Relay Switching Controller with Finite Time Tracking for Rigid Robotic Manipulators

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Abstract A global relay switching control scheme with finite time convergence (FTC) is proposed for multi-joint rigid robotic manipulator systems with uncertain dynamics. For a general finite time controller, the control signals may tend to infinity in finite time when the initial states of the system are in some specified area, causing the singularity problem of the closed-loop system and finite escape. The design scheme for finite time tracking control uses a relay switching control method so that the boundedness of the control signal is guaranteed and the singularity phenomenon is avoided. The validity of our scheme is demonstrated by simulation results.

Key words Finite time tracking, relay switching controller, terminal sliding mode (TSM), variable structure control

1 Introduction

For the tracking control of robot manipulators, many robust control strategies have been proposed in the literature (see [1] for a survey). The uncertainties in practical case of the robot manipulators are frequently encountered due to the unknown payload, friction, backlash, flexible joints, damping etc. The variable structure control approach has been widely used to deal with the uncertain systems and successfully applied to the rigid robotic manipulator systems [1∼6]. To speed up the error convergence on the sliding surface, the sliding mode parameters must be chosen such that the poles of the sliding mode dynamics are far away from the origin on the left-half of the s-plane. This will result in a high gain of the controller. In practice, the finite time tracking of the target is required for the rigid robotic manipulators. To get fast tracking error convergence, a TSM control scheme was proposed [7∼10] by employing a nonlinear switching surface, and a solution of the finite time tracking problem was derived. For multi-link rigid robotic manipulator systems with uncertainties, a robust MIMO TSM control scheme was developed in [11] and the FTC was reached by designing the controller and the switching plane variables. In [12∼14], finite time switching control problem was further investigated for the rigid robotic manipulator systems with physical constraints and obstacles using Lyapunov technique as added penalty terms. However, the control signal given in [11] could only be guaranteed bounded on the TSM surfaces. In transient process to the nonlinear switching surfaces, the singularity of closed-loop systems may occur if the initial states of the error systems are in some specified areas, resulting in the control law sufficiently large.

To avoid singularity, a new global TSM based nonsingular controller was designed in [15] for the second order rigid robotic manipulator systems. But it is not applicable to higher order systems. In this paper, a fast tracking control scheme with the FTC is proposed for an n-link rigid robotic manipulator systems with uncertainties. A relay switching control approach is given in which a linear switching plane for an augmented error system and a nonlinear switching surface are introduced. The use of the linear switching plane is to transfer the trajectories of the error dynamics to the small neighborhood of a specified fixed point in the error state space. Once the trajectory arrives at the neighborhood of the fixed point, the TSM control is activated such that the trajectories continuously move towards the nonlinear switching surface without incurring the singularity. On the nonlinear switching surface, the trajectory goes to zero in finite time. The globally asymptotic stability of the closed-loop system is guaranteed, and the output tracking error can reach zero in finite time. The proposed finite time tracking controller is robust for the system uncertainties.

1) Supported by National Natural Science Foundation of P. R. China (60174042, 69934010) and Natural Science Foundation of Shandong Province (Y2003G02)

Received May 14, 2004; in revised form September 28, 2004

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2 Problem formulation

The dynamic system of an n-joint robotic manipulator system can be described by the following equation

\[
M(q)\ddot{q} + F(q, \dot{q})\dot{q} + D\dot{q} + G(q) = u(t)
\]  

(1)

where \(q(t)\) is an \(n \times 1\) joint angular positions, \(u(t)\) is the \(n \times 1\) joint torques, \(M(q)\) is \(n \times n\) symmetric positive definite inertia matrix, \(F(q, \dot{q})\) represents the coriolis and centrifugal torques, and \(D\dot{q}\) is the viscous friction and \(G(q)\) is the gravitational torques. We assume \(u(t), q, \dot{q}\) are measurable. And the following assumptions are made:

- **A.** There exists a known positive constant \(a_1 > 0\) such that \(\lambda_{\text{min}}[M(q)^{-1}] \geq a_1 > 0\);
- **B.** An upper bound \(d_2 > 0\) of \(M(q)^{-1}\) is known;
- **C.** \(\|F(q, \dot{q})\dot{q} + D\dot{q} + G(q)\| < H(q, \dot{q}), H(q, \dot{q})\) is a known nonnegative function.

For practical robotic manipulators, the assumptions A1 ∼ A3 are reasonable. In fact, the stronger conditions are required by [16]. To make \(q, \dot{q}\) track the ideal reference model states \(q_m, \dot{q}_m\) a series of transformations for system (1) are employed. Define \(x = [q^T, \dot{q}^T]^T\); then system (1) can be expressed as

\[
\dot{x} = \begin{bmatrix} M(q)^{-1}[\ddot{t} - F(q, \dot{q})\dot{q} - G(q)] \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} M(q)^{-1} u(t)
\]

(2)

The reference model is chosen as\(^{[11]}\)

\[
\begin{bmatrix} \dot{q}_m \\ \dot{q}_m 
\end{bmatrix} = \begin{bmatrix} 0 & I \\ R & Q \end{bmatrix} \begin{bmatrix} q_m \\ \dot{q}_m 
\end{bmatrix} + \begin{bmatrix} 0 \\ B_1 \end{bmatrix} r(t) \triangleq A x_m + B r(t)
\]

(3)

where \(R, Q, B_1\) are constant matrices such that system (3) is stable, \(r(t), q_m(t), \dot{q}_m(t)\) are measurable signals, and \(x_m \triangleq [q_m^T, \dot{q}_m^T]^T\). The dynamic equation (3) is equivalent to the second order system \(\ddot{q}_m = R q_m + Q \dot{q}_m + B_1 r(t)\) with the similar form of system (1). Define a tracking error as

\[
\varepsilon(t) \triangleq q(t) - q_m(t), \quad e(t) \triangleq x(t) - x_m(t) = [\varepsilon^T, \dot{\varepsilon}^T]^T
\]

(4)

Defining \(B = [0, I]^T\), from (2) and (3), we get

\[
\begin{align*}
\dot{\varepsilon}(t) & = A \varepsilon(t) + B[M(q)^{-1}u(t) + g(q, \dot{q}, r)] \\
g(q, \dot{q}, r) & = M(q)^{-1}[\ddot{t} - F(q, \dot{q})\dot{q} - G(q)] - Rq - Q\dot{q} - B_1 r(t)
\end{align*}
\]

(5)

3 Controller design

To make \(e(t)\) reach zero in finite time to realize complete output tracking, we define a fast switching surface as

\[
s_i = \varepsilon_i + c_i \varepsilon_i^{p/q}, \quad \varepsilon = [\varepsilon_1, \cdots, \varepsilon_n]^T
\]

(7)

where \(c_i\)'s are positive constants, and \(p, q\) are odd positive integers satisfying \(p < q\) and \(2p > q\). Define

\[
S = [s_1, s_2, \cdots, s_n]^T, \quad C = \text{diag}(c_1, c_2, \cdots, c_n)
\]

(8)

The FTC control law \(u(t)\) which makes the tracking error \(e(t)\) reach zero in finite time is taken as

\[
u_i(t) = -\frac{S_i}{a_i\|S\|} \rho(t), \quad i = 1, 2, \cdots, n
\]

(9)

\[
\rho(t) = \left\|C \frac{q}{q} \text{diag}(\varepsilon_i^{p/q-1}) \varepsilon_i \right\| + \left\|B[M(q)^{-1}u(t) + g(q, \dot{q}, r)] \right\| + a_2 H(q, \dot{q}) + 1
\]

(10)

In what follows, let us analyze the closed-loop systems (5)∼(9). Choose a candidate Lyapunov function \(V\) as

\[
V = 0.5 S^T S
\]

(11)

Its time derivative along (5) together with (9) and (10) satisfies the following inequality

\[
\dot{V} = S^T \left( C \frac{q}{q} \text{diag}(\varepsilon_i^{p/q-1}) \varepsilon_i + S^T [0, I][A \varepsilon(t) + B[M(q)^{-1}u(t) + g(q, \dot{q}, r)]] \right) \leq
\]

\[
\|S^T\| \left(C \frac{q}{q} \text{diag}(\varepsilon_i^{p/q-1}) \varepsilon_i + a_2 \|S\| \|\rho(t)\| S^T\right)\times
\]

\[
\|S^T\| \left(C \frac{q}{q} \text{diag}(\varepsilon_i^{p/q-1}) \varepsilon_i + a_2 \|S\| \|\rho(t)\| S^T\right)\times
\]
\[
\|(0, I)A\hat{e}(t) - (Rq + Q\dot{q} + Br)\| \leq -\|S\| = -\sqrt{2V}
\]

(12)

Since \(V\) is positive definite, the inequality (12) implies that \(V\) reaches zero in finite time and keeps zero forever. This guarantees that \(S(t)\) tends to zero in finite time, i.e., the error dynamics \(\epsilon(t)\) of (5) reaches the fast nonlinear switching surface \(S = 0\). By (7), on the surface \(S = 0\), one obtains that

\[
\dot{\epsilon}(t) = -CE^{p/q}(t)
\]

where \(E^{p/q} = [\epsilon_1^{p/q}, \epsilon_2^{p/q}, \ldots, \epsilon_n^{p/q}]^T\). One can easily obtain that the dynamics \(\epsilon(t)\) of (13) attains zero in finite time and maintains zero forever. When \(\epsilon(t)\) is zero, its derivative \(\dot{\epsilon}(t)\) also becomes zero, henceforth \(\epsilon(t)\) converges to zero in finite time. So after a finite time, the error dynamics \(\epsilon(t)\) tends to zero. This shows that the joint angular positions of the robotic manipulator and its velocity completely track the ideal model.

**Remark 1.** The FTC controller (9) may cause the closed-loop (5)–(10) singularity. In fact, since there exists a term diag\(\epsilon_i^{p/q-1}\) in control signal (9) and \(p < q\), it is possible that diag\(\epsilon_i^{p/q-1}\) is sufficiently large if a certain \(\epsilon_i(t)\) is sufficiently small. But on the sliding surfaces, the singularity does not occur. Since in the sliding mode, \(S = 0\) implies that (13) holds. Then diag\(\epsilon_i^{p/q-1}\) can be expressed by

\[
\text{diag}(\epsilon_i^{p/q-1})\dot{\epsilon} = -[c_1\epsilon_1^{2p/q-1}, \ldots, c_n\epsilon_n^{2p/q-1}]^T
\]

(14)

This shows that each component \(c_i\epsilon_i^{2p/q-1}\) in diag\(\epsilon_i^{p/q-1}\) is bounded as \(\epsilon(t)\) is sufficiently small for \(2p > q\). Therefore, once the TSM is realized, the control law is bounded, not causing the singularity. For \(\epsilon(t)\) of the error dynamics (5) with certain specified initial value \(\epsilon(0)\), the trajectory can be divided into two stages in which it firstly moves to the switching surface in certain time interval \([0, t_1]\) and then slides along \(S = 0\) to the origin at finite time \(t_2\).

In the interval \([0, t_1]\), when \(\epsilon(t)\) moves to the switching surface, singularity may occur. In order to avoid this phenomenon, we introduce a kind of relay switching control scheme. Let us firstly select a fixed point \(\epsilon^*\) on the switching surface \(S = 0\) such that the trajectories starting from a sufficiently small neighborhood \(\Omega\) of \(\epsilon^*\) firstly reach \(S = 0\) and then move to origin along the surface \(S = 0\) under the control action (9). Here, \(\Omega\) is defined as

\[
\Omega = \{\epsilon : \|\epsilon - \epsilon^*\| < \epsilon_0 \ll 1\}
\]

(15)

It is worth noting that such a small neighborhood \(\Omega\) satisfying above requirement is existent. One can prove that under control law (9) the trajectories \(\epsilon(t)\) initiating from the open set \(\Omega\) cannot escape to infinity in finite time and tend to zero along the surface \(S = 0\). For the purpose, we assume that at some time instant \(t_0\), \(\epsilon(t_0) = \epsilon_0^*\) where \(\epsilon_0^* \in \Omega\). Under the control law (9), by (11) and (12), \(\|S(\epsilon(t))\|\) decreases and is close to zero in finite time. Since \(S(\epsilon^*) = 0\), \(\epsilon^*_0 \in \Omega\) and \(\epsilon_0 \ll 1\) is sufficiently small, \(\|S(\epsilon(t_0))\|\) is very small, which guarantees that \(\|S(\epsilon(t))\|\) is sufficiently small. Therefore, by (7), one has

\[
\dot{\epsilon}(t) = -CE^{p/q}(t) + S(t) \approx -CE^{p/q}(t)
\]

(16)

When \(\epsilon^*\) is away from the origin, \(\epsilon_0^*\) is also away from the origin owing to \(\epsilon_0 \ll 1\). This shows that the initial value \(\epsilon(t_0^*)\) is away from zero. By solving the inequalities (16) and (12) in time interval \([t_0, t]\) and considering the initial values \(\epsilon(t_0^*)\) and \(\|S(\epsilon(t_0^*))\|\), we conclude that \(\|S(\epsilon(t))\|\) reaches zero firstly before \(\epsilon_i(t)\) becomes very small. As shown in previous, on the switching surface \(S = 0\), the control signal is bounded. This illustrates that the control law \(u(t)\) given in (9) and (10) is bounded if the starting state of the trajectories \(\epsilon(t)\) is in a sufficiently small neighborhood of \(\Omega\) and in this sense the singularity is avoided.

In what follows, we will design a pre-TSM controller to make the trajectories of system (5) initiating from arbitrary point of the space \(R^n\) arrive at \(\Omega\) in finite time \(t_f\). Whenever the trajectory \(\epsilon(t)\) enters the region \(\Omega\), the control signal is switched to sliding mode control law (9). Let us construct an augmented linear system as

\[
\dot{z} = Az + Bv
\]

(17)

\[
v(t) = B^T\exp(-A^T t)G_e^{-T}(0, t_f)[\exp(-At_f)z(t_f) - z(0)]
\]

(18)
and $G_x(0, t_f)$ is the controllability Grammian matrix of linear system (15) with the form

$$G_x(0, t_f) = \int_0^{t_f} e^{-At} BB^T e^{-A^Tt} \, dt$$

(19)

and $z(0), z(t_f)$ are the initial state and final state of system (17), respectively. Based on the knowledge of linear system theory, under the control law $v(t)$ given by (18), $z(t)$ starting from any initial state vector $z(0)$ is transferred to any preset final state $z(t_f)$ at time $t_f$. Here, we let the final state $z(t_f)$ be on the nonlinear switching surface $S = 0$ and $z(t_f) = e^*$ so that if the designed pre-terminal controller drives the tracking error $e(t)$ starting from any initial state to a neighborhood of $z(t_f)(= e^*)$ at time $t_f$ then $e(t_f)$ is in the domain $\Omega$. Therefore, if at time $t_f$ the control law is switched from the pre-terminal controller to the finite time tracking controller (9), the tracking error $e(t)$ firstly reaches $S = 0$ and then converges to zero in finite time and the singularity does not occur.

The remaining task is to design the pre-terminal controller to satisfy the required conditions. Define

$$\eta(t) = e(t) - z(t), \quad \eta(0) = e(0) = \eta_0$$

(20)

(5) and (17) give rise to

$$\dot{\eta}(t) = A\eta(t) + B[M^{-1}(q)u(t) + g(q, \dot{q}, r) - v(t)]$$

(21)

Since the matrix $A$ is stable, there exists a positive definite matrix $P$ such that

$$A^TP + PA = -I$$

(22)

The pre-terminal controller is taken as

$$u(t) = -\left(\frac{\eta^T P B}{a_1 \|\eta^T P B\|}\right)^T J_v(t) + a_2 H(q, \dot{q}) + \|Rq + Q\dot{q} + B_1 r(t)\| + 1$$

(23)

where $a_1, a_2$ are the same as in the control law (9). The candidate Lyapunov function for system (21) is

$$V = 0.5\eta^T P \eta$$

(24)

The time derivative of $V$ along system (21) satisfies

$$\dot{V} = -\frac{1}{2} \eta^T \eta + \eta^T P B (M^{-1}(q)u(t) + g(q, \dot{q}, r) - v(t)) \leq -\frac{1}{2} \eta^T \eta - \|\eta^T P B\| \times \left[\|v(t)\| + a_2 H(q, \dot{q}) + \|Rq + Q\dot{q} + B_1 r(t)\| + 1\right] + \left(\eta^T P B\right)\|g(q, \dot{q}, r) - v(t)\| \leq -\frac{1}{2} \eta^T \eta \leq -\frac{1}{\lambda_{\text{max}}(P)} V$$

(25)

(25) implies that $\eta(t)$ converges to zero exponentially. This shows that there exists a finite time $T_0 = T_0(\eta_0)$ such that when $t \geq T_0$, $\|\eta(\eta_0, t)\| \leq \varepsilon_0$. In selection of the time $t_f$ in (18) and (19), one can let $t_f = T_0$ so that

$$\eta(\eta_0, t_0) = \eta(t_f) = e(t_f) - z(t_f) = e(t_f) - e^*, \quad e(t_f) \in \Omega$$

(26)

This shows that under the action of the pre-terminal control law (23), $e(t)$ enters the small neighborhood $\Omega$ of $e^*$ after time $t_f$. We consider the trajectories of (5) in time interval $[0, t_f]$ and $[t_f, \infty)$, respectively. According to the above analysis, as the time $t_f$ is fixed, the time interval $[0, t_f]$ is a finite one. (24) and (25) guarantee that $\eta(t)$ as well as $e(t)$ can not escape to infinite in $[0, t_f]$ so that all signals keep bounded in $[0, t_f]$. When $t \geq t_f$ the controller is switched to the TSM controller (9) under which the trajectory firstly arrives at the switching surface $S = 0$ and moves along this surface to the origin in finite time. Since on the switching surface $S = 0$, the TSM controller (9) has been proved to be bounded in time interval $[t_f, \infty)$ all the signals in the closed-loop are bounded.

The above analysis is summarized into the following theorem.

**Theorem.** For the tracking error system (5), if the assumption conditions A1)~A3) are satisfied, and the relay switching controller is given by (9) and (23), then the output tracking error $e(t)$ converges to zero in finite time and all the signals in closed-loop systems (1), (9) and (23) are bounded.
Now, let us recapitulate how the system evolves.

1) According to (21), design a pre-terminal controller (23) such that the trajectory of (5) enters a small neighborhood \( \Omega \) of \( z(t_f) = e^* \) in given finite time \( t_f \).

2) Design the TSM controller (9) such that the trajectory of system (5) starting from a small neighborhood \( \Omega \) of \( z(t_f) = e^* \) firstly reaches switching surface \( S = 0 \) and then moves to zero in finite time along this surface.

**Remark 2.** Since \( e^* \) is a specified point on \( S = 0 \), we let it be away from the origin so that a small neighborhood \( \Omega \) of \( e^* \) does not include the origin. However, a small \( \Omega \) means that the time for \( e(t) \) to take from the point \( e(t_f) \in \Omega \) to \( S = 0 \) is also very small. The total time from the initial instant to the final time when \( e(t) = 0 \) is divided into three parts: a) the time \( t_f \) from the initial value \( e(0) \) to \( e(t_f) \in \Omega \), relying on the initial value \( e(0) \); b) the time from \( e(t_f) \) to the point \( e(t) \in (S(e(t_f)) = 0) \); c) the sliding time \( t' \) from \( e(t_f) \) to the origin.

The proposed relay switching scheme in this paper is different from the one given in [7] in which the term \( \bar{\rho}(\rho_s) \) reaches \( S \) in given finite time \( \epsilon \), \( \bar{\rho}(\rho_s) \) is a specified point on \( S = 0 \). It is easy to check that such an \( e^* \) on the switching surface which simplifies the design of the controller.

4 Simulation example

To illustrate the effect of the controller given in this paper, a simulation example for a two-link robotic manipulator used in [11] is studied. The dynamics (1) is of the following form

\[
\begin{bmatrix}
    m_{12}(q_2)q_{12}(q_2) \\
    m_{12}(q_2)q_{22}
\end{bmatrix}
\begin{bmatrix}
    \dot{q}_1 \\
    \dot{q}_2
\end{bmatrix}
= \begin{bmatrix}
    \beta_2 q_2 \dot{q}_1^2 + 2\beta_2 q_2 \dot{q}_1 \dot{q}_2 \\
    -\beta_2 q_1 \dot{q}_2^2
\end{bmatrix}
+ \begin{bmatrix}
    \lambda_1(q_1 q_2) \dot{q}_1 \\
    \lambda_2(q_1, q_2) \dot{q}_2
\end{bmatrix}
+ \begin{bmatrix}
    u_1 \\
    u_2
\end{bmatrix}
\quad (27)
\]

where \( b_1, b_2, b_3 \) are positive constants. The parameter values in the system are selected as

\[
\begin{align*}
    r_1 = 1m, & \quad r_2 = 0.8m, \quad J_1 = 5kg \cdot m^2, \quad J_2 = 5kg \cdot m^2, \quad m_1 = 0.5kg, \quad m_2 = 1.5kg
\end{align*}
\]

Ignoring any friction torque and attenuation effects, the relay switching control algorithm was tested using Matlab toolbox. Therein, the term \( s_i/\|S\| \) is replaced by \( (s_i/(\|S\| + 0.005)) \) in order to eliminate the effects of the chattering. The reference model (3) for the manipulator to follow is defined by

\[
\begin{bmatrix}
    \dot{\bar{q}}_m \\
    \dot{\bar{q}}_m
\end{bmatrix}
= \begin{bmatrix}
    0 & I \\
    R & Q
\end{bmatrix}
\begin{bmatrix}
    \bar{q}_m \\
    \dot{\bar{q}}_m
\end{bmatrix}
+ \begin{bmatrix}
    0 \\
    B_1
\end{bmatrix}
\]

\[
r(t) = A\bar{x}_m + \bar{B}r(t), \quad A = \begin{bmatrix}
    0 & 0 & 1 & 0 \\
    0 & 0 & 0 & 1 \\
    0 & -4 & 0 & -5 \\
    0 & -4 & 0 & -5
\end{bmatrix}, \quad \bar{B} = \begin{bmatrix}
    0 & 0 \\
    0 & 0 \\
    1 & 0 \\
    0 & 1
\end{bmatrix}
\]

Here, we choose the upper bound function as \( H(q, \dot{q}) = b_1 + b_2 \|q\| + b_3 \|\dot{q}\|^2 \), \( b_1 = 35, b_2 = 1, b_3 = 8 \). It can be verified that the inequality in assumption A3 holds. We select \( a_1 = 0.05, a_2 = 2, p = 3, q = 5, c_1 = 1, t_f = 10 \). \( e^* \) is given by \( e^* = [-1, -1, 1, 1]^T \). It is easy to check that such an \( e^* \) is on the surface \( S = 0 \) defined by (7) and (8). The \( \Omega \) can be determined by \( \epsilon_0 = 0.005 \). The simulation results show that at \( t_f = 6 \), the \( e(t_f) \) is in \( \Omega \). It also can be seen from Fig. 1, Fig. 2, Fig. 5, and Fig. 6. The pre-terminal controller and FTC controller are given from Fig. 3, Fig. 4, Fig. 7, and Fig. 8. The simulations illustrate that the proposed relay switching control scheme is very effective, and the complete output tracking is achieved. In the time interval \([0, 6] \), the control signal is computed according to (23). And \( e(t) \) goes to \( \Omega \) from \( e(0) \) at time instant \( t_f = 6 \). With the convergence of \( e(t) \) to the sliding mode \( \mathbf{S} = 0 \) under the control signal (9) in approximate time interval \([6, 8, 5] \), the term \( s_i/\|S\| \) and \( \rho(t) \) in (9) and (10) keep bounded. At approximate time instant \( t = 8.5 \), the error dynamics \( e(t) \) reaches \( \mathbf{S} = 0 \), so the control
signal probably has abrupt changes. With the sliding mode control taking place and the tracking error becoming zero, the control signal tends to the steady state.

5 Conclusion

In this paper, a design scheme of variable structure relay switching controller guaranteeing the system global stability and FTC has been proposed for the n-link rigid robotic manipulator systems with unknown parameters and uncertain dynamics. Singularity phenomenon usually associated with
FTC is avoided and in this sense the control signal maintains bounded. The controller with FTC is a kind of fast response one with short transient time and can guarantee complete tracking. This is conformable to practical control objective of output tracking of rigid robotic manipulators, for instance, if it is asked that the robotic manipulator tracks a motion object in finite time, the performance with FTC is obviously superior to the one with asymptotical stability. Generally, finite time zero error tracking usually cannot be reached in practice due to the inexact switching happening around the surface. However, the proposed method in this paper is a kind of fast convergence control scheme.

References

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