Robust Dissipative Control for Linear Multi-variable Systems

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Abstract Robust quadratic dissipative control for a class of linear multi-variable systems with parameter uncertainties is considered, where the uncertainties are expressed in a linear fractional form.

Key words Dissipative, dissipative control, positive real, uncertain systems, LMI

1 Introduction

The notion of dissipative systems plays important roles in system and control theory and its applications extensively in stability analysis, nonlinear control, adaptive control system design and so on. For last decades, passive control and $H_\infty$ control have received much attention. However, passive control may lead to conservative results because only the phase information of systems is taken into account, while $H_\infty$ control can also get conservative results because only the gain information of systems is used for the control. By contrast, dissipative control may lead to less conservative results since dissipativeness is a generalization of passivity and $H_\infty$ performance, and provides a flexible tradeoff between gain and phase. This is the reason why dissipative control problems have attracted much interest in recent years.

In this paper, we will address quadratic dissipative analysis and control problems for linear systems with norm-bounded parameter uncertainties. First we will investigate conditions under which the uncertain system is robust quadratic dissipative. Then we will design both state-feedback and output feedback controllers such that the closed-loop system is robust quadratic dissipative.

The rest of the paper is organized as follows. Section 2 gives system description and preliminaries. In Section 3, we present the necessary and sufficient conditions for the uncertain system to be robust quadratic dissipative. Robust quadratic dissipative control problems will be discussed in Section 4. Finally some concluding remarks are drawn in Section 5.

2 System description and preliminaries

Consider the uncertain linear system described by

$$\dot{x}(t) = A_\Delta x(t) + B_\Delta \omega(t), \quad x(0) = 0$$

$$z(t) = C_\Delta x(t) + D_\Delta \omega(t)$$

where $x(t) \in \mathbb{R}^n$ is the state, $\omega(t) \in \mathbb{R}^m$ the exogenous input and $z(t) \in \mathbb{R}^p$ the controlled output. $A_\Delta, B_\Delta, C_\Delta$ and $D_\Delta$ are uncertain parameter matrices satisfying the following assumptions

\begin{equation}
\begin{bmatrix}
A_\Delta & B_\Delta \\
C_\Delta & D_\Delta 
\end{bmatrix} = \begin{bmatrix}
A & B \\
C & D 
\end{bmatrix} - \Delta(t) \begin{bmatrix}
H \\
H_1 
\end{bmatrix}
\end{equation}

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where $\Delta(t)$ depends on uncertain matrix $F(t) \in R^{n \times j}$:

$$\Delta(t) = F(t)[I + JF(t)]^{-1}, \quad F^T(t)F(t) \leq I, \quad J^T J < I$$

and $A, B, C, D, H, H_1, E, E_1, J$ are known constant matrices.

In the following the parameter uncertainties are said to be admissible if both (1) and (2) hold.

Now let’s recall the notion of quadratic dissipativeness. Consider the nominal system of system $(\Sigma_\Delta)$, that is, system $(\Sigma_\Delta)$ satisfying $F(t) = 0$:

$$(\Sigma_1) : \dot{x}(t) = Ax(t) + B\omega(t), \quad x(0) = 0$$

$$(z(t) = Cx(t) + D\omega(t)$$

with its transfer function matrix

$$G(s) = D + C(sI - A)^{-1}B$$

It is assumed that system $(\Sigma_1)$ is controllable and observable. Given $Q \in R^{p \times p}, S \in R^{n \times m}$ and $R \in R^{m \times m}$ with $Q = Q^T$ and $R = R^T$. Denote

$$M(s) = G^*(s)QG(s) + S^TG(s) + G^*(s)S + R$$

Then, based on the definition of quadratic dissipativeness in [8], we give a notation of strictly quadratic dissipativeness as follows.

**Definition 1.** Assume that $A$ is asymptotically stable. System $(\Sigma_1)$ is said to be $(Q, S, R)$ - dissipative if $M(j\omega) \geq 0, 0 < \omega \leq \omega < \infty$ and system $(\Sigma_1)$ is said to be strictly $(Q, S, R)$ - dissipative if $M(j\omega) \succ 0, 0 < \omega \leq \omega < \infty$. For short $(Q, S, R)$ - dissipative and strictly $(Q, S, R)$ - dissipative are referred to as quadratic dissipative and strictly quadratic dissipative, respectively.

When $Q = R = 0, S = I, p = m$, Definition 1 reduces to the following definition:

**Definition 2.** Assume that $A$ is asymptotically stable. System $(\Sigma_1)$ is said to be strictly positive real if $G(j\omega) + G^*(j\omega) \succeq 0, 0 \leq \omega \leq \infty$ and system $(\Sigma_1)$ is said to be extended strictly positive real (ESPR) if $G(j\omega) + G^*(j\omega) \succ 0, 0 \leq \omega \leq \infty$.

The strictly $(Q, S, R)$ - dissipative includes not only extended strictly positive realness but also the standard $H_\infty$ performance as special cases. When $Q = -I, S = 0$ and $R = I$, the strictly $(Q, S, R)$ - dissipative reduces to the standard $H_\infty$ performance. Noting that $Q \leq 0$ holds in these two cases, in the following we suppose

$$Q = -Q \succeq 0$$

Definition 2 is a frequency domain characterization of ESPR systems. The state space characterization of ESPR systems can be given by

**Lemma 1** [9]. Given system $(\Sigma_1)$ with $p = m$, the following statements are equivalent:

a) $(\Sigma_1)$ is ESPR;

b) There exists $0 < P \in R^{n \times m}$ such that

$$\begin{bmatrix} A^T P + PA & C^T - PB \\ C - B^T P & - (D^T + D) \end{bmatrix} < 0.$$ 

[8] obtained a state space characterization of quadratic dissipative systems by spectral factorization. For the strictly quadratic dissipative case we will give a corresponding result, which is essential to deal with strictly quadratic dissipative control problems. Augmenting system $(\Sigma_1)$, a system is given as follows.

$$(\Sigma_2) : \dot{x}(t) = Ax(t) + [B \quad 0]\omega(t)$$

$$z(t) = \begin{bmatrix} S^TC \\ -Q^{1/2}C \end{bmatrix} x(t) + \begin{bmatrix} R/2 + S^TD & 0 \\ -Q^{1/2}D & I_p/2 \end{bmatrix} \bar{\omega}(t)$$

we have

**Lemma 2.** Consider systems $(\Sigma_1)$ and $(\Sigma_2)$, the following statements are equivalent:

a) $(\Sigma_1)$ is strictly $(Q, S, R)$ - dissipative

b) $(\Sigma_2)$ is ESPR
c) There exists $0 < P \in \mathbb{R}^{n \times n}$ such that

$$
\begin{bmatrix}
A^T P + PA & PB - C^T S \\
B^T P - S^T C & -D^T S - S^T D - R & D^T \bar{Q}^{1/2}
\end{bmatrix} < 0
$$

(3)

**Proof.** The transfer function matrix of $(\Sigma_2)$ is

$$
T(s) = \begin{bmatrix} R/2 + S^T G(s) & 0 \\
-\bar{Q}^{1/2} G(s) & I_p/2 \end{bmatrix}
$$

Hence, $\forall \epsilon > 0$ we have $\epsilon > 0$ such that

$$
\begin{bmatrix}
R + S^T G(j\omega) + G^*(j\omega) S & -G^*(j\omega) \bar{Q}^{1/2} \\
-\bar{Q}^{1/2} G(j\omega) & I_p
\end{bmatrix} > 0, \forall \omega \leq \infty
$$

$\iff$ $A$ is asymptotically stable and

$$
G^*(j\omega)QG(j\omega) + S^T G(j\omega) + G^*(j\omega) S + R > 0, \forall \omega \leq \infty
$$

$\iff$ a).

By Lemma 1, b) $\Rightarrow$ c). This completes the proof. \qed

3 Robustly dissipative analysis problem

Now we investigate conditions under which the uncertain system $(\Sigma_\Delta)$ is strict quadratic dissipative for all admissible uncertainties.

**Definition 3.** System $(\Sigma_\Delta)$ satisfying assumption A.1 is said to be *robustly strict dissipative* if there exists $0 < P \in \mathbb{R}^{n \times n}$ such that the following LMI holds:

$$
\begin{bmatrix}
A_\Delta^T P + PA_\Delta & PB_\Delta - C_\Delta S & C_\Delta^T \bar{Q}_\Delta^{1/2} \\
B_\Delta^T P - S^T C_\Delta & -D^T_\Delta S - S^T D_\Delta - R & D^T_\Delta \bar{Q}_\Delta^{1/2} \\
\bar{Q}_\Delta^{1/2} C_\Delta & -I
\end{bmatrix} < 0
$$

(4)

for all admissible uncertainties.

For short *robustly strict $(Q, S, R)$ dissipative* is said to be robustly strict quadratic dissipative. Now that condition (4) is difficult to verify because of the uncertainties, we will reduce the robustly dissipative analysis problem of $(\Sigma_\Delta)$ to that of a system without uncertainties. Introducing a system associated with $(\Sigma_\Delta)$:

$$(\Sigma_\epsilon) : \dot{x}(t) = Ax(t) + B_\epsilon \bar{\omega}(t) = Ax(t) + [B \in H] \bar{\omega}(t), \ x(0) = 0$$

$$(\Sigma_\epsilon) : \dot{z}(t) = C_\epsilon x(t) + D_\epsilon \bar{\omega}(t) = [\epsilon^{-1} \ I \ C] x(t) + [\epsilon^{-1} \ I \ D \in H] \bar{\omega}(t)$$

where $\epsilon > 0$ will be specified later.

Denoting

$$
\tilde{Q} = \begin{bmatrix} 0 & I \\
-1 & \ 0 \end{bmatrix}, \ \tilde{S} = \begin{bmatrix} 0 & 0 \\
0 & S \end{bmatrix}, \ \tilde{R} = \begin{bmatrix} R & 0 \\
0 & I \end{bmatrix}, \ \tilde{\bar{Q}} = - \tilde{Q}
$$

then, we have

**Theorem 1.** System $(\Sigma_\Delta)$ is robustly strict $(Q, S, R)$ dissipative if and only if there exists a scalar $\epsilon > 0$ such that $(\Sigma_\epsilon)$ is strictly $(\tilde{Q}, \tilde{S}, \tilde{R})$ dissipative.

**Proof.** By Definition 3, $(\Sigma_\Delta)$ is robustly strict $(Q, S, R)$ dissipative if and only if there exists $0 < P \in \mathbb{R}^{n \times n}$ such that

$$
\begin{bmatrix}
A_\Delta^T P + PA_\Delta & PB_\Delta - C_\Delta S & C_\Delta^T \bar{Q}_\Delta^{1/2} \\
B_\Delta^T P - S^T C_\Delta & -D^T_\Delta S - S^T D_\Delta - R & D^T_\Delta \bar{Q}_\Delta^{1/2} \\
\bar{Q}_\Delta^{1/2} C_\Delta & -I
\end{bmatrix} < 0
$$
for all admissible uncertainties. Observing (1), it is easy to see that

\[
\begin{bmatrix}
A^T + PA & PB - CT S & C^T \hat{Q}^{1/2} \\
B^T P - S^T C & -D^T S - S^T D - R & D^T \hat{Q}^{1/2} \\
\hat{Q}^{1/2} C & -I & -I
\end{bmatrix}

\begin{bmatrix}
PH \\
-SP^T H_1 \\
\hat{Q}^{1/2} H_1
\end{bmatrix}
\Delta(t)[E \ E_1] 0

- \begin{bmatrix}
PH \\
-SP^T H_1 \\
\hat{Q}^{1/2} H_1
\end{bmatrix}
\Delta(t)[E \ E_1] 0
\end{bmatrix}^T < 0
\]

This LMI is equivalent to

\[
\begin{bmatrix}
A^T P + PA & PB - CT S & \varepsilon PH & \varepsilon^{-1} E^T_1 & C^T \hat{Q}^{1/2} \\
B^T P - S^T C & -D^T S - S^T D - R & \varepsilon PH & -\varepsilon S^T H_1 & \varepsilon^{-1} E^T_1 \\
\varepsilon H^T P & -\varepsilon H^T S & -I & J^T & \varepsilon H^T \hat{Q}^{1/2} \\
\varepsilon^{-1} E & 0 & J & -I & 0 \\
\hat{Q}^{1/2} C & 0 & -I
\end{bmatrix}
\]

i.e.,

\[
\begin{bmatrix}
A^T P + PA & PB - CT S & \varepsilon PH & \varepsilon^{-1} E^T_1 & C^T \hat{Q}^{1/2} \\
B^T P - S^T C & -D^T S - S^T D - R & \varepsilon PH & -\varepsilon S^T H_1 & \varepsilon^{-1} E^T_1 \\
\varepsilon H^T P & -\varepsilon H^T S & -I & J^T & \varepsilon H^T \hat{Q}^{1/2} \\
\varepsilon^{-1} E & 0 & J & -I & 0 \\
\hat{Q}^{1/2} C & 0 & -I
\end{bmatrix}
\]

Thus, \((\Sigma_i)\) is strictly dissipative. □

In view of Theorem 1, the robustly strict quadratic dissipative analysis of system \((\Sigma_\Delta)\) is simplified to that of \((\Sigma_i)\) without uncertainties. The latter can be solved using LMI approach.

**Theorem 2.** \((\Sigma_i)\) is strictly dissipative if and only if there exists a scalar \( \mu > 0 \) and a matrix \( X > 0 \) such that

\[
\begin{bmatrix}
XA^T + AX & B - XC^T S & \mu H & XE^T & X \hat{Q}^{1/2} \\
B^T - S^T CX & -D^T S - S^T D - R & -\mu S^T H_1 & E^T_1 & D^T \hat{Q}^{1/2} \\
\mu H^T & -\mu H^T S & -\mu I & J^T & \mu H^T \hat{Q}^{1/2} \\
E^T & E_1 & \mu J & -\mu I & 0 \\
\hat{Q}^{1/2} C & 0 & -I & -I & 0
\end{bmatrix}
\]

\( < 0 \)  \( (5) \)

**Proof.** By Lemma 2, \((\Sigma_i)\) is strictly dissipative if and only if there exists a scalar \( \varepsilon > 0 \) and a matrix \( P > 0 \) such that

\[
\begin{bmatrix}
A^T P + PA & PB - CT S & \varepsilon PH & \varepsilon^{-1} E^T_1 & C^T \hat{Q}^{1/2} \\
B^T P - S^T C & -D^T S - S^T D - R & \varepsilon PH & -\varepsilon S^T H_1 & \varepsilon^{-1} E^T_1 \\
\varepsilon H^T P & -\varepsilon H^T S & -I & J^T & \varepsilon H^T \hat{Q}^{1/2} \\
\varepsilon^{-1} E & 0 & J & -I & 0 \\
\hat{Q}^{1/2} C & 0 & -I
\end{bmatrix}
\]

\( < 0 \)

Pre-multiply and post-multiply the last matrix by \( diag(P^{-1}, I, \varepsilon I, \varepsilon I, I) \) and let \( \varepsilon^2 = \mu \) and \( P^{-1} = X \). The desired result follows immediately. □

4 Robustly dissipative control problem

Consider the following uncertain system

\[
(\Sigma_\Delta): \dot{x}(t) = A_\Delta x(t) + B_\omega(t) + B_1 u(t), \quad x(0) = 0
\]
4.1 State-feedback design

Suppose the state can be measured, the robustly dissipative control problem via state feedback can be stated as follows: design a state feedback controller \( u(t) = Kx(t) \) for uncertain system \((\Sigma)\) such that the closed-loop system is robustly strictly dissipative. It is expected to reduce the robustly dissipative control problem via state feedback to that for a system without uncertainties. To this end, an auxiliary system associated with \((\Sigma)\) is introduced.

\[
\begin{align*}
(\Sigma_{ex}) : \dot{x}(t) &= \Delta \dot{x}(t) + B_1 \Delta \omega(t) + B_1 u(t), \quad x(0) = 0 \\
z(t) &= C_i x(t) + D_i \Delta \omega(t) + D_{12} \Delta u(t)
\end{align*}
\]

where \( D_i^T = [\varepsilon^{-1} E_i^T \quad D_i^T] \).

**Theorem 3.** There exists a state-feedback controller \( u(t) = Kx(t) \) for system \((\Sigma)\) such that the resulting closed-loop system is **robustly strictly** dissipative if and only if there exists a scalar \( \varepsilon > 0 \) such that the closed-loop system composed of system \((\Sigma_{ex})\) and the controller is **strictly** dissipative.

**Proof.** Applying \( u(t) = Kx(t) \) to \((\Sigma_{ex})\), we get the closed-loop system

\[
\begin{align*}
\dot{x}(t) &= \Delta \dot{x}(t) + B_1 \Delta \omega(t), \quad x(0) = 0 \\
z(t) &= \tilde{C}_\Delta x(t) + D_\Delta \tilde{\omega}(t)
\end{align*}
\]

where

\[
\tilde{A}_\Delta = \bar{A} - H \Delta(t) \bar{E}, \quad \tilde{C}_\Delta = \bar{C} - H_1 \Delta(t) \bar{E}
\]

and

\[
\bar{A} = A + B_1 K, \quad \bar{C} = C + D_{12} K, \quad \bar{E} = E + E_2 K
\]

Applying \( u(t) = Kx(t) \) to \((\Sigma_{ex})\), we obtain

\[
\begin{align*}
(\Sigma_{ex}) : \dot{x}(t) &= \tilde{A} x(t) + B_1 \tilde{\omega}(t) \\
z(t) &= \tilde{C}_\Delta x(t) + D_\Delta \tilde{\omega}(t)
\end{align*}
\]

where \( \tilde{C}_\Delta^T = [\varepsilon^{-1} \tilde{E}_\Delta^T \quad \tilde{C}_\Delta^T] \).

Consider the two closed-loop systems above and the desired result is readily obtained using Theorem 1.

By Theorem 3, in order to solve the robustly dissipative control problem for uncertain system \((\Sigma_{ex})\), we only need to solve that for \((\Sigma_{ex})\).

**Theorem 4.** There exists a state-feedback controller \( u(t) = Kx(t) \) for system \((\Sigma_{ex})\) such that the resulting closed-system is **strictly** dissipative if and only if there exists a scalar \( \mu > 0 \) and matrices \( W \in R^{n \times n} \) and \( 0 < X \in R^{n \times n} \) such that

\[
\begin{bmatrix}
X A^T + W B_1^T + A X + B_1 W & B - (C X + D_{12} W)^T S & \mu H & (E X + E_2 W)^T & (C X + D_{12} W)^T \bar{Q}^{1/2} \\
B^T - S^T (C X + D_{12} W) & -D^T S - S D - R & -\mu S H_1 & E_1^T & D^T \bar{Q}^{1/2} \\
\mu H^T & -\mu H_1^T S & -\mu I & \mu J^T & \mu H_1^T \bar{Q}^{1/2} \\
E X + E_2 W & \mu J & -\mu J & 0 & 0 \\
\bar{Q}^{1/2} (C X + D_{12} W) & \bar{Q}^{1/2} D & \mu \bar{Q}^{1/2} H_1 & 0 & -I
\end{bmatrix} < 0
\]

Moreover, if \( X^*, W^* \) and \( \mu^* \) is a solution to (6), \( u(t) = W^* (X^*)^{-1} x(t) \) is a desired controller.

**Proof.** (Similar to the proof of Theorem 2, it is omitted.)

4.2 Dynamic output feedback design

Consider the following system

\[
\begin{align*}
(\Sigma_{df}) : \dot{x}(t) &= A_\Delta x(t) + B_\Delta \omega(t) + B_{12} u(t), \quad x(0) = 0 \\
z(t) &= C_\Delta x(t) + D_\Delta \omega(t) + D_{12} u(t) \\
y(t) &= C_1 x(t) + D_{21} \omega(t) + D_{22} u(t)
\end{align*}
\]
where $y(t) \in \mathbb{R}^r$ is the measured output

$$
[C_1 \Delta \ D_{21} \Delta \ D_{22} \Delta] = [C_1 \ D_{21} \ D_{22}] - H_2 \Delta(t)[E \ E_1 \ E_2]
$$

$H_2, C_1, D_{21}$ and $D_{22}$ are known matrices, and the rest matrices are the same in ($\Sigma_{\Delta s}$).

In this section, we will address the robustly dissipative control problem via output feedback, that is, design of a dynamic output feedback controller of the form

$$(\Sigma_f) : \xi(t) = F_0 \xi(t) + G_0 y(t)$$

$u(t) = K_0 \xi(t)$

for ($\Sigma_f$) such that the resulting closed-loop system is \textit{robustly strict} $(Q, S, R)$ – dissipative.

Similar to the case of state feedback design, an auxiliary system related to ($\Sigma_{\Delta f}$) is constructed

$$(\Sigma_{\Delta f}) : \bar{x}(t) = \bar{A} \bar{x}(t) + \bar{B}_\Delta \bar{\omega}(t) + \bar{B}_1 u(t)$$

$z(t) = C_1 x(t) + D_{21} \bar{\omega}(t) + D_{22} u(t)$

where $D_{2\Delta} = [D_{21} \ \varepsilon H_2]$, and the other matrices are defined as in ($\Sigma_{\Delta s}$).

The following theorem gives the main result of this paper.

\textbf{Theorem 5.} Dynamic output feedback controller ($\Sigma_f$) for ($\Sigma_{\Delta f}$) achieves \textit{robustly strict} $(Q, S, R)$ – dissipative if and only if the controller ($\Sigma_f$) for ($\Sigma_{\Delta s}$) achieves \textit{strictly} $(\bar{Q}, \bar{S}, \bar{R})$ – dissipative for some $\varepsilon > 0$.

\textbf{Proof.} On the one hand, the closed-loop system of ($\Sigma_{\Delta f}$) together with ($\Sigma_f$) is

$$(\Sigma_{\Delta f}) : \dot{\eta}(t) = \bar{A} \eta(t) + \bar{B}_\Delta \bar{\omega}(t), \quad x(0) = 0$$

$z(t) = \bar{C}_\Delta \eta(t) + D_\Delta \bar{\omega}(t)$

where

$$\eta(t) = [x^T(t) \ \xi^T(t)]^T, \quad \bar{C}_\Delta = \bar{C} - H_1 \Delta(t) \bar{E}, \quad \bar{C} = [C \ D_{12} K_0]$$

$$\bar{E} = [E \ E_2 K_0], \quad [\bar{A}_\Delta \ \bar{B}_\Delta] = [\bar{A} \ \bar{B} - \bar{H} \Delta(t)] [\bar{E} \ E_1]$$

$$\bar{A} = \begin{bmatrix} A & B_1 K_0 \\ G_0 C_1 & F_0 + G_0 D_{22} K_0 \end{bmatrix}, \quad \bar{B} = \begin{bmatrix} B \\ G_0 D_{21} \end{bmatrix}, \quad \bar{H} = \begin{bmatrix} H \\ G_0 H_2 \end{bmatrix}$$

on the other hand, the closed-loop system composed of ($\Sigma_{\Delta s}$) and ($\Sigma_f$) is

$$(\Sigma_{\Delta s}) : \dot{\eta}(t) = \bar{A}_\Delta \eta(t) + \bar{B}_\Delta \bar{\omega}(t), \quad x(0) = 0$$

$$z(t) = \bar{C}_\Delta \eta(t) + D_\Delta \bar{\omega}(t)$$

where $\bar{B}_\Delta = [\bar{B} \ \varepsilon \bar{H}], \quad \bar{C}_\Delta^T = [\varepsilon^{-1} \bar{E}^T \ \bar{C}^T]$.

For systems ($\Sigma_{\Delta s}$) and ($\Sigma_{\Delta f}$) using Theorem 1, the desired result is readily obtained. \hfill $\Box$

Theorem 5 provides necessary and sufficient conditions for robustly dissipative control problems via output feedback. From Theorem 5 together with Theorem 3, it can be concluded that robustly dissipative control problems can be converted to those without uncertainties.

\section{5 Conclusions}

In this paper we have studied strictly quadratic dissipative analysis and control problems for a class of uncertain systems. Necessary and sufficient conditions have been obtained for the uncertain systems to be robustly strict quadratic dissipative. It has been shown that robust strict quadratic dissipative control problems for the uncertain systems can be reduced to those for some related systems without uncertainties. The latter can be solved using an LMI approach. As a byproduct, it has been shown that strictly quadratic dissipativeness of a linear system is equivalent to extended strictly positive realness of a related system. By the equivalence, strictly quadratic dissipativeness characterization of linear systems has been derived. The proof, compared with that of [8], is simple.
References

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