Robust Nonlinear PID Controllers for Anti-windup Design of Robot Manipulators with an Uncertain Jacobian Matrix

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Abstract As for Cartesian regulation of robot manipulators with uncertain Jacobian matrix, an anti-windup scheme of robust nonlinear PID (RN-PID) controllers is proposed to solve the practical problems arising from integral action and integrator windup in PID-like control systems. Asymptotic stability is guaranteed while position and joint velocity measurements are only required; robustness of the resulting closed-loop system is also guaranteed due to the constraints acting on integral action. Especially, compared with other anti-windup approachs, the proposed algorithm is simpler and more effective for anti-windup design.

Key words Robot manipulator, robust nonlinear PID, anti-windup, robustness, uncertain Jacobian

PID controllers have existed for over half a century and can be found today in virtually all control systems. Despite of the remarkable theoretical progress and break-throughs in advanced control techniques so far, PID controllers more hold their own against harder-to-implement advanced control systems since the benefits from the latter have never been quite obvious. This is especially true under practical operating conditions of industrial control systems, where harsh and varying environment, imperfect system models, generally modest operator expertise available, and short breakdown recovery tolerance usually render the application and implementation of advanced control very difficult.

It is well known that PID controllers can effectively deal with nonlinearity and uncertainties of dynamics[1−3], and asymptotic stability is achieved accordingly[4−7]. Various schemes and their modification appear in the literature. For example, PID controller consisting of saturated-P and saturated-D plus a gravity compensation[4−5], and PID-like controller viz. linear PD plus an integral action of a nonlinear function of position errors[6] were presented recently. In the presence of Jacobian uncertainty, Cheah derived nonlinear PD plus adaptive or perfect gravity compensation[8−9], an approximate Jacobian matrix PD (PID) control law[10] and PID controllers[11] for task space set-point problem of robot manipulator; they also solved H_∞ tuning problem for PID control of task space[12]. Huang[13] presented a class of transpose Jacobian-based NPID regulators, which include a nonlinear function of errors in proportional and integral actions.

Due to the physical system restriction, output torque of robot actuator is always limited. Thus, saturation is probably the most widely encountered in control system when such limitation is not taken into acount during the controller design. As we know, actuator saturation deteriorates the performance of the control system and even leads to instability[14]. Therefore, constrained control has received more and more attention in [15−17] .

Anti-windup control is a popular approach to dealing with control saturation[18−22]. Its main idea is to design a normal controller without considering the saturation first and then take a compensation to minimize the adverse effects of the saturation on the closed-loop performance. Many anti-windup approaches have been presented based on robustness and H_∞ optimal control[23−24] and extensive numerical design algorithms have been developed based on LMI tools[25−26].

As far as PID control is concerned, integrator windup may occur since control signals are always bounded in physical systems. It results in large overshot and long setting time for a set point response. Various anti-windup schemes have been proposed to deal with integrator windup[27−34] which are classified into two different approaches[33−34]: 1) conditional integration, in which the value of integrators is frozen when certain conditions are verified; 2) back-calculation, in which the difference between the controller output and the actual plant input is fed back to the integrator. However, there have been relatively few attempts to analyze the resulting closed-loop behavior viz. stability, especially for the former approach[24]. Some conditional integration may not guarantee zero steady error for the step reference input[35]. As for back-calculation approach, the compensation for integrators is active whenever actuators are saturated; since its compensation is activated after saturation, integrator windup cannot be completely avoided. When an estimated limitation is embedded in the controller, a too "restrictive" estimation results in the oscillatory system[30]. Although a unified framework for anti-windup in [28] can be used as a design method, it is only applicable to linear time-invariant plants and as a result, it becomes practically difficult in the case of robot systems because of the high nonlinearity.

By means of some restraints on integral action, an anti-windup scheme of robust nonlinear PID controllers is proposed in this paper to tackle the integrator windup problem; actually, it provides a simple and effective alternative of anti-windup scheme for PID control. In the presence of uncertain robot Jacobian, asymptotic stability is proved and some issues including singularity, anti-windup and robustness are discussed as well.

1 Preliminaries

Throughout this paper, $\lambda_M\{\cdot\}$ ($\lambda_m\{\cdot\}$) denotes the maximal (minimal) eigenvalue of matrix. The norm of vector is defined by $\|\boldsymbol{x}\| = \sqrt{\boldsymbol{x}^{\mathrm{T}}\boldsymbol{x}}$ and the norm of matrix is defined by the corresponding induced norm $\|M\| = \sqrt{\lambda_M\{M^{\mathrm{T}}M\}}$.

On the basis of a set of all continuous piecewise-differentiable increasing functions introduced by [13, 24], it is reformulated as follows.

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Definition 1. $\mathscr{F}(\boldsymbol{m},\boldsymbol{\rho},\boldsymbol{\varepsilon},\boldsymbol{x})$ denotes the set of all continuous piecewise-differentiable increasing functions vectors $\boldsymbol{s}(\boldsymbol{x})$ (its entry is shown in Fig. 1) with constant vectors $\boldsymbol{m},\boldsymbol{\rho}$, and $\boldsymbol{\varepsilon}$, where

$$\boldsymbol{s}(\boldsymbol{x}) = [s_1(x_1) \quad s_2(x_2) \quad \cdots \quad s_n(x_n)]$$

and $m_i, \rho_i, \varepsilon_i$ $(i = 1, 2, \cdots, n)$ such that

$$\rho_i|\,x_i| \geq |\,s_i(x_i)| \geq m_i|\,x_i|, \forall x_i \in \mathbf{R} : |\,x_i| < \boldsymbol{\varepsilon}_i$$

$$\rho_i\varepsilon_i \geq |\,s_i(x_i)| \geq m_i\varepsilon_i, \forall x_i \in \mathbf{R} : |\,x_i| < \varepsilon_i$$

$$0 \leq \frac{\mathrm{d}s_i(\boldsymbol{x})}{\mathrm{d}x_i} \leq \rho_i, \forall x_i \in \mathbf{R}$$

where $\boldsymbol{x} \in \mathbf{R}^n$, $|\cdot|$ stands for the absolute value. Examples of $s_i(\boldsymbol{x})$, such as the Arimoto′s sine function, the tangent hyperbolic function, the saturated function, etc., can be found in [6−13].

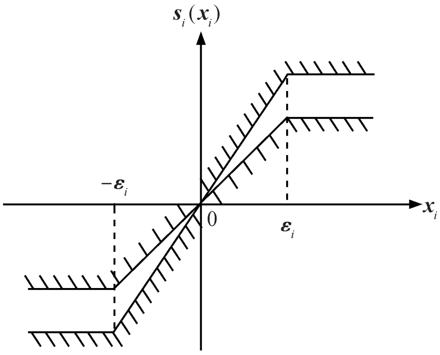


Fig. 1 Entry $s_i(\boldsymbol{x})$ of function vectors $\boldsymbol{s}(\boldsymbol{x})$

Define matrix

$$T_s(\boldsymbol{x}) = \frac{\partial \boldsymbol{s}(\boldsymbol{x})}{\partial \boldsymbol{x}^{\mathrm{T}}} = \mathrm{diag}\{\frac{\partial s_i(\boldsymbol{x})}{\partial x_i},\ i = 1, \cdots, n\} \tag{1}$$

It is easy to verify that

$$T_s(\boldsymbol{x}) \geq 0, \quad \|\boldsymbol{s}(\boldsymbol{x})\| \leq \bar{\rho}\bar{\varepsilon}\sqrt{n}, \quad \|T_s(\boldsymbol{x})\| \leq \bar{\rho} \tag{2}$$

where $\bar{\rho} = \max\limits_i\{\rho_i\}$ and $\bar{\varepsilon} = \max\limits_i\{\varepsilon_i\}$.

In the absence of friction and other disturbances, the dynamics of a rigid serial n-link robot manipulator is given in the joint space as follows

$$M(\boldsymbol{q})\,\ddot{\boldsymbol{q}} + C(\boldsymbol{q},\dot{\boldsymbol{q}})\,\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{\tau} \tag{3}$$

where $\boldsymbol{q}$ is the $n \times 1$ vector of joint angle, $M(\boldsymbol{q})$ is the $n \times n$ inertial matrix; $C(\boldsymbol{q},\dot{\boldsymbol{q}})\,\dot{\boldsymbol{q}}$, $\boldsymbol{g}(\boldsymbol{q})$ and $\boldsymbol{\tau} \in \mathbf{R}^n$ denote the centrifugal Coriolis force, the gravitational force, and control inputs, respectively.

Assumption 1. It is assumed that the robot manipulator is operating in the finite workspace Ω, where the Jacobian matrices $J(\boldsymbol{q})$ and $\hat{J}(\boldsymbol{q})$ are non-singular. Furthermore, there only exists an unknown equlibrium configuration $\boldsymbol{q}_e$ that corresponds to the desired posture $\boldsymbol{x}_d$ of end-effector in the task-space under kinematic uncertainties.

Therefore, we can define error signals as

$$\tilde{\boldsymbol{q}} = \boldsymbol{q} - \boldsymbol{q}_e, \quad \tilde{\boldsymbol{x}} = \boldsymbol{x} - \boldsymbol{x}_d \tag{4}$$

Let $\boldsymbol{x} = [\boldsymbol{\chi}^{\mathrm{T}}\ \boldsymbol{\phi}^{\mathrm{T}}]^{\mathrm{T}} \in \mathbf{R}^n$ be the actual posture in the task space, which is directly measurable, where $\boldsymbol{\chi}$ describes the end-effector position and $\boldsymbol{\phi}$ describes its orientation. Usually, Euler or RPY angle is adopted to describe the orientation. The analytical Jabobian matrix is given as

$$\dot{\boldsymbol{x}} = \frac{\partial\,\boldsymbol{\Phi}(\boldsymbol{q})}{\partial \boldsymbol{q}} \cdot \dot{\boldsymbol{q}} = J(\boldsymbol{q}) \cdot \dot{\boldsymbol{q}} \tag{5}$$

In the following, Jacobian denotes the analytical Jacobian unless otherwise noted.

A list of properties of the robot dynamic model (3) is recalled as[13, 36]

$$\lambda_m\{M(\boldsymbol{q})\}I \leq M(\boldsymbol{q}) \leq \lambda_M\{M(\boldsymbol{q})\}I \tag{6}$$

$$\boldsymbol{y}^{\mathrm{T}}[\dot{M}(\boldsymbol{q}) - 2C(\boldsymbol{q},\dot{\boldsymbol{q}})\,]\boldsymbol{y} = 0,\ \exists\ \boldsymbol{y} \in \mathbf{R}^n \tag{7}$$

There exist positive constants k_c, k_g, $\bar{k}_g$, $k_{\hat{J}}$, and $\bar{k}_{\hat{J}}$ such that

$$\|C(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}}\| \leq k_c\,\|\dot{\boldsymbol{q}}\|^2 \tag{8}$$

$$\left\|\hat{J}^{-\mathrm{T}}(\boldsymbol{q})\ [\boldsymbol{g}(\boldsymbol{q}) - \boldsymbol{g}(\boldsymbol{q}_e)]\right\| \leq k_g\,\|\tilde{\boldsymbol{x}}\| \tag{9}$$

$$\left\|\hat{J}^{-\mathrm{T}}(\boldsymbol{q})\ [\boldsymbol{g}(\boldsymbol{q}) - \boldsymbol{g}(\boldsymbol{q}_e)]\right\| \leq \bar{k}_g \tag{10}$$

$$\left\|\hat{J}^{-\mathrm{T}}(\boldsymbol{q})\ [\,I - \hat{J}(\boldsymbol{q})\hat{J}^{-\mathrm{T}}(\boldsymbol{q}_e)]\ \boldsymbol{g}(\boldsymbol{q}_e)\right\| \leq k_{\hat{J}}\,\|\tilde{\boldsymbol{x}}\| \tag{11}$$

$$\left\|\hat{J}^{-\mathrm{T}}(\boldsymbol{q})\ [\,I - \hat{J}(\boldsymbol{q})\hat{J}^{-\mathrm{T}}(\boldsymbol{q}_e)]\,\boldsymbol{g}(\boldsymbol{q}_e)\right\| \leq \bar{k}_{\hat{J}} \tag{12}$$

where $\hat{J}^{-\mathrm{T}}$ denotes $(\hat{J}^{\mathrm{T}})^{-1}$.

According to (9) ~ (12), we have the following result.

Proposition 1. For function vectors $\boldsymbol{\vartheta}(\tilde{\boldsymbol{q}})$ and $\boldsymbol{\Upsilon}(\tilde{\boldsymbol{x}})$, where

$$\boldsymbol{\vartheta}(\tilde{\boldsymbol{q}}) = \boldsymbol{g}(\boldsymbol{q}) - \boldsymbol{g}(\boldsymbol{q}_e) + [\,I - \hat{J}^{\mathrm{T}}(\boldsymbol{q})J^{-\mathrm{T}}(\boldsymbol{q}_e)]\,\boldsymbol{g}(\boldsymbol{q}_e)$$

and

$$\boldsymbol{\Upsilon}(\tilde{\boldsymbol{x}}) = \hat{J}^{-\mathrm{T}}(\boldsymbol{q})\,\boldsymbol{\vartheta}(\tilde{\boldsymbol{q}}) = \hat{J}^{-\mathrm{T}}(\boldsymbol{q})\,\boldsymbol{g}(\boldsymbol{q}) - \hat{J}^{-\mathrm{T}}(\boldsymbol{q}_e)\,\boldsymbol{g}(\boldsymbol{q}_e) \tag{13}$$

there exist constants μ_i $(i = 1, \cdots, n)$ and function vector

$$\boldsymbol{s}(\tilde{\boldsymbol{x}}) \in \mathscr{F}(\boldsymbol{m},\ \boldsymbol{\rho},\ \boldsymbol{\varepsilon},\ \tilde{\boldsymbol{x}}) \tag{14}$$

such that, for every i $(i = 1, \cdots, n)$

$$|\,\boldsymbol{\Upsilon}_i(\tilde{\boldsymbol{x}})| \leq \mu_i\,|\boldsymbol{s}_i(\tilde{\boldsymbol{x}})| \tag{15}$$

From (15), it can be easily verified that

$$\|\,\boldsymbol{\Upsilon}(\tilde{\boldsymbol{x}})\,\| \leq \bar{\mu}\,\|\boldsymbol{s}(\tilde{\boldsymbol{x}})\|\,, \int \boldsymbol{\Upsilon}^{\mathrm{T}}(\tilde{\boldsymbol{x}})\mathrm{d}\tilde{\boldsymbol{x}} \leq \bar{\mu}\int \boldsymbol{s}^{\mathrm{T}}(\tilde{\boldsymbol{x}})\mathrm{d}\tilde{\boldsymbol{x}} \tag{16}$$

where $\bar{\mu} = \max\limits_i\{\mu_i\}$.

2 RN-PID control laws

Being a physical system, the signals in control loops are always limited. Suppose that each joint actuator has a maximum torque $\bar{\boldsymbol{\tau}}_i$, namely,

$$|\boldsymbol{\tau}_i| \leq \bar{\boldsymbol{\tau}}_i, \quad i = 1, 2, \cdots, n \tag{17}$$

After the control input increases up to its upper bound, the control signal is not large enough to suppress the error; at the same time, the integral action of PID controller continuous to rise. As a result, the control signal stays at its limit for a long time before it unwinds. The problem is known as integrator or reset wind-up.

Some PID-like control or adaptive PD control with uncertain gravitational force[9−10], such as PD-like control with perfect or adaptive gravitational compensation, has been presented. Unfortunately, due to the unlimited integral, such PID-like controllers cannot deal with the problem of stability and integrator windup arising from integrator. For example, adaptive PD control with uncertain gravitational force in [9] is given as

$$\boldsymbol{\tau} = -\hat{J}^{\mathrm{T}}(\boldsymbol{q}) \cdot [K_p\, \boldsymbol{s}(\tilde{\boldsymbol{x}}) + K_v \dot{\boldsymbol{x}}\,] + Z(\boldsymbol{q})\, \hat{\boldsymbol{\theta}} \tag{18}$$

$$\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\theta}}(0) - \int_0^t L \cdot Z^{\mathrm{T}}(\boldsymbol{q}) \cdot \{\, \dot{\boldsymbol{q}} + \alpha \hat{J}^{-1}(\boldsymbol{q})\, \boldsymbol{s}(\tilde{\boldsymbol{x}})\}\mathrm{d}\zeta \tag{19}$$

where $Z(\boldsymbol{q})$ is the gravity regressor, the estimation of a set of parameters; L is a positive definite matrix. A similar result like (18) and (19) was presented in [10]. As we can see, $\boldsymbol{s}(\cdot)$ defined in [9−10] belongs to $\mathscr{F}(\boldsymbol{m}, \boldsymbol{\rho}, \boldsymbol{\varepsilon}, \cdot)$. Actually, adaptive PD control with uncertain gravitational force presented in [9−10] can be conceived as PID-like control. Therefore, PID-like controllers such as (18) and (19) are apt to give rise to integrator windup due to their unlimited integral action.

Based on the above facts, we get intuitively an idea that integrator windup may be overcome if the integral action is restrained properly. This is the basic idea of anti-windup for PID-like controllers, the corresponding control scheme is given below.

2.1 RN-PID controller

The robust nonlinear anti-windup PID control laws are proposed as

$$\boldsymbol{\tau} = -K_v \dot{\boldsymbol{q}} - \hat{J}^{\mathrm{T}}(\boldsymbol{q}) K_p \boldsymbol{s}(\tilde{\boldsymbol{x}}) - \hat{J}^{\mathrm{T}}(\boldsymbol{q}) \cdot K_I \boldsymbol{s}_I(\boldsymbol{w}) \tag{20}$$

$$\boldsymbol{w} = \int_0^t [\hat{J}(\boldsymbol{q}(\varsigma))\dot{\boldsymbol{q}}(\varsigma) + \alpha \boldsymbol{s}(\tilde{\boldsymbol{x}}(\varsigma))]\mathrm{d}\varsigma \tag{21}$$

where

$$\boldsymbol{s}(\cdot) \in \mathscr{F}(\boldsymbol{m}, \boldsymbol{\rho}, \boldsymbol{\varepsilon}, \cdot) \tag{22}$$

$$\boldsymbol{s}_I(\cdot) \in \mathscr{F}(\boldsymbol{m}_I, \boldsymbol{\rho}_I, \boldsymbol{\varepsilon}_I, \cdot) \tag{23}$$

α is a positive constant, K_p, K_v, and K_I are all positive definite matrices.

The difference between RN-PID control and other PID-like controls is that in the former, the output of integrator is shaped by function $\boldsymbol{s}_I(\cdot)$ before it enters the controller. As a result, the integral action keeps limited in the framework of (20). When the difference between $\bar{\boldsymbol{\tau}}_i$ and $\sup_{\tilde{\boldsymbol{x}}}\{\boldsymbol{s}_{I,i}(\tilde{\boldsymbol{x}})\}$, i.e., $(\bar{\boldsymbol{\tau}}_i - \sup\{\boldsymbol{s}_{I,i}(\cdot)\})$ is large enough, the control output always responds rapidly to error variation, and the control signal would not retain its limit no matter how large the integrator output is. In other words, windup does not occur in the resulting system of (20) and (21).

Remark 1. It is worthy to note a saturated linear PID controller i.e., saturated-PI plus saturated-D feedback in the joint-space[37]. Unfortunately, when PI action is saturated, the corresponding closed-loop system behaves as derivative feedback if integral prevails over the proportional action. Hence, integrator windup may come up.

2.2 Asymptotic stability of RN-PID controller

The closed-loop system dynamics is obtained by substituting the control action $\boldsymbol{\tau}$ into the dynamics of robot manipulator (3)

$$\begin{aligned} &M(\boldsymbol{q})\ddot{\boldsymbol{q}} + C(\boldsymbol{q}, \dot{\boldsymbol{q}})\,\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) - \hat{J}^{\mathrm{T}}(\boldsymbol{q})\hat{J}^{-\mathrm{T}}(\boldsymbol{q}_e)\, \boldsymbol{g}(\boldsymbol{q}_e) = \\ &-K_v \dot{\boldsymbol{q}} - \hat{J}^{\mathrm{T}}(\boldsymbol{q})\, K_p\, \boldsymbol{s}(\tilde{\boldsymbol{x}}) - \hat{J}^{\mathrm{T}}(\boldsymbol{q}) \cdot K_I\, \boldsymbol{\Psi}(\boldsymbol{\varpi}) \end{aligned} \tag{24}$$

where

$$\boldsymbol{\varpi} = \boldsymbol{w} - \boldsymbol{w}_0 \tag{25}$$

$$\boldsymbol{\Psi}(\boldsymbol{\varpi}) = \boldsymbol{s}_I(\boldsymbol{w}) + K_I^{-1}\hat{J}^{-\mathrm{T}}(\boldsymbol{q}_e) \cdot \boldsymbol{g}(\boldsymbol{q}_e) \tag{26}$$

and $\boldsymbol{w}_0$ satisfies the following condition

$$\boldsymbol{s}_I(\boldsymbol{w}_0) = -K_I^{-1}\hat{J}^{-\mathrm{T}}(\boldsymbol{q}_e) \cdot \boldsymbol{g}(\boldsymbol{q}_e) \tag{27}$$

Suppose that for every i $(i = 1, \cdots, n)$,

$$|s_{I,i}(\boldsymbol{w}_0)| \le m_{I,i} \cdot \varepsilon_{I,i} \tag{28}$$

We can define the scalar function $U(\boldsymbol{\varpi})$ as

$$\nabla_{\varpi} U(\boldsymbol{\varpi}) = K_I \boldsymbol{\Psi}(\boldsymbol{\varpi}) \tag{29}$$

Proposition 2. The scalar function $U(\boldsymbol{\varpi})$ defined in (29) satisfies

$$\begin{cases} U(\boldsymbol{\varpi}) > 0, & \text{if} \quad \boldsymbol{\varpi} \ne 0 \\ U(\boldsymbol{\varpi}) = 0, & \text{iff} \quad \boldsymbol{\varpi} = 0 \end{cases} \tag{30}$$

$$U(\boldsymbol{\varpi}) \to +\infty, \text{when } |\boldsymbol{\varpi}| \to \infty \tag{31}$$

provided that (28) is valid.

Remark 2. From the physical point of view, (28) means that the minimum of the equilibrium integral action $\boldsymbol{s}_I(\cdot)$ in controller (20) should be sufficient to eliminate the steady error.

Note that function vector $\boldsymbol{\Psi}(\boldsymbol{\varpi})$ may satisfy all conditions presented in Definition 1, but it usually is not centrosymmetric with respect to the origin. However, it is still a continuous piecewise-differentiable increasing function vector when condition (28) is valid since so does the function vector $\boldsymbol{s}_I(\cdot)$ in (23).

Theorem 1. Consider the dynamics of robot manipulator (3) with uncertain Jacobian matrix that can be described as

$$\left\| J(\boldsymbol{q}) - \hat{J}(\boldsymbol{q}) \right\| \le p \tag{32}$$

where p is a positive constant; RN-PID control law (20) and (21) make system (3) asymptotically stable with equilibrium point $[\dot{\boldsymbol{q}}^{\mathrm{T}} \;\; \tilde{\boldsymbol{x}}^{\mathrm{T}}]^{\mathrm{T}} = \boldsymbol{0}$ provided that there exist positive constant α, function vectors $\boldsymbol{s}(\tilde{\boldsymbol{x}})$ and $\boldsymbol{s}_I(\tilde{\boldsymbol{x}})$ in (22) and (23), such that the following inequalities are all satisfied

$$(K_p + \hat{J}^{-\mathrm{T}}K_v\hat{J}^{-1} - \bar{\mu}I)T_s > \alpha^2 T_s \hat{J}^{-\mathrm{T}} M \hat{J}^{-1} T_s \tag{33}$$

$$\begin{aligned} &4\alpha[K_p - \bar{\mu}\, I] > p^2[\lambda_M\{K_p\} + \\ &\qquad \alpha\lambda_M\{\hat{J}^{-\mathrm{T}}K_v\hat{J}^{-1}\} + \bar{\mu}]^2 \cdot [K_v - \alpha\eta I]^{-1} \end{aligned} \tag{34}$$

where $\bar{\mu}$ is showed in (16), T_s is defined in (1) and

$$\eta = k_c\bar{\rho}\bar{\varepsilon}\sqrt{n} \cdot \|\hat{J}^{-1}(\boldsymbol{q})\| + \bar{\rho}[p + \|\hat{J}^{\mathrm{T}}(\boldsymbol{q})\|] \cdot \|\hat{J}^{-\mathrm{T}}(\boldsymbol{q})M(\boldsymbol{q})\|_M$$

$$\|\hat{J}^{-\mathrm{T}}(\boldsymbol{q})\; M(\boldsymbol{q})\|_M = \sup_{\boldsymbol{q} \in \mathbf{R}^n}\{\|\hat{J}^{-\mathrm{T}}(\boldsymbol{q})M(\boldsymbol{q})\|\}$$

Proof. Two steps are carried out for the proof.

1) Positive definition of Lyapunov function candidate

Consider the Lyapunov function candidate

$$V(\dot{\boldsymbol{q}}, \tilde{\boldsymbol{x}}, \boldsymbol{\varpi}) = V_0(\dot{\boldsymbol{q}}, \tilde{\boldsymbol{x}}) + U(\boldsymbol{\varpi}) \tag{35}$$

where

$$\begin{aligned} V_0 = &\textstyle\int \{K_p\boldsymbol{s}(\tilde{\boldsymbol{x}}) + \hat{J}^{-\mathrm{T}}(\boldsymbol{q})\, \boldsymbol{g}(\boldsymbol{q}) - \hat{J}^{-\mathrm{T}}(\boldsymbol{q}_e)\, \boldsymbol{g}(\boldsymbol{q}_e)\, \}^{\mathrm{T}}\, \mathrm{d}\tilde{\boldsymbol{x}} + \\ &\frac{1}{2}\, \dot{\boldsymbol{q}}^{\mathrm{T}} M(\boldsymbol{q})\dot{\boldsymbol{q}} + \alpha\boldsymbol{s}^{\mathrm{T}}(\tilde{\boldsymbol{x}}) \cdot \int \hat{J}^{-\mathrm{T}}(\boldsymbol{q})\; M(\boldsymbol{q})\mathrm{d}\dot{\boldsymbol{q}} + \\ &\alpha \textstyle\int \boldsymbol{s}^{\mathrm{T}}(\tilde{\boldsymbol{x}}) \cdot \hat{J}^{-\mathrm{T}}(\boldsymbol{q})K_v\hat{J}^{-1}(\boldsymbol{q})\mathrm{d}\tilde{\boldsymbol{x}} \end{aligned} \tag{36}$$

The Hassian matrix H_V of function V_0 is

$$H_V = \begin{bmatrix} M & \alpha M \hat{J}^{-1} T_s \\ \alpha T_s \hat{J}^{-\mathrm{T}} M & (K_p + \hat{J}^{-\mathrm{T}} K_v \hat{J}^{-1} - \bar{\mu} I) T_s \end{bmatrix} \tag{37}$$

According to Lemma A1 in Appendix, $H_V > 0$ provided that (33) is valid. With the help of convex function condition, it means that $V_0 > 0$.

Resorted to (30) and (31), then $V > 0$ and hence $V(\dot{\boldsymbol{q}}, \tilde{\boldsymbol{x}}, \boldsymbol{\varpi})$ is a radially unbounded positive definite function.

2) Time derivative of Lyapunov function candidate

The time derivative of the Lyapunov function is

$$\begin{aligned}\dot{V} = & [\dot{\boldsymbol{q}} + \alpha\hat{J}^{-1}(\boldsymbol{q})\boldsymbol{s}(\tilde{\boldsymbol{x}})]^{\mathrm{T}} M\, \ddot{\boldsymbol{q}} + \alpha\dot{\tilde{\boldsymbol{x}}}^{\mathrm{T}} T_s \int \hat{J}^{-\mathrm{T}}(\boldsymbol{q}) M\, \mathrm{d}\dot{\boldsymbol{q}} + \\ & \dot{\tilde{\boldsymbol{x}}}^{\mathrm{T}}[K_p \boldsymbol{s}(\tilde{\boldsymbol{x}}) + \hat{J}^{-\mathrm{T}}(\boldsymbol{q})\, \boldsymbol{g}(\boldsymbol{q}) - \hat{J}^{-\mathrm{T}}(\boldsymbol{q}_e)\, \boldsymbol{g}(\boldsymbol{q}_e)] + \\ & \frac{1}{2}\dot{\boldsymbol{q}}^{\mathrm{T}} \dot{M} \dot{\boldsymbol{q}} + [\dot{\tilde{\boldsymbol{x}}} + \alpha \boldsymbol{s}(\tilde{\boldsymbol{x}})]^{\mathrm{T}} K_I\, \boldsymbol{\Psi}(\boldsymbol{\varpi}) + \\ & \alpha\, \dot{\tilde{\boldsymbol{x}}}^{\mathrm{T}} \cdot \hat{J}^{-\mathrm{T}}(\boldsymbol{q})\ K_v \hat{J}^{-1}(\boldsymbol{q}) \cdot \boldsymbol{s}(\tilde{\boldsymbol{x}})\end{aligned} \tag{38}$$

By means of (7), function (38) along the trajectory of the closed system (24) can be written as

$$\begin{aligned}\dot{V} = & -\dot{\boldsymbol{q}}^{\mathrm{T}} K_v \dot{\boldsymbol{q}} - \alpha\dot{\boldsymbol{q}}^{\mathrm{T}} C^{\mathrm{T}} \hat{J}^{-1} \boldsymbol{s}(\tilde{\boldsymbol{x}}) + \alpha\dot{\tilde{\boldsymbol{x}}}^{\mathrm{T}} T_s \int \hat{J}^{-\mathrm{T}} M \mathrm{d}\dot{\tilde{\boldsymbol{q}}} + \\ & \dot{\boldsymbol{q}}^{\mathrm{T}}[J - \hat{J}\,]^{\mathrm{T}} \cdot [K_p \boldsymbol{s}(\tilde{\boldsymbol{x}}) + \hat{J}^{-\mathrm{T}} \boldsymbol{g}(\boldsymbol{q}) - \hat{J}^{-\mathrm{T}}(\boldsymbol{q}_e)\, \boldsymbol{g}(\boldsymbol{q}_e)] - \\ & \alpha \boldsymbol{s}^{\mathrm{T}}(\tilde{\boldsymbol{x}})\, K_p\, \boldsymbol{s}(\tilde{\boldsymbol{x}}) + \alpha\dot{\boldsymbol{q}}^{\mathrm{T}}(J - \hat{J})^{\mathrm{T}} \hat{J}^{-\mathrm{T}} K_v \hat{J}^{-1} \boldsymbol{s}(\tilde{\boldsymbol{x}}) - \\ & \alpha \boldsymbol{s}^{\mathrm{T}}(\tilde{\boldsymbol{x}}) \cdot [\hat{J}^{-\mathrm{T}}(\boldsymbol{q})\, \boldsymbol{g}(\boldsymbol{q}) - \hat{J}^{-\mathrm{T}}(\boldsymbol{q}_e)\, \boldsymbol{g}(\boldsymbol{q}_e)]\end{aligned} \tag{39}$$

With the help of inequalities (2), (6), (8), (16), (30), and the following mathematics procedures, one has

$$\begin{aligned}& \dot{\tilde{\boldsymbol{x}}}^{\mathrm{T}} T_s \int \hat{J}^{-\mathrm{T}}(\boldsymbol{q}) M \mathrm{d}\dot{\tilde{\boldsymbol{q}}} = \\ & \dot{\boldsymbol{q}}^{\mathrm{T}}[J(\boldsymbol{q}) - \hat{J}(\boldsymbol{q}) + \hat{J}(\boldsymbol{q})]^{\mathrm{T}} \cdot T_s \cdot \int \hat{J}^{-\mathrm{T}}(\boldsymbol{q})\, M \mathrm{d}\dot{\tilde{\boldsymbol{q}}} \le \\ & \bar{\rho}\,[\, p + \|\hat{J}^{\mathrm{T}}(\boldsymbol{q})\|\,] \cdot \|\, \hat{J}^{-\mathrm{T}}(\boldsymbol{q})\, M\|_M \cdot \|\dot{\boldsymbol{q}}\|^2\end{aligned} \tag{40}$$

$$\|\, \dot{\boldsymbol{q}}^{\mathrm{T}} C^{\mathrm{T}} \hat{J}^{-1}(\boldsymbol{q})\, \boldsymbol{s}(\tilde{\boldsymbol{x}})\, \| \le k_c \bar{\rho} \bar{\varepsilon} \sqrt{n} \cdot \|\, \hat{J}^{-1}(\boldsymbol{q})\, \| \tag{41}$$

yielding

$$\begin{aligned}\dot{V} \le & -\dot{\boldsymbol{q}}^{\mathrm{T}}[K_v - \alpha\eta I]\dot{\boldsymbol{q}} - \alpha \boldsymbol{s}^{\mathrm{T}}(\tilde{\boldsymbol{x}})\ [K_p - \bar{\mu}\, I\,]\, \boldsymbol{s}(\tilde{\boldsymbol{x}}) + \\ & p[\lambda_M\{K_p\} + \alpha\lambda_M\{\hat{J}^{-\mathrm{T}} K_v \hat{J}^{-1}\} + \bar{\mu}] \cdot \|\dot{\boldsymbol{q}}\| \cdot \|\boldsymbol{s}(\tilde{\boldsymbol{x}})\| = \\ & -\frac{1}{2}\boldsymbol{y}^{\mathrm{T}} R\, \boldsymbol{y}\end{aligned} \tag{42}$$

where

$$\boldsymbol{y} = [\,\dot{\boldsymbol{q}}^{\mathrm{T}} \quad \boldsymbol{s}^{\mathrm{T}}(\tilde{\boldsymbol{x}})]^{\mathrm{T}}$$

$$R = \begin{bmatrix} 2[K_v - \alpha\eta I] & p[\lambda_M\{K_p\} + \alpha\lambda_M\{\hat{J}^{-\mathrm{T}} K_v \hat{J}^{-1}\} + \bar{\mu}]I \\ p[\lambda_M\{K_p\} + \alpha\lambda_M\{\hat{J}^{-\mathrm{T}} K_v \hat{J}^{-1}\} + \bar{\mu}]I & 2\alpha[K_p - \bar{\mu} I] \end{bmatrix}$$

Making use of Lemma A1 in Appendix again, we obtain

$$\dot{V} \le 0 \tag{43}$$

when (34) is valid.

On the basis of Lyapunov′s stability theory, the Lyapunov-stability of the equilibrium is meant by the aforementioned fact that the Lyapunov function candidate is a radially unbounded globally positive definite function and its time derivative is a negative semi-definite function. Using LaSalle invariance principle, then

$$[\boldsymbol{s}^{\mathrm{T}}(\tilde{\boldsymbol{x}})\ \ \dot{\boldsymbol{q}}^{\mathrm{T}}]^{\mathrm{T}} \to 0 \text{ as } t \to +\infty \tag{44}$$

and hence

$$[\tilde{\boldsymbol{x}}^{\mathrm{T}}\ \ \dot{\boldsymbol{q}}^{\mathrm{T}}]^{\mathrm{T}} \to 0 \text{ as } t \to +\infty \tag{45}$$

□

Remark 3. A simplified form of RN-PID controller can be employed for the controlled system

$$\boldsymbol{\tau} = -K_v\, \dot{\boldsymbol{q}} - \hat{J}^{\mathrm{T}}(\boldsymbol{q}) K_p\, \tilde{\boldsymbol{x}} - \hat{J}^{\mathrm{T}}(\boldsymbol{q}) \cdot K_I \cdot \boldsymbol{s}_I(\boldsymbol{w}) \tag{46}$$

and a similar result can be derived under some sufficient conditions. The proof can follow the similar argument and procedure. It is omitted because of the limited space.

Remark 4. Compared with PID-like controllers, the essential dissimilarity is that controller (20) introduces function $\boldsymbol{s}_I(\cdot)$ for the integrator. As a result, the integral output is restrained forcibly when it approaches to the upper bound $\boldsymbol{s}_I(\cdot)$, and hence windup would not occur under suitable parameter selection in $\boldsymbol{s}_I(\cdot)$. In other words, RN-PID control proposed in this paper has an advantage of anti-windup over others PID-like controllers.

Remark 5. Conditions (33) and (34) show the relationship between bound of Jacobian uncertainty p and the control gains. (33) indicates that the control gains should exceed some lower bound so as to suppress and eliminate the error. Besides, (34) is valid when the Jacaobian uncertainty p is within some range; in this case, increase of controller gains makes (34) valid as p becomes larger. When p goes beyond this range, (34) is never satisfied no matter what control gains are chosen; in this case, maybe performance of the resulting system will be deteriorated.

Remark 6. Note that inequalities (6), (10), and (12) hold only for robot manipulators with all revolute joints. In this case, we can always find some positive constant $\bar{\mu}$ and some function $\boldsymbol{s}(\cdot)$ in Proposition 1 so as to make (16) valid even if it is difficult to do so. After all, some conservative estimates of k_c, k_g, $\bar{k_g}$, $k_{\hat{J}}$, $\bar{k}_{\hat{J}}$ and hence conservativeness can be found to satisfy (16), (33), and (34) consequently after the careful calibration of controller parameters.

2.3 Controller tuning and its anti-windup design

The proposed controller seems to be complicated to implement in practical engineering, but its alternative form (46) can easily be implemented. In a word, the tuning procedure is composed of two steps: 1) tuning of controller parameters like the normal approach of PID; 2) choosing parameter of $\boldsymbol{s}_I(\cdot)$ so as make (28) and (52) (or (53)) below valid. Hence the know-how of PID controllers remains in some sense. The detailed analysis for parameter tuning is presented below.

Equation (28) implies that for the attaining asymptotic stability, the integral action must be large enough to overcome the gravitational force. When some (constant) disturbance $\boldsymbol{\tau}_d$ exists, the dynamics of robot manipulator can be written as

$$M(\boldsymbol{q})\, \ddot{\boldsymbol{q}} + C(\boldsymbol{q}, \dot{\boldsymbol{q}})\, \dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{\tau} + \boldsymbol{\tau}_d \tag{47}$$

then condition (28) is changed to

$$\left|s'_{I,i}(\boldsymbol{w}_0)\right| \le m_{I,i}\, \varepsilon_{I,i} \tag{48}$$

for every i, $i = 1, \cdots, n$, where

$$\boldsymbol{s}'_I(\boldsymbol{w}_0) = K_I^{-1} \hat{J}^{-\mathrm{T}}(\boldsymbol{q}_e) \cdot [\boldsymbol{\tau}_d - \boldsymbol{g}(\boldsymbol{q}_e)] \tag{49}$$

or

$$\boldsymbol{s}'_I(\boldsymbol{w}_0) = K_I^{-1} \hat{J}^{-\mathrm{T}}(\boldsymbol{q}_e) \cdot [\boldsymbol{\tau}_d + \hat{\boldsymbol{g}}(\boldsymbol{q}_e) - \boldsymbol{g}(\boldsymbol{q}_e)] \tag{50}$$

where (50) is for the case that the estimated gravity compensation is provided for the controller law (20) in practice, i.e.,

$$\boldsymbol{\tau} = \hat{\boldsymbol{g}}(\boldsymbol{q}) - K_v \dot{\boldsymbol{q}} - \hat{J}^{\mathrm{T}}(\boldsymbol{q})\, K_p\, \boldsymbol{s}_p(\tilde{\boldsymbol{x}}) - \hat{J}^{\mathrm{T}}(\boldsymbol{q})\, K_I \boldsymbol{s}_I(\boldsymbol{w}) \tag{51}$$

In the course of controller parameter tuning, parameters of function vector $\boldsymbol{s}_I(\cdot)$ in (20) are chosen to satisfy conditions in (28) or (48), so as to guarantee asymptotic stability in the case of non-disturbance or to eliminate steady-error and some constant disturbances.

On the other hand, in practice, it is always designed to assure some so-called actuator-margin $\tilde{\boldsymbol{\tau}}$ so as to guarantee the responding capability of the controller, i.e., it requires some torque margin except for the counteract of self-gravity

$$\tilde{\boldsymbol{\tau}} = \bar{\boldsymbol{\tau}} - \hat{J}^{\mathrm{T}}(\boldsymbol{q}_e)\, K_I \cdot \boldsymbol{\pi} \geq \boldsymbol{\delta} \tag{52}$$

for the case of parameters satisfied (28), or

$$\tilde{\boldsymbol{\tau}} = \bar{\boldsymbol{\tau}} - |\hat{\boldsymbol{g}}(\boldsymbol{q})|_{\max} - \hat{J}^{\mathrm{T}}(\boldsymbol{q}_e)\, K_I \cdot \boldsymbol{\pi} \geq \boldsymbol{\delta} \tag{53}$$

for the case of parameter satisfied (48), where

$$\pi_i = \rho_{I,i}\, \varepsilon_{I,i}, \quad i = 1, \cdots, n \tag{54}$$

and $\boldsymbol{\delta}$ is a positive constant vector. When either condition (52) for controller (20) and (21), or (53) for controller (21) and (51) is valid, integral action of RN-PID controller keeps away from the upper bound of actuator torque and hence integrator windup does not occur.

For the sake of presentation convenience, we suppose that, without loss of generalization, saturated functions are chosen for every entry of function vector $\boldsymbol{s}_I(\cdot)$ in the following. When outputs of integrators reach the upper bound of the each saturated function $s_{I,i}(\cdot)$ and increase, the closed-loop system with RN-PID controllers is equivalent to nonlinear PD feedback with disturbance $\boldsymbol{\tau}'_d$, where

$$\boldsymbol{\tau}'_d = \boldsymbol{\tau}_d + \hat{\boldsymbol{g}}(\boldsymbol{q}_e) - \boldsymbol{g}(\boldsymbol{q}_e) - \hat{J}^{\mathrm{T}}(\boldsymbol{q}_e) K_I \cdot \boldsymbol{\pi} \tag{55}$$

The resulting system is closed-loop feedback, instead of open-loop system of other PID control scheme at the time of integrator windup. Hence, the rapid response can be guaranteed, while the system with anti-windup scheme such as back-calculation may still suffer from windup for finite time since its compensation is always lagged and hence the integrator continues to accumulate.

3 Discussion

3.1 Limitation of the proposed law at singularity

In general, the Jacobian matrix is a function of the arm configuration $\boldsymbol{q}$ (or along with kinematic parameters for some manipulators). For a given robot manipulator (it means a given robot Jacobian $J(\boldsymbol{q})$), the estimated Jacobian $\hat{J}(\boldsymbol{q})$ can be select to keep away from some singularities that are depended on kinematic parameters. However, those singularities, which are only related to configuration $\boldsymbol{q}$, cannot be shunned regardless of any difference form selection of the Jacobian $\hat{J}(\boldsymbol{q})$. It is indicated that it is impossible to design $\hat{J}(\boldsymbol{q})$ so that it is full rank at singular points that are independent of robot kinematic parameters.

From (9) $\sim$ (12) and the proof of Theorem 1, we observe that configurations at singular points cannot be used as control target. Furthermore, coefficients such as k_g, $\bar{k}_g$, $k_{\hat{J}}$, $\bar{k}_{\hat{J}}$ become larger when the configuration approaches to these singular points; this means these singularities affect negatively the attraction domain of the controller and performance of the closed-loop system accordingly, although RN-PID control law (20), (21), or (51), is not dependent on the inverse matrix of $\hat{J}(\boldsymbol{q})$.

On the other hand, when configuration approaches the singular points, the value of $s'_{I,i}(\boldsymbol{w}_0)$ in (49) or (50) becomes larger and hence the value of $m_{I,i} \cdot \varepsilon_{I,i}$ in (48) (so does $(\rho_{I,i} \cdot \varepsilon_{I,i})$) becomes larger; as a consequence, actuator margin becomes small, which implies that effectiveness of anti-windup function of the proposed control law declines. When the margin increases up to the critical point "zero", then function of anti-windup of RN-PID loses.

3.2 Robustness of the resulting system

In the structure of PID-like controllers, the integrator is an inherently unstable device. It leads the resulting closed-loop system to instability. For example, its response to step input and a bounded signal is a ramp and an unbounded signal, respectively. In the presence of some plant uncertainties, exogenous disturbances, and poor PID parameters tuning, they lead to poor performance or even instability attributable to over action of integrator. In the framework of RN-PID controller (20), the output of integrators is shaped by the continuous piecewise-differentiable increasing function $s_{I,i}(\cdot)$, the resulting system can be conceived as PD feedback with bounded disturbance when output of integrator $\boldsymbol{w}$ in (21) exceeds the upper bound of $s_{I,i}(\cdot)$. This implies the bounded integral action for the closed-system and negative effect of over integrator action is diminished or even eliminated accordingly. As a result, performance of the closed-loop system is achieved.

In the case of practical engineering, performance indices such as rise time, peak time, maximum overshoot, setting time, steady state error, etc., are used in the controlled system analysis. Similarly, these analysis techniques can be applied to RN-PID control in terms of the tuning procedure in Section 2.3; the detailed analysis is omitted due to the space limitation. Actually, based on the aforementioned analysis, the RN-PID controller has the desirable feature and advantages over a standard PID-like one:

1) The proposed control tackles the problem of windup of PID-like control system. In comparison with other approaches, RN-PID control design provides a simpler and more effective alternative scheme for anti-windup; unlike some anti-windup approach, conditional integration loses null steady error, asymptotic stability is achieved when parameters of integrators are chosen to satisfy (28) or (48).

2) Robustness of the closed-loop system is guaranteed even in the presence of poor PID parameter tuning, inexact knowledge of plant or disturbance. When the constraints imposed by $\boldsymbol{s}(\cdot)$ is too restrictive to make condition (28) or (43) satisfied, unlike back-calculation approach, the resulting system is practically stable. In this case, actually, the resulting closed-loop system can be conceived as a PD feedback control one if actuator-margin, i.e., (52) or (53) is still available. As for back-calculation approach, the closed-loop system is oscillatory in the case of too "restrictive" estimated limitation.

Remark 7. It seems that this RN-PID controller is similar to the result proposed by [11]: as for the latter, a limited term is included as an input of the integrator, hence the output of the integrator still goes to infinity; therefore, robustness of the latter is poor and windup is to occur.

4 Numeric example

Friction $\boldsymbol{\tau}$ is considered in robot model (3), namely

$$M(\boldsymbol{q})\,\ddot{\boldsymbol{q}} + C(\boldsymbol{q}, \dot{\boldsymbol{q}})\,\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) + \boldsymbol{\tau}_f = \boldsymbol{\tau} \tag{56}$$

The friction model of $\boldsymbol{\tau}_f$ is given as [38]:

$$\tau_{f,i} = \sigma_0 z + \sigma_1 \frac{\mathrm{d}z}{\mathrm{d}t} + \sigma_2 \dot{q}_i$$

$$\frac{\mathrm{d}z}{\mathrm{d}t} = \dot{q}_i - \frac{|\dot{q}_i| \cdot z}{h(\dot{q}_i)}$$

$$\sigma_0 h(\dot{q}_i) = f_c + (f_s - f_c) \cdot \exp\left\{ -\left(\frac{\dot{q}_i}{v_s}\right)^2 \right\}$$

where f_c is the Coulomb friction level, f_s is the level of the stiction force, and v_s is the Stribeck velocity; coefficient σ_0, σ_1, and σ_2 are constant. The following friction parameters are used in all simulations:

$$\sigma_0 = 10^5, \quad \sigma_1 = \sqrt{10^5}, \quad \sigma_2 = 0.4$$

$$f_c = 4, \quad f_s = 6, \quad v_s = 0.001$$

A two-link manipulator shown in Fig. 2 is considered, whose parameters are given as (Fig. 3 (a)):

$$M(\boldsymbol{q}) = \begin{bmatrix} 6.4 & 3.2\cos(q_2 - q_1) \\ 3.2\cos(q_2 - q_1) & 3.2 \end{bmatrix}$$

$$C(\boldsymbol{q}, \dot{\boldsymbol{q}}) = -3.2\sin(q_2 - q_1)\begin{bmatrix} 0 & \dot{q}_2 \\ \dot{q}_1 & 0 \end{bmatrix}$$

$$\boldsymbol{g}(\boldsymbol{q}, \dot{\boldsymbol{q}}) = \begin{bmatrix} 78.4\sin(q_1) \\ 4\sin(q_2) \end{bmatrix}$$

Actual length $\quad l_1 = 0.8\,\mathrm{m}, \quad l_2 = 0.8\,\mathrm{m}$
Estimation $\quad \hat{l}_1 = 0.9\,\mathrm{m}, \quad \hat{l}_2 = 0.7\,\mathrm{m}$

Cartesian coordinate reference in Fig. 2 is established. The estimated Jacobian matrix is

$$\hat{J}(q) = \begin{bmatrix} \hat{l}_1\cos q_1 & \hat{l}_2\cos q_2 \\ -\hat{l}_1\sin q_1 & -\hat{l}_2\sin q_2 \end{bmatrix}$$

The initial condition

$$\boldsymbol{x}(0) = [0.2364\ 1.5643]^{\mathrm{T}}\mathrm{m}, \quad \dot{\boldsymbol{q}}(0) = [0\ \ 0]^{\mathrm{T}}\mathrm{rad/s}$$

$$\bar{\boldsymbol{\tau}}_i = 85\,\mathrm{N\cdot m}$$

The desired position

$$\boldsymbol{x}_d = [1.2\ 0.9]^{\mathrm{T}}\mathrm{m}$$

Disturbance $\boldsymbol{\tau}_d(t)$ is a random signal (mean is zero, variance is $1(\mathrm{N\cdot m})^2$) in the first joint and square wave (amplitude is 1 N, frequency is 0.5 Hz) plus Chirp signal in the second joint.

Every entry of $\boldsymbol{s}(\tilde{\boldsymbol{x}})$ and $\boldsymbol{s}_I(\tilde{\boldsymbol{x}})$ is saturated function. Some controller parameters are chosen as $\alpha = 2$, $\boldsymbol{m} = \boldsymbol{\rho} = [20\ \ 20]$, $\boldsymbol{\varepsilon} = [0.5\ \ 0.5]$, $\boldsymbol{m}_I = \boldsymbol{\rho}_I = [3\ \ 3]$.

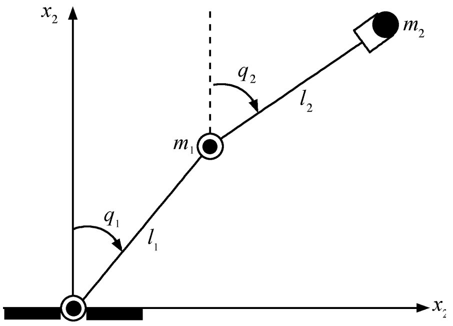
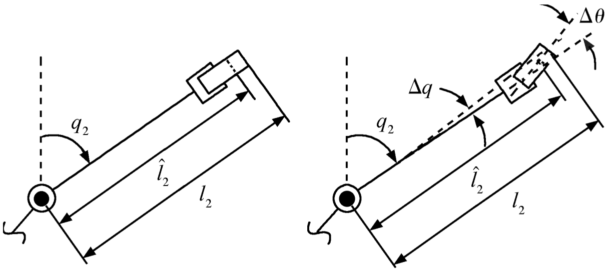


Fig. 2 The two-link robot manipulator



(a) Uncertain length (b) Uncertain length & orientation

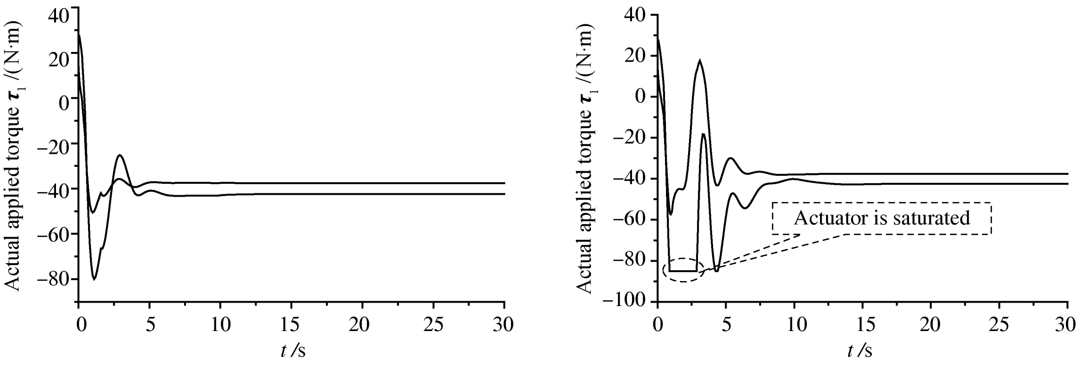
Fig. 3 Kinematic uncertainty of manipulation

Simulation results (Figs. 4 ~ 9) give comparisons between RN-PID controller (46) (denotes Sch.-A marked with "solid" curves) and transpose Jacobian-based NPID controller (denotes Sch.-B, marked with "dot". where $\varepsilon_I = \infty$, other parameters are the same as Sch.-A), with two cases:

Case 1: $K_p = 3I, K_v = 30I, K_I = I$, Sch.-$A$: $\boldsymbol{\varepsilon}_I = [5\ \ 5]$
Case 2: $K_p = 1.5I, K_v = 25I, K_I = 2I$, Sch.-$A$: $\boldsymbol{\varepsilon}_I = [2.3\ \ 2.3]$

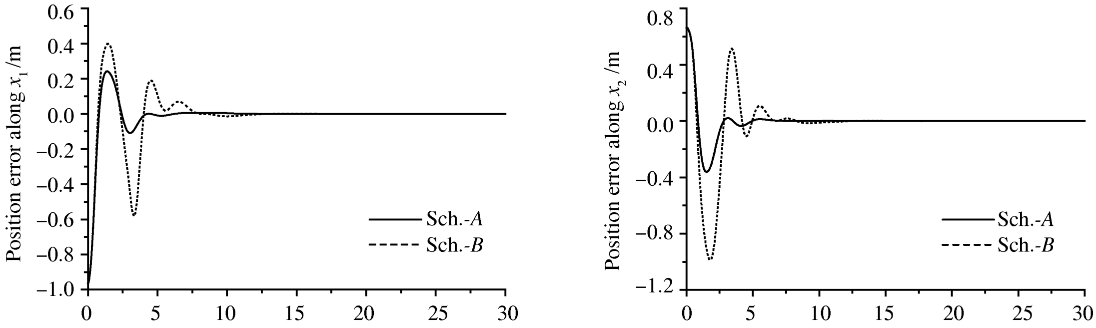
For Case 1, Fig. 4 indicates integrator windup occurs for Sch.-B when applied torque is saturated, and it results in large overshoot and long setting time in Fig. 5. So does Fig. 6, in the presence of disturbance, measurement noise and friction force. While actuators are not saturated in the case of Sch.-A, windup is prevented and performance is guaranteed.

On the other hand, when parameters are chosen as Case 2, Sch.-A makes the resulting system asymptotically stable without disturbance, noise or friction force (Fig. 7), but it



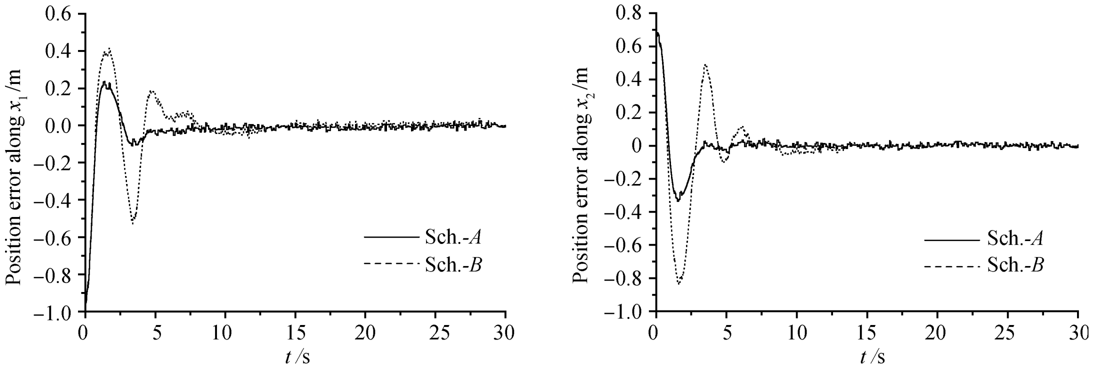
(a) Actual torque of Sch.-A (b) Actual torque of Sch.-B

Fig.4 Actual torque of Case 1 without $\boldsymbol{\tau}_d$, noise, and friction



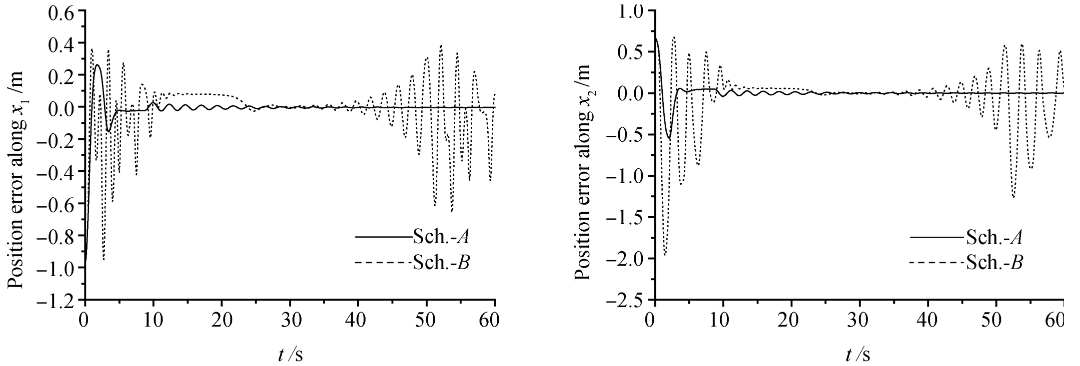
(a) Position error $\tilde{x}_1$ (b) Position error $\tilde{x}_2$

Fig.5 Comparison of Case 1 without $\boldsymbol{\tau}_d$, noise, or friction



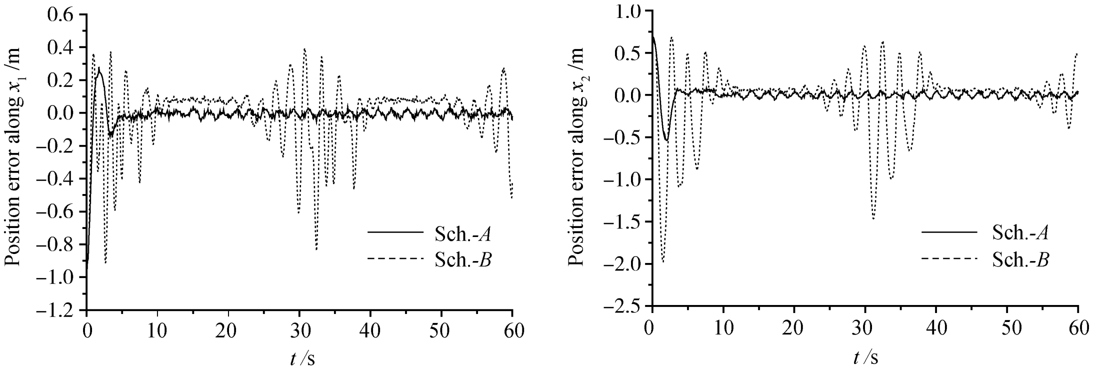
(a) Position error $\tilde{x}_1$ (b) Position error $\tilde{x}_2$

Fig.6 Comparison of Case 1 in the presence of $\boldsymbol{\tau}_d$, noise, and friction



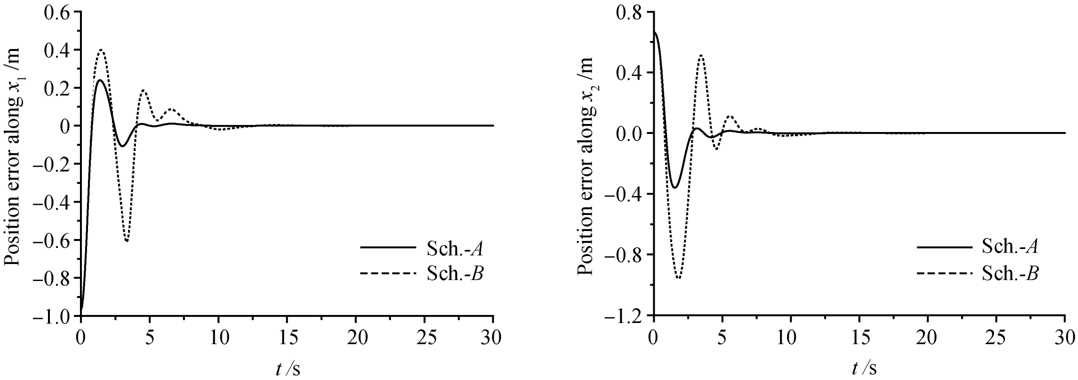
(a) Position error $\tilde{x}_1$ (b) Position error $\tilde{x}_2$

Fig.7 Comparison of Case 2 without $\boldsymbol{\tau}_d$, noise, or friction



(a) Position error $\tilde{x}_1$ (b) Position error $\tilde{x}_2$

Fig.8 Comparison of Case 2 in the presence of $\boldsymbol{\tau}_d$, noise, and friction



(a) Position error $\tilde{x}_1$ (b) Position error $\tilde{x}_2$

Fig.9 Comparison of Case 1 without $\boldsymbol{\tau}_d$, noise, or friction under length and orientation uncertainty

tends to instability for Sch.-B because of its unlimited integral action (Figs.7 ∼ 8). Meanwhile, Sch.-A makes the closed-loop system stable in the presence of disturbance, measurement noise, and friction force (Fig. 8).

In all above simulations, uncertain length is considered (see Fig. 3 (a)). If both uncertain length and uncertain orientation are taken into account (see Fig. 3 (b)), then the results in Fig. 9 shows the effectiveness of the proposed controller, in which all parameters are the same as those of Fig. 4. In Fig. 3 (b), joint angle error $\Delta q = 10°$ arises from the uncertain orientation $\Delta\theta$ of the object.

5 Conclusion

As for the task of Cartesian regulations, this paper proposes a class of robust nonlinear anti-windup PID controllers to resolve the control problem arising from integral actions and integrator windup. It is a recommendable alternative scheme for anti-windup that overcomes the shortcomings of the existing anti-windup approaches. Due to the suitable constraint on integral actions, these controllers can deal with integrator windup effectively; careful controller tuning avoids leading to windup. Asymptotic stability is guaranteed in the presence of uncertain Jacobian matrix while the measurements of joint angle, joint angle velocity and the end-effector posture in task space are only required. Besides, in the presence of poor PID tuning as well as inexact knowledge of plant or disturbance, robustness of the closed-loop system is guaranteed.

Appendix

Lemma A1. Suppose that $P = \begin{bmatrix} A & B \\ B^{\mathrm{T}} & C \end{bmatrix}$ is square. Matrix P is positive definite if and only if

$$A > 0\,,\ C - B^{\mathrm{T}}A^{-1}B > 0$$

$$\text{or}\quad C > 0\,,\ A - BC^{-1}B^{\mathrm{T}} > 0$$

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