Fault Detection System Design for Actuator of a Thermal Process Using Operator Based Approach

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Abstract This paper proposes an operator based fault detection method for an actuator fault of an aluminum plate thermal process with input constraints. Operator-based robust right coprime factorization (RCF) approach is utilized in this method. After developing a mathematical model of the thermal process, a robust tracking operator system is designed for the process with input constraints. Following this, design of the fault detection system is given by using operator-based robust RCF approach. Finally, experiment is conducted to support the proposed design method.

Key words Operator, robust right coprime factorization (RCF), fault detection, input constraints

Abstract Based on operator theory, the stability and well-posedness of the system are given for stabilizing the system by robust RCF methods. As for fault detection, a large number of interesting design methods have been researched. For example, [1−3], identities are applied to an uncertain process. The following lemmas of an RCF are employed based on [5,6]. Let us consider a set of operators called unimodular operators. Then, the feedback control system is denoted as \(\{P, K\}\) shown in Fig. 1, where \(P\) is the process and \(K\) is the controller.

The following lemmas of an RCF are employed based on the result in [4].

**Lemma 1.** Given \(\{P, K\}\), \(P = ND^{-1}\), and \(K = SR^{-1}\) are the RCF of the process and controller, respectively, then \(\{P, K\}\) is well-posed if and only if

\[
\begin{pmatrix}
D \\
-N \\
-R
\end{pmatrix}
\begin{pmatrix}
-S \\
R
\end{pmatrix}^{-1}
\]

(2)

exists and is internally stable if and only if (2) is BIBO stable.

Hence, the stability and well-posedness of the system depend on the existence and stability of operator (2).
Lemma 2. Suppose \( P = ND^{-1} \) and \( K = SR^{-1} \) such that the operators \( D, N, S, \) and \( R \) are BIBO stable. Then, these are RCFs for \( P \) and \( K \) if they satisfy (2).

Lemma 3. Suppose that Lemmas 1 and 2 are satisfied. Then, the system is overall stable if and only if the operator \( M \) is a unimodular operator, namely, \( M \in U(W, U) \).

1.2 Experimental setup

There are three main parts for the experimental setup. They are shown as follows: 1) aluminum part; 2) interface part; 3) computer part.

The first part consists of an aluminum board divided into three blocks hypothetically, that is, three thermal sensors and three heaters whose maximum outputs are 40 W. Besides, both the sensors and the heaters are fixed on three spots of those three blocks.

For measuring the temperature of an aluminum plate, thermal sensors are used and the measured temperatures are output as voltages. Then, a computer is used to convert binary data into base 10. The model of this setup is shown in Fig. 2, and the real picture of the process is shown in Fig. 3, where DIO board means digital input and output board.

Fig. 2 General diagram of the thermal process

Fig. 3 Photo of the thermal process

The configuration of the aluminum plate thermal process is shown in Fig. 4, the detailed explanations of the variables in Fig. 4 are shown in [4]. Fourier’s law of heat conduction, Newton’s law of cooling, and the equation relating heat capacity and specific heat of objects are used in the development of the mathematical model. Using these three laws, the thermal process is described as follows:

\[
y(t) = \frac{1}{cm} e^{-At} \int e^{A\tau} u_d(\tau) d\tau
\]  

\[(3)\]

Consider the nominal thermal process described by the following RCF:

\[
y(t) = P(u_d)(t) = ND^{-1}(u_d)(t)
\]  

\[(5)\]

where \( D^{-1} \) is invertible; the following two equations are given

\[
D(w)(t) = cmw(t), \quad D^{-1}(u_d)(t) = \frac{1}{cm} u_d(t)
\]  

\[(6)\]

\[
N(w)(t) = e^{-At} \int e^{A\tau} w(\tau) d\tau
\]  

\[(7)\]

where \( A \) is heat transfer coefficient. \( A \) is defined as follows:

\[
A = \frac{\alpha(4S_1 + 2S_2 + 4S_3 + S_4 + 2S_5 - S_6)}{cm}
\]  

\[(4)\]

In a real system, a thermal process necessarily contains uncertainties or disturbances, and the perturbations should have an effect on \( D \) or \( N \). Therefore, in this paper we suppose that the process \( P \) has a bounded perturbation \( \Delta P \) and assume that only \( N \) has a bounded perturbation \( \Delta N \), and the operator \( N + \Delta N \) is stable such that

\[
P + \Delta P = (N + \Delta N)D^{-1}
\]  

\[(8)\]

\[
(N + \Delta N)(w)(t) = (e^{-At} + \Delta_1) \int e^{A\tau} w(\tau) d\tau
\]  

\[(9)\]

where \( D: W \to U \) and \( N: W \to Y \) are stable operators, and the space \( W \) is called a quasi-state space of \( P \). \( D \) is invertible. \( \Delta_1 \) in (9) is regarded as an uncertainty created by the approximation of the aluminum plate in modeling. The modelling of the uncertain factor is described in [4].

Here, define that the process input \( u_d(t) \) is subject to the following constraint on its magnitude,

\[
u_d(t) = \sigma(u_1(t))
\]  

\[(10)\]

where \( u_1(t) \) is the control input before the constraint.

2 Operators’ design

To control the thermal process, two stable operators \( S : Y \to U \) and \( R : U \to U \) are required to be designed under the condition of well-posedness and that of \( N \) and
Algorithm of the fault detection system

In [4], we proposed a fault detection system with relevance to the tracking operator \( M \). One advantageous point of the method was that the fault signal in the tracking operator can be obtained without using a large number of sensors. The fault detection system can be meaningfully provided that the tracking operator works on hardware. Besides, the fault detection system proposed in this paper may be more useful than that in [4], since it is applied to an actuator fault and does not depend on what kind of the operators are, i.e., software or hardware.

For detecting fault signal, first three operators \( R_0, S_0, \) and \( D \) represented in Figs. 6 and 7 are designed.

For Fig. 5, three sorts of Bezout identities, given in (21) ~ (23), are satisfied.

\[ (SN + RD)(w)(t) = I(w)(t) \quad (21) \]
\[ (S(N + \Delta N) + RD)(w)(t) = I(w)(t) \quad (22) \]
\[ (S(N + \Delta N) + RD)(w)(t) = I(w)(t) \quad (23) \]

When \( R_0 \) and \( S_0 \) are designed to satisfy the following Bezout identity:

\[ (S_0N + R_0D)(w)(t) = I(w)(t) \quad (24) \]

\( S_0 \) and \( R_0 \) are obtained as follows:

\[ S_0(y_a)(t) = (1 - K_0) \left( \frac{dy(t)}{dt} + Ay(t) \right) \quad (25) \]
\[ R_0(u_a)(t) = \frac{K_0}{cm} u_a(t) \quad (26) \]

where \( K_0 \) is a constant. Assume that the sum of the output of \( S_0 \) and that of \( R_0 \) is a mapping from space \( W \) to \( U \). That is, the sum \( u_0 \in U \) is represented as follows:

\[ u_0 = R_0(u_a)(t) + S_0(y_a)(t) \quad (27) \]
Moreover, process input $u_d$ becomes

$$u_d = R^{-1}(e)(t) + \text{fault}$$

(28)

It may be understandable from (21) that signal $w$ is equivalent to $u$, because the Bezout identity is identity mapping from $W$ to $U$. Similarly, (24) implies that signal $w$ equals to $u_0$. However, we have not made sure whether $u = u_0$. If so, (24) can be written as

$$(S_0N + R_0D)(w)(t) = M(w)(t)$$

(29)

where $M(w)(t)$ is unimodular, though in this paper, $M(w)(t)$ is set as $I(w)(t)$. Consequently, (29) can be written as

$$L(R_0D + S_0N) = I$$

(30)

where $N, S_0$ and $R_0$ are invertible. Eventually, the difference between the actuator’s signal (before affected by the fault signal, i.e. the output of operator $R_1$) and $y_d$ results in the actuator faults. In other words, the fault signal can be detected by $\text{abs}(R^{-1}(e)(t) - y_d)$. Unless there are no actuator faults, the fault signal is 0.

4 Experiments and discussions

4.1 Experimental results

An experiment to show the effectiveness of the fault detection system is conducted. Table 1 represents parameters utilized in the simulation, and the initial temperature and desired temperature of the aluminum plate are 28.3°C and 26.8°C, respectively. Other detailed information is given in [4].

Fig. 8 shows process input and its output, and Fig. 9 expresses the output of $R^{-1}, y_d$, and detected fault signal in $0 < t \leq 1800$ s when $0 < t \leq 400$ s, the fault signal in the actuator is 0 W; when $400 < t < 500$ s, it is 5.0 W; and when $500 \leq t < 1800$ s, it is 0 W. Moreover, Figs. 10 and 11 show the comparison of $y_d$ and $u_d$, and detected fault signals, respectively. Figs. 12 and 13 are magnified figures of the process input and the detected fault signal, respectively. Finally, the output of tracking operator, $u$ is compared with $u_0$ in Fig.14, which is the sum of $R_0$ and $S_0$.

From Fig. 8, it is shown that process output almost tracks the desired temperature before the fault signal happened and after actuator fault was fixed, the temperature is back to the desired temperature. Furthermore, the designed fault detection system can detect similar fault signal added according to Figs. 11 and 13. Detailed discussion in regards to the simulation will be given in next subsection.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum power of heater</td>
<td>40 W</td>
</tr>
<tr>
<td>Reference input $r$</td>
<td>$r = 1.5$</td>
</tr>
<tr>
<td>Constant $B$</td>
<td>$B = 0.7$</td>
</tr>
<tr>
<td>Proposed gain $K_p$</td>
<td>$K_p = 3.2$</td>
</tr>
<tr>
<td>Gain for fault detection $K_f$</td>
<td>$K_f = 0.95$</td>
</tr>
<tr>
<td>Simulation time</td>
<td>$1800$ s</td>
</tr>
</tbody>
</table>

Table 1 Parameters for experiment

![Fig. 8 y and u_d](image)

![Fig. 9 Plant inputs without faults](image)

![Fig. 10 y_d and u_d](image)
4.2 Discussions

From Figs. 8 and 12, process output \( y \) tracks the desired temperature in approximately 100 s and after adding the fault signal, its output is back to the desired temperature. As for process input \( u_d \), when \( 400 < t < 500 \) s, the value of its input increases owing to the influence from the fault signal, and its value is 5 W; this result is similar to the simulation result shown in Fig. 15, where the other simulation results including control are omitted for brevity. According to the simulation result, due to feedback from process output including an actuator fault, the impact from an actuator fault seems to be mitigated. However, the actuator fault affects the actuator directly in this experiment. This may be because of process input constraints. Steady state value of process input is 0, which is equivalent to minimum input constraints. Consequently, the actuator faults cannot be reduced. Furthermore, a differentiator in the operator \( S \) affects the signal whose amplitude is 40 W for the second graph \( (u_d) \) of Fig. 8.

The third graph from the top in Figs. 9 and 11 imply the effectiveness of the designed fault detection system, since the fault signal detected and that added are similar, though the former one is slightly larger (see Fig. 13). This might be the result from gain \( K_0 \) as it is not a proper number. Another possible reason will surface when we discuss the difference between \( y_d \) and \( u_d \). From Fig. 9, the designed system can detect fault signal, as signal \( y_d \) could be as the same as signal \( u_d \) if \( w = w_0 \) is satisfied. However, according to Fig. 10, these two signals are not exactly the same. This may be the outcome of process input constraints and (31). If the input constraints are somehow considered and the equation is satisfied as \( L(R_O D + S_0 N) = I \), where \( L \) is unimodular, this problem would be solved.

Final argumentative point is with regards to the error between \( u \) and \( u_0 \). It is clear from Fig. 7 that \( u_0 \) is the sum of \( S_0(y)(t) \) and \( R_0(u_d)(t) \). From Fig. 14, signal \( u \) does not equal to signal \( u_0 \). The impulse signals result when the changes in temperature of the aluminum plate are created.
by a differentiator in operator $S_0$, which originally come from low temperature resolution of CMOS sensors. Besides, the output of operator $R_0$ is restricted by process input constraints. For these two reasons, the value of $u_0$ is different from that of $u$, and this also could lead to data mismatching between $y_d$ and $u_d$.

During the experiment, the environment temperature is fixed. As a result, the heat loss to the environment can be calculated. Based on the result in [4], including the heat loss, the uncertain factor in (10) is modeled as
\[
\Delta_1 = e^{-At} \left( \int e^{A\tau} w(\tau) d\tau \right)^{-1}
\]
The uncertain effect can be controlled and removed by stability condition (14) and fault detecting conditions (25) and (31). However, for the case of the uncertain factor being unknown, this issue will be the future work.

5 Conclusion

In this paper, using the method of an operator-based robust RCF factorization approach, a fault detection system for an actuator was considered. The effectiveness of the proposed fault detection system is confirmed by the experimental result.

References


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