\sin\beta_1 c_{\Delta 2} + \sin\beta_2 c_{\Delta 1}) \\ v_1 \sin(\beta_2 - \beta_1) \\ v_2 \\ v_3 \end{bmatrix} \tag{8}$$

where θ_m is expressed by θ. For $i = 1, 2$, denote

$$\begin{aligned} s_{\Lambda i} &= \sin(2\theta - \theta_0 + \beta_i), \quad s_\Theta = \sin(2x_0 - \theta_0) \\ c_{\Lambda i} &= \cos(2\theta - \theta_0 + \beta_i), \quad c_\Theta = \cos(2x_0 - \theta_0) \end{aligned} \tag{9}$$

and consider the following expressions

$$\sin\theta c_{\Delta i} = -\frac{1}{2}\sin(\beta_i - \theta_0) + \frac{1}{2}s_{\Lambda i}, \quad i = 1, 2$$

$$\cos\theta c_{\Delta i} = \frac{1}{2}\cos(\beta_i - \theta_0) + \frac{1}{2}c_{\Lambda i}, \quad i = 1, 2$$

Then, note that

$$\sin\beta_1\sin(\beta_2 - \theta_0) + \sin\beta_2\sin(\beta_1 - \theta_0) = 2\sin\beta_1\sin\beta_2\cos\theta_0 - \sin\theta_0\sin(\beta_1 + \beta_2)$$

$$\sin\beta_1\cos(\beta_2 - \theta_0) + \sin\beta_2\cos(\beta_1 - \theta_0) = 2\sin\beta_1\sin\beta_2\sin\theta_0 + \cos\theta_0\sin(\beta_1 + \beta_2)$$

and

$$\sin\beta_1 s_{\Lambda 2} + \sin\beta_2 s_{\Lambda 1} = 2\sin\beta_1\sin\beta_2 c_\Theta + \sin(\beta_1 + \beta_2)s_\Theta$$

$$\sin\beta_1 c_{\Lambda 2} + \sin\beta_2 c_{\Lambda 1} = -2\sin\beta_1\sin\beta_2 s_\Theta + \sin(\beta_1 + \beta_2)c_\Theta$$

Hence, by taking the following state and input transformations

$$\begin{cases} x_0 = \theta \\ x_1 = x_m \cos\theta + y_m \sin\theta \\ x_2 = -x_m \sin\theta + y_m \cos\theta - 2L\dfrac{\sin\beta_1 \sin\beta_2}{\sin(\beta_2-\beta_1)} \\ x_3 = x_m \sin\theta - y_m \cos\theta \\ x_4 = x_m \cos\theta + y_m \sin\theta - L\dfrac{\sin(\beta_1+\beta_2)}{\sin(\beta_2-\beta_1)} \\ u_0 = v_1 \sin(\beta_2-\beta_1) \\ u_1 = -x_4 v_1 \sin(\beta_2-\beta_1) - \dfrac{2Lv_2\sin^2\beta_2}{\sin^2(\beta_2-\beta_1)} + \dfrac{2Lv_3\sin^2\beta_1}{\sin^2(\beta_2-\beta_1)} \\ u_2 = x_2 v_1 \sin(\beta_2-\beta_1) - \dfrac{Lv_2\sin(2\beta_2)}{\sin^2(\beta_2-\beta_1)} + \dfrac{Lv_3\sin(2\beta_1)}{\sin^2(\beta_2-\beta_1)} \end{cases} \tag{10}$$

One obtains the uncertain chained system[22]

$$\begin{cases} \dot{x}_0 = u_0 \\ \dot{x}_1 = x_2u_0 + (x_1 - x_4)(\alpha\sin\theta_0)u_0 - \\ \qquad (x_2+x_3)(1-\alpha\cos\theta_0)u_0 \\ \dot{x}_2 = u_1 - (x_2+x_3)(\alpha\sin\theta_0)u_0 - \\ \qquad (x_1-x_4)(1-\alpha\cos\theta_0)u_0 \\ \dot{x}_3 = x_4u_0 + (x_2+x_3)(\alpha\sin\theta_0)u_0 + \\ \qquad (x_1-x_4)(1-\alpha\cos\theta_0)u_0 \\ \dot{x}_4 = u_2 + (x_1-x_4)(\alpha\sin\theta_0)u_0 - \\ \qquad (x_2+x_3)(1-\alpha\cos\theta_0)u_0 \end{cases} \tag{11}$$

where $u_0 = v_1\sin(\beta_2-\beta_1)$ and $\sin(\beta_2-\beta_1) \neq 0$.

In contrast to canonical chained form (3), model (11) has two new parameters α and θ_0. In practice, they are usually uncalibrated. Comparing with model (3), the first term on the right side of each equation of (11) is identical except uncertain coefficient gains. Note that the second and third terms on the right side of the second equation are dependent on x_1, x_2, x_3 and x_4. Therefore, (11) does not satisfy the so-called triangular structure[8] which is required in many papers. So (11) is called an uncertain chained system.

For uncertain system (11), the tracking problem is how to design u_0, u_1 and u_2 such that the trajectory $(x_0, x_1, x_2, x_3, x_4)$ can track a desired reference trajectory $(x_{0r}, x_{1r}, x_{2r}, x_{3r}, x_{4r})$. In the work-space of type $(1,2)$ mobile robot, the adaptive dynamic tracking problem is how to design adaptive control law and dynamic feedback control to make the trajectory $q = (x, y, \theta)$ tracking a desired reference trajectory $q_r = (x_r, y_r, \theta_r)$ in the work-space of the robot with the help of the reference trajectories in the image space.

3 Tracking controller design

In this section, our objective is to design adaptive dynamic feedback tracking controllers to solve the tracking problem for uncertain chained system (11) and the type $(1,2)$ mobile robot in the work-apace. In order to design the controller, three Assumptions and a Lemma are needed as follows.

Assumption 1. Uncertain chained system (11) satisfies $u_0 \neq 0$.

Assumption 2. θ_0 is known, and $\alpha_1 = \alpha_2 = \alpha$ are unknown. There exist two constants $\underline{\alpha}$ and $\bar{\alpha}$ such that $\underline{\alpha} \leq \alpha \leq \overline{\alpha}$.

Assumption 3. x_{ir} $(i = 0, 1, \cdots, 4)$ are bounded. u_{0r} $(u_{0r} \neq 0)$, u_{1r}, u_{2r} and their derivatives are all bounded too.

Remark 2. System (11) is based on the assumption $\sin(\beta_2-\beta_1) \neq 0$. This means $u_0 \neq 0$ for (11).

Remark 3. For Assumption 2, $\alpha_1 = \alpha_2 = \alpha$ means that the scale factor along i_1 axis is the same with that one along j_1 axis. Some CCD cameras are made like this. However, $\alpha_1 = \alpha_2 = \alpha$ are limitations. As for the tracking problem of the case $\alpha_1 \neq \alpha_2$ and unknown, we will further investigate it in the future.

Remark 4. Assumption 3 is rational. Commonly, the positive upper and lower bounds of the scale factor can be estimated in advance. In practice, the robot often has the same structure feature with a reference target when the robot tracks the reference trajectory.

Remark 5. Consider system (11) under Assumptions $1\sim3$. By substituting $\theta - \theta_0$ for θ, (11) will become the system with $\theta_0 = 0$. This implies that the direction of j axis is identical to that one of X axis. Hence, we only need to discuss the case: $\theta_0 = 0$, $\alpha_1 = \alpha_2 = \alpha$ are unknown for (11).

Based on the Assumptions $1\sim3$ and the analysis above, system (11) can be rewritten as

$$\begin{cases} \dot{x}_0 = u_0 \\ \dot{x}_1 = [x_2 - (x_2+x_3)(1-\alpha)]u_0 \\ \dot{x}_2 = u_1 - [(x_1-x_4)(1-\alpha)]u_0 \\ \dot{x}_3 = [x_4 + (x_1-x_4)(1-\alpha)]u_0 \\ \dot{x}_4 = u_2 - [(x_2+x_3)(1-\alpha)]u_0 \end{cases}$$

where α is the unknown camera parameter, u_0, u_1 and u_2 are the control inputs to be designed. It can also be rewritten as

$$\begin{cases} \dot{x}_0 = u_0 \\ \dot{x}_1 = -x_3u_0 + \alpha x_{23}u_0 \\ \dot{x}_2 = u_1 + (1-\alpha)x_{41}u_0 \\ \dot{x}_3 = x_1u_0 + \alpha x_{41}u_0 \\ \dot{x}_4 = u_2 - (1-\alpha)x_{23}u_0 \end{cases} \tag{12}$$

where

$$x_{23} = x_2 + x_3, \quad x_{41} = x_4 - x_1$$

The desired reference system for (12) is

$$\begin{cases} \dot{x}_{0r} = u_{0r} \\ \dot{x}_{1r} = -x_{3r}u_{0r} + \alpha x_{23r}u_{0r} \\ \dot{x}_{2r} = u_{1r} + (1-\alpha)x_{41r}u_{0r} \\ \dot{x}_{3r} = x_{1r}u_{0r} + \alpha x_{41r}u_{0r} \\ \dot{x}_{4r} = u_{2r} - (1-\alpha)x_{23r}u_{0r} \end{cases} \tag{13}$$

where

$$x_{23r} = x_{2r} + x_{3r}, \quad x_{41r} = x_{4r} - x_{1r}$$

Denote

$$\begin{cases} e_i = x_i - x_{ir}, \quad i = 0, 1, 2, 3, 4 \\ e_{23} = e_2 + e_3, \quad e_{41} = e_4 - e_1 \end{cases} \tag{14}$$

By using (12) and (13), the following kinematic tracking error system is obtained

$$\begin{cases} \dot{e}_0 = p \\ \dot{e}_1 = -(x_3p + e_3u_{0r}) + \alpha(x_{23}p + e_{23}u_{0r}) \\ \dot{e}_2 = u_1 - u_{1r} + (1-\alpha)(x_{41}p + e_{41}u_{0r}) \\ \dot{e}_3 = (x_1p + e_1u_{0r}) + \alpha(x_{41}p + e_{41}u_{0r}) \\ \dot{e}_4 = u_2 - u_{2r} - (1-\alpha)(x_{23}p + e_{23}u_{0r}) \end{cases} \tag{15}$$

where $p := u_0 - u_{0r}$.

Based on the idea of backstepping and the structure of model (15), choose two new transformations as

$$\xi = e_{23} + k_1 e_1 u_{0r}, \quad \eta = e_{41} + k_3 e_3 u_{0r} \tag{16}$$

where k_1 and k_3 are positive constant control gains. Then, we have

$$\begin{aligned} e_{23} &= -k_1 e_1 u_{0r} + \xi \\ e_{41} &= -k_3 e_3 u_{0r} + \eta \\ \dot{e}_{23} &= u_1 - u_{1r} + x_4 p + e_4 u_{0r} \\ \dot{e}_{41} &= u_2 - u_{2r} - x_2 p - e_2 u_{0r} \end{aligned}$$

Choose the Lyapunov function candidate

$$\begin{aligned} V = {} & V_1 + V_2 + V_3 = \\ & \frac{1}{2}\left(k_0 e_0^2 + e_1^2 + e_3^2\right) + \frac{1}{2}\left(\xi^2 + \eta^2\right) + \frac{1}{2}\left(p^2 + \Lambda\tilde{\alpha}^2\right) \end{aligned} \tag{17}$$

where k_0 is a positive constant gain. $\tilde{\alpha}$ is the parameter error defined as $\tilde{\alpha} = \alpha - \hat{\alpha}$. $\hat{\alpha}$ is the estimation of α.

By using (16), we have

$$\begin{aligned} \dot{V}_1 = {} & k_0 e_0 \dot{e}_0 + e_1 \dot{e}_1 + e_3 \dot{e}_3 = \\ & k_0 e_0 p + e_1[(-x_3 p - e_3 u_{0r}) + \alpha(x_{23} p + e_{23} u_{0r})] + \\ & e_3[(x_1 p + e_1 u_{0r}) + \alpha(x_{41} p + e_{41} u_{0r})] = \\ & (k_0 e_0 - e_1 x_3 + e_3 x_1) p + \alpha(e_1 x_{23} + e_3 x_{41}) p - \\ & k_1 \alpha e_1^2 u_{0r}^2 - k_3 \alpha e_3^3 u_{0r}^2 + \alpha u_{0r}(e_1 \xi + e_3 \eta) \end{aligned}$$

$$\begin{aligned} \dot{V}_2 = {} & \xi\dot{\xi} + \eta\dot{\eta} = \\ & \xi[(\dot{e}_2 + \dot{e}_3) + k_1(\dot{e}_1 u_{0r} + e_1 \dot{u}_{0r})] + \\ & \eta[(\dot{e}_4 - \dot{e}_1) + k_3(\dot{e}_3 u_{0r} + e_3 \dot{u}_{0r})] = \\ & \xi[(u_1 - u_{1r} + x_4 p + e_4 u_{0r}) + \\ & k_1 u_{0r}(-x_3 p - e_3 u_{0r} + \alpha x_{23} p + \alpha e_{23} u_{0r}) + \\ & k_1 e_1 \dot{u}_{0r}] + \eta[(u_2 - u_{2r} - x_2 p - e_2 u_{0r}) + \\ & k_3 u_{0r}(x_1 p + e_1 u_{0r} + \alpha x_{41} p + \alpha e_{41} u_{0r}) + k_3 e_3 \dot{u}_{0r}] \end{aligned}$$

$$\dot{V}_3 = p\dot{p} + \Lambda\tilde{\alpha}\dot{\tilde{\alpha}}$$

Hence, the time derivative of V along the solution of (15) satisfies

$$\begin{aligned} \dot{V} = {} & \dot{V}_1 + \dot{V}_2 + \dot{V}_3 = \\ & - k_1 \alpha e_1^2 u_{0r}^2 - k_3 \alpha e_3^3 u_{0r}^2 + \\ & (k_0 e_0 - e_1 x_3 + e_3 x_1) p + \alpha(e_1 x_{23} + e_3 x_{41}) p + \\ & \alpha u_{0r}(e_1 \xi + e_3 \eta) + \xi[(u_1 - u_{1r} + x_4 p + e_4 u_{0r}) + \\ & k_1 u_{0r}(-x_3 p - e_3 u_{0r} + \alpha x_{23} p + \alpha e_{23} u_{0r}) + \\ & k_1 e_1 \dot{u}_{0r}] + \eta[(u_2 - u_{2r} - x_2 p - e_2 u_{0r}) + \\ & k_3 u_{0r}(x_1 p + e_1 u_{0r} + \alpha x_{41} p + \alpha e_{41} u_{0r}) + \\ & k_3 e_3 \dot{u}_{0r}] + p\dot{p} + \Lambda\tilde{\alpha}\dot{\tilde{\alpha}} = \\ & - k_1 \alpha e_1^2 u_{0r}^2 - k_3 \alpha e_3^2 u_{0r}^2 + p[k_0 e_0 - e_1 x_3 + \\ & e_3 x_1 + \hat{\alpha}(e_1 x_{23} + e_3 x_{41}) + (x_4 \xi - x_2 \eta) + \\ & u_{0r}(-k_1 x_3 \xi + k_3 x_1 \eta) + \hat{\alpha} u_{0r}(k_1 x_{23} \xi + k_3 x_{41} \eta) + \dot{p}] + \\ & \tilde{\alpha}[(e_1 x_{23} + e_3 x_{41}) p + u_{0r}(e_1 \xi + e_3 \eta) + \\ & u_{0r} p(k_1 x_{23} \xi + k_3 x_{41} \eta) + u_{0r}^2(k_1 e_{23} \xi + k_3 e_{41} \eta) - \\ & \Lambda\dot{\hat{\alpha}}] + \xi(u_1 - u_{1r} + e_4 u_{0r} + \hat{\alpha} e_1 u_{0r} - \\ & k_1 e_3 u_{0r}^2 + k_1 \hat{\alpha} e_{23} u_{0r}^2 + k_1 e_1 \dot{u}_{0r}) + \\ & \eta(u_2 - u_{2r} - e_2 u_{0r} + \hat{\alpha} e_3 u_{0r} + \\ & k_3 e_1 u_{0r}^2 + k_3 \hat{\alpha} e_{41} u_{0r}^2 + k_3 e_3 \dot{u}_{0r}) \end{aligned}$$

where α is a constant and $\dot{\tilde{\alpha}} = -\dot{\hat{\alpha}}$.

Take the adaptive law and dynamic feedback controller as follows

$$\begin{cases} \dot{\hat{\alpha}} = \Lambda^{-1}[(e_1 x_{23} + e_3 x_{41}) p + u_{0r}(e_1 \xi + e_3 \eta) + \\ \qquad u_{0r} p(k_1 x_{23} \xi + k_3 x_{41} \eta) + \\ \qquad u_{0r}^2(k_1 e_{23} \xi + k_3 e_{41} \eta)] \\ \dot{p} = -k_5 p - k_0 e_0 + e_1 x_3 - e_3 x_1 - \\ \qquad \hat{\alpha}(e_1 x_{23} + e_3 x_{41}) - (x_4 \xi - x_2 \eta) - \\ \qquad u_{0r}(-k_1 x_3 \xi + k_3 x_1 \eta) - \hat{\alpha} u_{0r}(k_1 x_{23} \xi + k_3 x_{41} \eta) \end{cases} \tag{18}$$

$$\begin{cases} u_0 = u_{0r} + p \\ u_1 = -k_2 \xi + u_{1r} - e_4 u_{0r} - \hat{\alpha} e_1 u_{0r} + k_1 e_3 u_{0r}^2 - \\ \qquad k_1 \hat{\alpha} e_{23} u_{0r}^2 - k_1 e_1 \dot{u}_{0r} \\ u_2 = -k_4 \eta + u_{2r} + e_2 u_{0r} - \hat{\alpha} e_3 u_{0r} - k_3 e_1 u_{0r}^2 - \\ \qquad k_3 \hat{\alpha} e_{41} u_{0r}^2 - k_3 e_3 \dot{u}_{0r} \end{cases} \tag{19}$$

One obtains

$$\dot{V} = -k_1 \alpha e_1^2 u_{0r}^2 - k_3 \alpha e_3^3 u_{0r}^2 - k_2 \xi^2 - k_4 \eta^2 - k_5 p^2 \tag{20}$$

where k_i $(i = 1, 2, 3, 4, 5)$ are positive constant control gains.

Now, in order to prove the convergence of e_i $(i = 0, 1, 2, 3, 4)$, an important lemma is introduced as follows. It is called "the extended Barbalat theorem". Its proof was given in [23].

Lemma 1. If the differentiable function $f(t)$ has a finite limit as $t \to \infty$, and $\dot{f}(t)$ can be divided into two parts, one is uniformly continuous and the other is convergent to zero as $t \to \infty$, then $\dot{f}(t) \to 0$, and the part of the uniform continuity tends to zero, too[23].

Theorem 1. Under Assumptions 1 ∼ 3, the adaptive law (18) and dynamic feedback controller (19) can guarantee that all the variables of the closed-loop system (15), (18) and (19) are bounded. In addition, p and kinematic tracking errors e_i $(i = 0, 1, 2, 3, 4)$ asymptotically converge to zero.

Proof. Considering (17), (20) and Assumptions 1 ∼ 3, we find that the Lyapunov function $V(t)$ is nonincreasing and converges to a limiting value ($\lim V(t) \geq 0$). This means that e_1, e_3, ξ, η and p are all bounded. Then, e_2, e_4 are all bounded too by using (16). x_i $(i = 1, 2, 3, 4)$ are bounded further by Assumption 3. In view of (15), (18) and (19) again, we have $\dot{e}_i$ $(i = 0, 1, 2, 3)$, $\dot{\xi}$, $\dot{\eta}$, $\dot{p}$, u_0, u_1, u_2 and $\ddot{V}$ are all bounded too. So, $\dot{V}$ is uniformly continuous. By using Lemma 1, we obtain that $\dot{V}$ tends to zero and

$$\lim_{t\to\infty} (e_1 u_{0r},\ e_3 u_{0r},\ \xi,\ \eta,\ p) = \mathbf{0}$$

Consider that u_{0r} is bounded and $u_{0r} \neq 0$ in Assumption 3. We have $(e_1, e_3) \to \mathbf{0}$ as $t \to \infty$. By the definitions of ξ and η, one obtains $(e_2, e_4) \to \mathbf{0}$ as $t \to \infty$. Using the Extended Barbalat Theorem in Lemma 1 on the second equation of (18), we have $\dot{p} \to 0$ and $k_0 e_0 \to 0$ where k_0 is a bounded control gain. Hence, we have $e_i \to 0$ $(i = 0, 1, 2, 3, 4)$ asymptotically as $t \to \infty$. □

Remark 6. For the uncertain chained system (12), the tracking problem can be solved by using the control law u_0, u_1 and u_2. However, in practice, the controllers usually are

v_1, v_2 and v_3 in the robot running place for (1) or (8). By using (8) and (10), they can be deduced as follows.

$$\begin{cases} v_1 = \frac{u_0}{\sin(\beta_2-\beta_1)} \\ v_2 = \frac{[u_2\sin\beta_1 - u_1\cos\beta_1 - (x_2\sin\beta_1 + x_4\cos\beta_1)u_0]\sin(\beta_2-\beta_1)}{2L\sin\beta_2} \\ v_3 = \frac{[u_2\sin\beta_2 - u_1\cos\beta_2 - (x_2\sin\beta_2 + x_4\cos\beta_2)u_0]\sin(\beta_2-\beta_1)}{2L\sin\beta_1} \\ \dot\beta_1 = v_2 \\ \dot\beta_2 = v_3 \end{cases} \tag{21}$$

Theorem 2. Under the Assumptions $1\sim3$, $e_i \to 0$ $(i = 0, 1, 2, 3, 4)$ ensure the trajectory (x, y, θ) of type $(1, 2)$ mobile robot in the task-place tracking the reference trajectory (x_r, y_r, θ_r) by using the controllers (18) and (19), or using control law (18), (19) and (21).

Proof. For (10), we have

$$\begin{cases} x_{0r} = \theta_r \\ x_{1r} = x_{mr}\cos\theta_r + y_{mr}\sin\theta_r \\ x_{2r} = -x_{mr}\sin\theta_r + y_{mr}\cos\theta_r - 2L\dfrac{\sin\beta_{1r}\sin\beta_{2r}}{\sin(\beta_{2r}-\beta_{1r})} \\ x_{3r} = x_{mr}\sin\theta_r - y_{mr}\cos\theta_r \\ x_{4r} = x_{mr}\cos\theta_r + y_{mr}\sin\theta_r - L\dfrac{\sin(\beta_{1r}+\beta_{2r})}{\sin(\beta_{2r}-\beta_{1r})} \\ u_{0r} = v_{1r}\sin(\beta_{2r}-\beta_{1r}) \\ u_{1r} = -x_{4r}v_{1r}\sin(\beta_{2r}-\beta_{1r}) - \\ \qquad \dfrac{2Lv_{2r}\sin^2\beta_{2r}}{\sin^2(\beta_{2r}-\beta_{1r})} + \dfrac{2Lv_{3r}\sin^2\beta_{1r}}{\sin^2(\beta_{2r}-\beta_{1r})} \\ u_{2r} = x_{2r}v_{1r}\sin(\beta_{2r}-\beta_{1r}) - \\ \qquad \dfrac{Lv_{2r}\sin(2\beta_{2r})}{\sin^2(\beta_{2r}-\beta_{1r})} + \dfrac{Lv_{3r}\sin(2\beta_{1r})}{\sin^2(\beta_{2r}-\beta_{1r})} \end{cases} \tag{22}$$

Considering (4), we have

$$\begin{bmatrix} x_{mr} \\ y_{mr} \end{bmatrix} = \alpha H(\theta_0)\left[\begin{bmatrix} x_r \\ y_r \end{bmatrix} - \begin{bmatrix} c_x \\ c_y \end{bmatrix}\right] + \begin{bmatrix} O_{c1} \\ O_{c2} \end{bmatrix} \tag{23}$$

Equation (4) minus (23) gives

$$\begin{bmatrix} x - x_r \\ y - y_r \end{bmatrix} = \alpha^{-1}H^{-1}(\theta_0)\begin{bmatrix} x_m - x_{mr} \\ y_m - y_{mr} \end{bmatrix} \tag{24}$$

where $H^{-1}(\theta_0) = H^{\rm T}(\theta_0)$.

Consider the second and forth equations in (10). We have

$$\begin{bmatrix} x_m \\ y_m \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix}\begin{bmatrix} x_1 \\ x_3 \end{bmatrix} \tag{25}$$

$$\begin{bmatrix} x_{mr} \\ y_{mr} \end{bmatrix} = \begin{bmatrix} \cos\theta_r & \sin\theta_r \\ \sin\theta_r & -\cos\theta_r \end{bmatrix}\begin{bmatrix} x_{1r} \\ x_{3r} \end{bmatrix} \tag{26}$$

It is obvious that

$$\begin{bmatrix} x_1 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_{1r} + e_1 \\ x_{3r} + e_3 \end{bmatrix} = \begin{bmatrix} x_{1r} \\ x_{3r} \end{bmatrix} + \begin{bmatrix} e_1 \\ e_3 \end{bmatrix} \tag{27}$$

Substracting (26) from (25), we obtain the following relationship

$$\begin{bmatrix} x_m - x_{mr} \\ y_m - y_{mr} \end{bmatrix} = \begin{bmatrix} e_1\cos\theta + e_3\sin\theta \\ e_3\sin\theta - e_3\cos\theta \end{bmatrix} + 2\sin\frac{e_0}{2}\begin{bmatrix} -x_{1r}\sin\left(\theta_r + \dfrac{e_0}{2}\right) + x_{3r}\cos\left(\theta_r + \dfrac{e_0}{2}\right) \\ x_{1r}\cos\left(\theta_r + \dfrac{e_0}{2}\right) + x_{3r}\sin\left(\theta_r + \dfrac{e_0}{2}\right) \end{bmatrix} \tag{28}$$

Then, for (24), we have

$$\begin{bmatrix} x - x_r \\ y - y_r \end{bmatrix} = \frac{1}{\alpha}\begin{bmatrix} e_1\cos(\theta-\theta_0) + e_3\sin(\theta-\theta_0) \\ e_1\sin(\theta-\theta_0) - e_3\cos(\theta-\theta_0) \end{bmatrix} + \frac{2}{\alpha}\sin\frac{e_0}{2}\begin{bmatrix} -x_{1r}\sin\left(\theta-\theta_0-\frac{e_0}{2}\right) + x_{3r}\cos\left(\theta-\theta_0-\frac{e_0}{2}\right) \\ x_{1r}\cos\left(\theta-\theta_0-\frac{e_0}{2}\right) + x_{3r}\sin\left(\theta-\theta_0-\frac{e_0}{2}\right) \end{bmatrix} \tag{29}$$

Note that $\theta = x_0 = x_{0r} + e_0 = \theta_r + e_0$. Then, we have $\theta \to \theta_r$, $\sin\frac{e_0}{2} \to 0$ and $|\frac{2}{\alpha}\sin\frac{e_0}{2}| \le \frac{|e_0|}{\alpha} \to 0$ because of $e_i \to 0$ $(i = 0, 1, 2, 3)$. For $\sin(\theta_r + \frac{e_0}{2})$, $\cos(\theta_r + \frac{e_0}{2})$, x_{1r}, x_{3r}, $\cos(\theta-\theta_0)$, $\sin(\theta-\theta_0)$, $\sin(\theta-\theta_0-\frac{e_0}{2})$, $\cos(\theta-\theta_0-\frac{e_0}{2})$ are all bounded. Therefore, $(x_m, y_m) \to (x_{mr}, y_{mr})$ and $(x, y) \to (x_r, y_r)$ by relationships (28) and (29). □

To sum up, under the Assumptions $1\sim3$, the trajectory (x, y, θ) for type $(1, 2)$ mobile robot in the task-space can track the reference trajectory of (x_r, y_r, θ_r) by using controller (18), (19) and (21). Simulation results are addressed in the next section.

4 Simulation

In this section, the simulations have been implemented mainly for the states e_i $(i = 0, 1, 2, 3, 4)$ of the error system (15), the adaptive law (18), the control law (19), the tracking errors $(e_{x_m}, e_{y_m}, e_{\theta_m}) = (x_m - x_{mr}, y_m - y_{mr}, \theta_m - \theta_{mr})$ in the image frame, and the tracking errors $(e_x, e_y, e_\theta) = (x - x_r, y - y_r, \theta - \theta_r)$ in the task-space for type $(1, 2)$ mobile robot. Two cases are considered for the different choices of the bounded control gains k_i $(i = 0, 1, 2, 3, 4, 5)$. Given a sensor noise on the velocities, the tracking simulations results are also implemented and addressed in Case 3.

Case 1. Consider (12), (15), (18) and (19). Then, take the initial error value $[e_0(0), e_1(0), e_2(0), e_3(0), e_4(0)] = [0.2, 0.4, 0.1, -0.2, 0]$ for the configuration of (15). Further, choose the parameters as $\theta_0 = \pi/3$, $u_{0r} = 0.1$, $u_{1r} = 1$, $u_{2r} = 1.5$, $k_0 = 500$, $k_1 = 12$, $k_2 = 20$, $k_3 = 15$, $k_4 = 18$, $k_5 = 20$ and the control gain $\Lambda = 1$. The trajectories of error states e_i $(i = 0, 1, 2, 3, 4)$ and the control inputs u_i $(i = 0, 1, 2)$ are plotted in Figs. $2\sim4$ respectively. The estimates

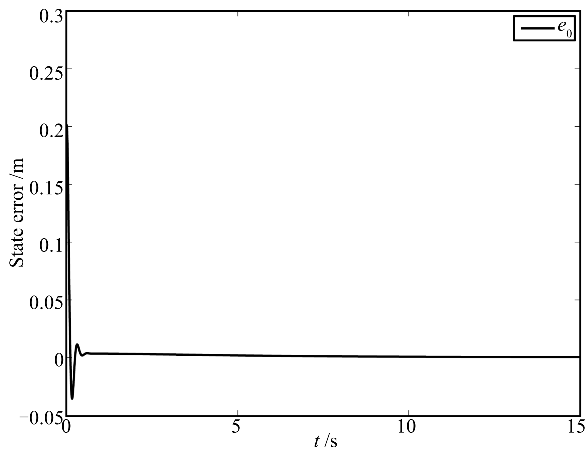

Fig. 2 The trajectory e_0 with respect to time for Case 1

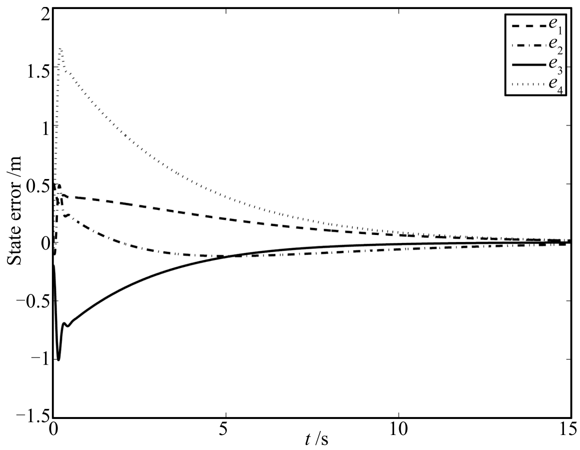

Fig. 3 The trajectories of e_i $(i = 1, \cdots, 4)$ for Case 1

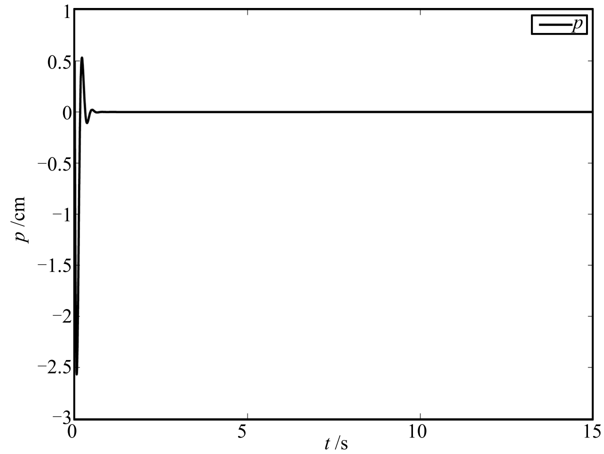

Fig. 6 The trajectory of p with respect to time for Case 1

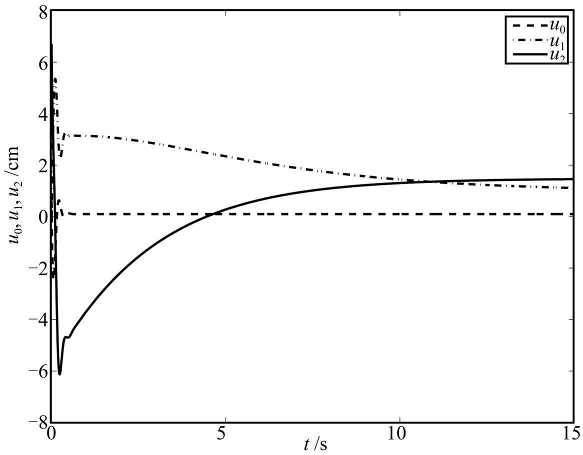

Fig. 4 The trajectories of u_i $(i = 0, 1, 2)$ for Case 1

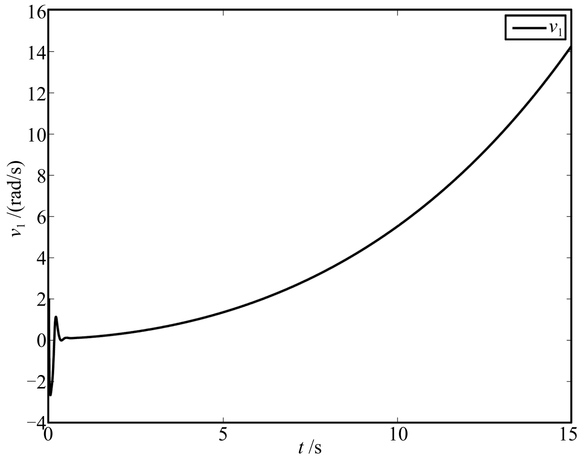

Fig. 7 The velocity v_1 of the mobile robot for Case 1

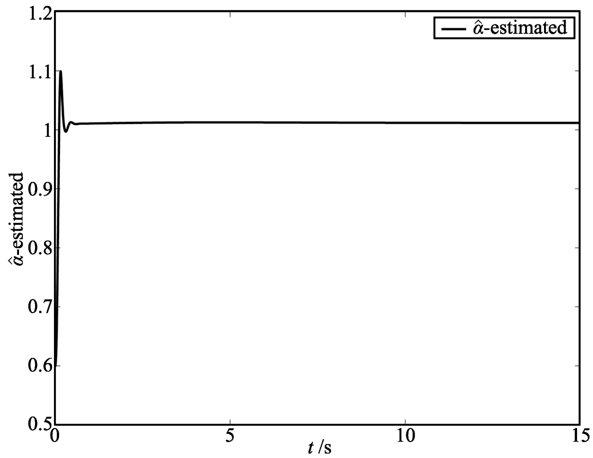

Fig. 5 The trajectory of $\hat{\alpha}$ with respect to time for Case 1

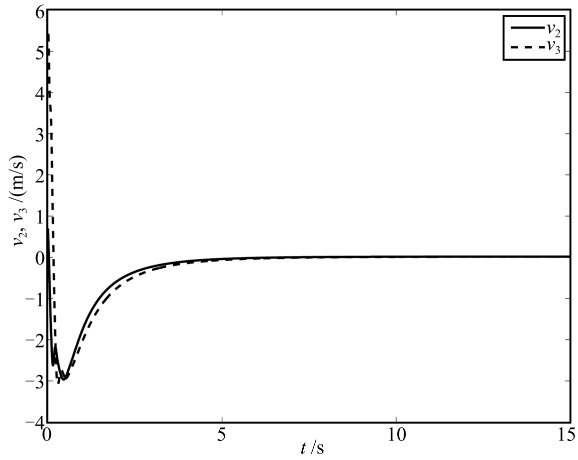

Fig. 8 The velocities v_2 and v_3 of the mobile robot for Case 1

of the parameter $\hat{\alpha}$ and the dynamic feedback factor p are plotted respectively in Figs. 5 and 6. In addition, the control laws v_1, v_2 and v_3 in the task-space are also plotted in Figs. 7 and 8 by using (12), (15), (19) and (21).

Assume that the reference trajectory of the mobile robot in the image frame is chosen as $x_{mr} = 2\cos\theta_r$, $y_{mr} = \sin\theta_r$. Then, $x_{1r} = \cos^2\theta_r$, $x_{3r} = \cos\theta_r \sin\theta_r$ by (22). The tracking error trajectories for $e_{x_m} = x_m - x_{mr}$, $e_{y_m} = y_m - y_{mr}$ and $e_{\theta_m} = \theta_m - \theta_{mr}$ in the image space are presented in Fig. 9. By using (4), (10), (15) and (29), the tracking error trajectories for $e_x = x - x_r$, $e_y = y - y_r$ and $e_\theta = \theta - \theta_r$ in the robot work-space are addressed in Fig. 10.

Case 2. Given the initial error values $[e_0(0), e_1(0), e_2(0), e_3(0), e_4(0)] = [0.2, 0.4, 0.1, -0.2, 0]$. Then, the parameters such as θ_0, u_{0r}, u_{1r}, u_{2r}, α, Λ and the reference trajectory of the mobile robot in the image frame are the same as in Case 1. However, choose $k_0 = 900$, $k_1 = 22$, $k_2 = 30$, $k_3 = 25$, $k_4 = 38$ and $k_5 = 50$. Then, e_i $(i = 0, 1, 2, 3, 4)$ are plotted in Fig. 11. The tracking error trajectories for e_{x_m}, e_{y_m} and e_{θ_m} in the image space are presented in Fig. 12. The tracking error trajectories for e_x, e_y and e_θ in the robot work-space are addressed in Fig. 13.

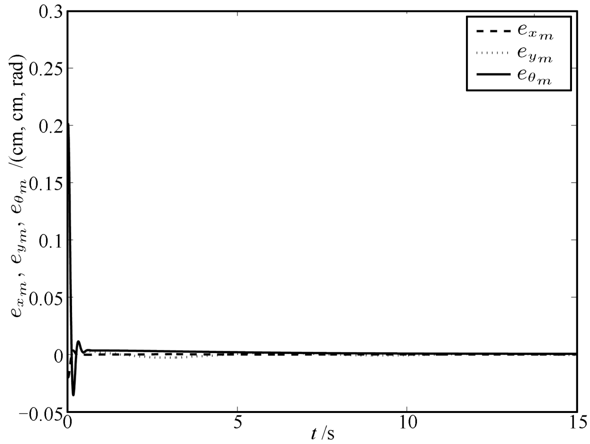

Fig. 9 The tracking error trajectories for e_{x_m}, e_{y_m} and e_{θ_m} in the image frame for Case 1

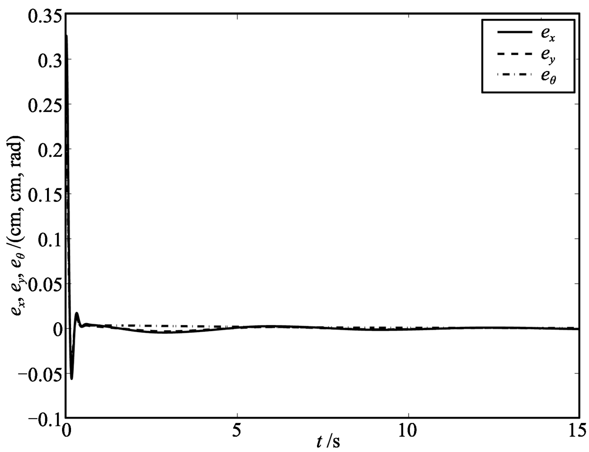

Fig. 10 The tracking error trajectories for e_x, e_y and e_θ in the robot task-space for Case 1

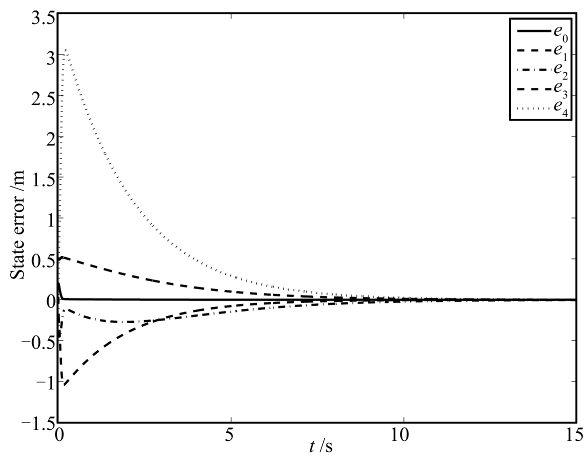

Fig. 11 The tracking errors in the robot task-space for Case 2

Case 3. For the type $(1, 2)$ mobile robot, assume $v_1 = v_2 = v_3 = 0$ in the initial state. The initial values for system (8) are $[x_m(0), y_m(0), \theta(0), \beta_1(0), \beta_2(0)] = [0, 0, 0, 0.2, 0.1]$. Given a sensor noise on the velocities $\Delta v_1 = 1$, $\Delta v_2 = 2$, $\Delta v_3 = 1$. By using (8), we have $[x_m, y_m, \theta, \beta_1, \beta_2] = [0.0165, 0.0042, -0.0046, 0.4, 0.2]$ when $t_0 = 0.1\,\text{s}$ (without loss of generality, t_0 is a finite constant). Then, $[x_0, x_1, x_2, x_3, x_4] = [-0.046, 0.0165, 0.7831, 0.0041, 5.7007]$ by (10). Take the reference trajectories in the image space $x_{mr} = 2\cos\theta_r$, $y_{mr} = \sin\theta_r$. $\beta_{1r} = \theta_r$ and $\beta_{2r} = 2\theta_r$. Considering (22), the reference states are $[x_{0r}, x_{1r}, x_{2r}, x_{3r}, x_{4r}] = [0.05, 1.9975, 0.1498, 0.0499, 7.9775]$ when $t_0 = 0.1\,\text{s}$. So, at this moment, we get new initial values $[e_0(t_0), e_1(t_0), e_2(t_0), e_3(t_0), e_4(t_0)] = [-0.056, -1.9810, 0.6333, -0.0458, -2.2096]$. Choose the controllers (18) and (19) where $k_0 = 1\,200$, $k_1 = 30$, $k_2 = 40$, $k_3 = 50$, $k_4 = 46$ and $k_5 = 100$. Then, e_i $(i = 0, 1, 2, 3, 4)$ converge to zero asymptotically. The trajectories are plotted in Fig. 14. The tracking error trajectories for e_{x_m}, e_{y_m} and e_{θ_m} in the image space are presented in Fig. 15. The tracking error trajectories for e_x, e_y and e_θ are addressed in the robot work-space in Fig. 16.

Remark 7. Comparing the tracking errors in Case 1 with those in Case 2, we find that the bigger gains k_i $(i = 0, 1, \cdots, 5)$ make the better convergence. However, they could not be big enough in the practice. Considering the tracking control problems in [16], an adaptive controller is

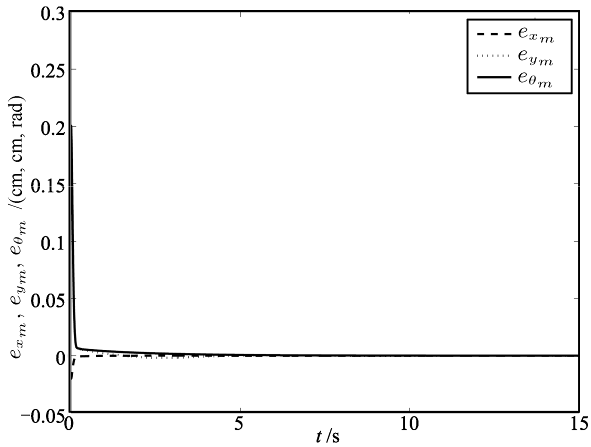

Fig. 12 The tracking errors in the image frame for Case 2

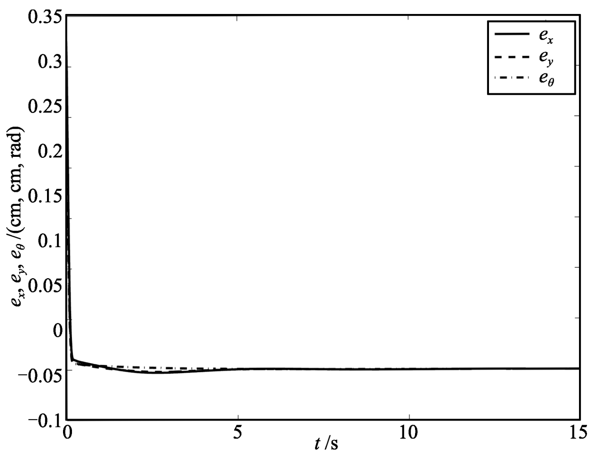

Fig. 13 The tracking error trajectories for e_x, e_y and e_θ in the robot task-space for Case 2

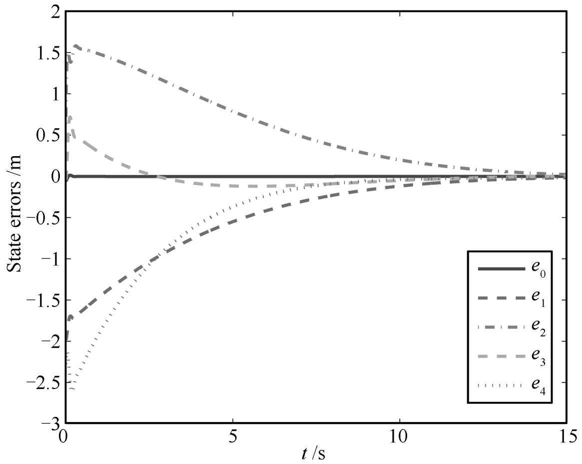

Fig. 14 The tracking errors in the robot task-space for Case 3

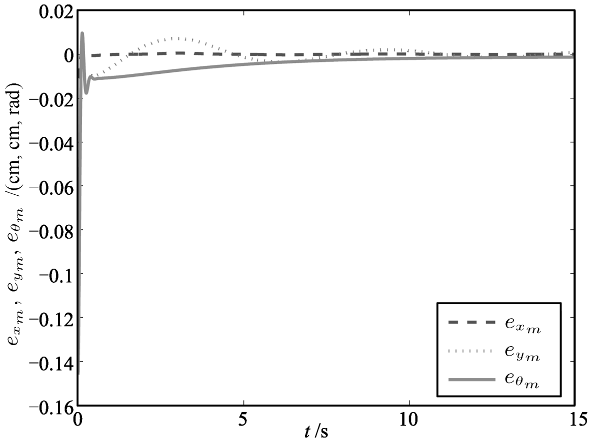

Fig. 15 The tracking errors in the image frame for Case 3

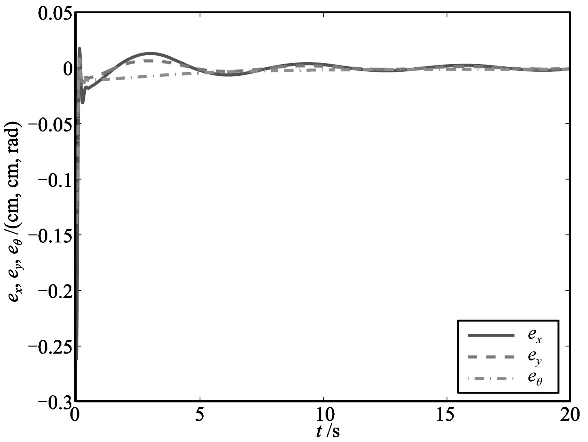

Fig. 16 The tracking error trajectories for e_x, e_y and e_θ in the robot task-space for Case 3

designed to compensate for uncertain camera and mechanical parameters in the kinematic and dynamic systems for type $(2, 0)$ mobile robot. The tracking errors for X-coordinate and Y-coordinate converged to zero within ten seconds. In our paper, two transformations are exploited based on the idea of backstepping with the help of camera-robot system. An adaptive control law and dynamic feedback robust controllers are designed to track the desired trajectory for the type $(1, 2)$ robot by using Lyapunov direct method and the extended Barbalat Lemma. The tracking errors for X-coordinate and Y-coordinate can also converge to zero within 10 seconds (see Figs. 12 and 13). Simulation results (Figs. $2 \sim 13$) demonstrate the feasibility of the proposed adaptive and dynamic feedback laws.

5 Conclusions and future work

Based on the visual servoing feedback and the transformations for the canonical chained form of type $(1, 2)$ mobile robot, we present an uncertain chained model of nonholonomic kinematic system. Then an adaptive law and dynamic feedback controller has been proposed for the kinematic error system of the nonholonomic mobile robot. The asymptotical convergence of closed-loop error system is rigorously proved by Lyapunov stability theory and the extended Barbalat Lemma. Simulation results illustrate the performance of the proposed controller.

In this paper, the adaptive dynamic feedback tracking controller is investigated for known θ_0, but $\alpha_1 = \alpha_2 = \alpha$ are unknown. As for other cases, such as θ_0, α_1 and α_2 all unknown, they will be dealt with in the future. In addition, dynamics tracking control problems with uncertain parameters are not neglected, and we will further investigate them.

References

1 Kolmanovsky I, McClamroch N H. Developments in nonholonomic control problems. *IEEE Control Systems Magazine*, 1995, **15**(6): 20−36

2 Wang C L. Semiglobal practical stabilization of nonholonomic wheeled mobile robots with saturated inputs. *Automatica*, 2008, **44**(3): 816−822

3 Leroquais W, d'Andréa-Novel B. Transformation of the kinematic models of restricted mobility wheeled mobile robots with a single platform into chain forms. In: Proceedings of the 34th Conference on Decision and Control. New Orleans, LA: IEEE, 1995. 3811−3816

4 Pang Hai-Long, Ma Bao-Li. Adaptive unified controller of arbitrary trajectory tracking for wheeled mobile robots with unknown parameters. *Control Theory and Applications*, 2014, **31**(3): 285−292 (in Chinese)

5 Cao K C. Global κ-exponential tracking control of nonholonomic systems in chained-form by output feedback. *Acta Automatica Sinica*, 2009, **35**(5): 568−576

6 Ma B L, Tso S K. Unified controller for both trajectory tracking and point regulation of second-order nonholonomic chained systems. *Robotics and Autonomous Systems*, 2008, **56**(4): 317−323

7 Campion G, Bastin G, Dandrea-Novel B. Structural properties and classification of kinematic and dynamic models of wheeled mobile Robots. *IEEE Transactions on Robotics and Automation*, 1996, **12**(1): 47−62

8 Jiang Z P. Robust exponential regulation of nonholonomic systems with uncertainties. *Automatica*, 2000, **36**(2): 189−209

9 Ma Bao-Li. Robust smooth time-varying exponential stabilization of dynamic nonholonomic mobile cart with parameter uncertainties. *Acta Automatica Sinica*, 2005, **31**(2): 314−319 (in Chinese)

10 Wang C L, Liang Z Y, Jia Q W. Dynamic feedback robust stabilization of nonholonomic mobile robots based on visual servoing. *Journal of Control Theory and Applications*, 2010, **8**(2): 139−144

11 Liang Z Y, Wang C L. Robust stabilization of nonholonomic chained form systems with uncertainties. *Acta Automatica Sinica*, 2011, **37**(2): 129−142

12 Dong W J. On trajectory and force tracking control of constrained mobile manipulators with parameter uncertainty. *Automatica*, 2002, **38**(9): 1475−1484

13 Wang Y N, Peng J Z, Sun W, Yu H S, Zhang H. Robust adaptive tracking control of robotic systems with uncertainties. *Journal of Control Theory and Applications*, 2008, **6**(3): 281−286

14 Allen P K, Timcenko A, Yoshimi B, Michelman P. Automated tracking and grasping of a moving object with a robotic hand-eye system. *IEEE Transactions on Robotics and Automation*, 1993, **9**(2): 152−165

15 Do K D, Jiang Z P, Pan J. Simultaneous tracking and stabilization of mobile robots: an adaptive approach. *IEEE Transactions on Automatic Control*, 2004, **49**(7): 1147−1152

16 Dixon W E, Dawson D M, Zergeroglu E, Behal A. Adaptive tracking control of a wheeled mobile robot via an uncalibrated camera system. *IEEE Transactions on Systems, Man, and Cybernetics — Part B: Cybernetics*, 2001, **31**(3): 341−352

17 Jia Bing-Xi, Liu Shan, Zhang Kai-Xiang, Chen Jian. Survey on robot visual servo control: vision system and control strategies. *Acta Automatica Sinica*, 2015, **41**(5): 861−873 (in Chinese)

18 Chen J, Dixon W E, Dawson M, McIntyre M. Homography-based visual servo tracking control of a wheeled mobile robot. *IEEE Transactions on Robotics*, 2006, **22**(2): 406−415

19 Wang H S, Liu Y H, Zhou D X. Dynamic visual tracking for manipulators using an uncalibrated fixed camera. *IEEE Transactions on Robotics*, 2007, **23**(3): 610−617

20 Wang C L, Mei Y C, Liang Z Y, Jia Q W. Dynamic feedback tracking control of non-holonomic mobile robots with unknown camera parameters. *Transactions of the Institute of Measurement and Control*, 2010, **32**(2): 155−169

21 Yang F, Wang C L. Adaptive tracking control for dynamic nonholonomic mobile robots with uncalibrated camera parameters. In: Proceedings of the 8th Asian Control Conference. Kaohsiung, China: IEEE, 2011. 269−274

22 Liang Z Y, Wang C L. Robust exponential stabilization of nonholonomic wheeled mobile robots with unknown visual parameters. *Journal of Control Theory and Applications*, 2011, **9**(2): 295−301

23 Samson C. Control of chained systems application to path following and time-varying point-stabilization of mobile robots. *IEEE Transactions on Automatic Control*, 1995, **40**(1): 64−77

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