Comments on “$H_{\infty}$ Output Feedback Control Design of Fuzzy Dynamic Systems Via LMI”

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Abstract We point out a mistake and the incorrect conclusion of the paper “$H_{\infty}$ output feedback control design of fuzzy dynamic systems via LMI” in Acta Automatica Sinica 2001, 27(4). We also give the correction of the mistake.

Key words Fuzzy dynamic systems, $H_{\infty}$ output feedback control, linear matrix inequality (LMI), asymptotical stability

1 Introduction

In the paper “$H_{\infty}$ output feedback control design of fuzzy dynamic systems via LMI” in Acta Automatica Sinica 2001, 27(4)[11], the authors discussed the $H_{\infty}$ output feedback control problem for fuzzy dynamic systems. Unfortunately, the conclusion of its Theorem 2, which refers to asymptotical stability of $H_{\infty}$ output feedback control problem for fuzzy dynamic systems, is incorrect.

2 Description of the problem

The mistake of the paper is found in the proof of its Theorem 2 regarding the asymptotical stability of the closed loop system, where the following equation is used (see equ. (37) of [1]):

$$\varepsilon_i^{-1}\Delta A_i^T \Delta A_i = \varepsilon_i^{-1}\begin{bmatrix} I & 0 \\ 0 & C_i \end{bmatrix}^T \begin{bmatrix} \Delta A_i & \Delta B_i \\ 0 & 0 \end{bmatrix} \begin{bmatrix} I & 0 \\ 0 & C_i \end{bmatrix} \begin{bmatrix} I & 0 \\ 0 & B_i \end{bmatrix} \begin{bmatrix} -\Delta A_i & \Delta B_i \\ 0 & 0 \end{bmatrix} \begin{bmatrix} I & 0 \\ 0 & C_i \end{bmatrix}$$

(1)

Directly calculating the right hand side of (1) yields

$$\varepsilon_i^{-1}\begin{bmatrix} I & 0 \\ 0 & C_i \end{bmatrix}^T \Delta A_i^T \Delta B_i \begin{bmatrix} I & 0 \\ 0 & C_i \end{bmatrix} = \begin{bmatrix} C_i^T \Delta B_i^T \Delta A_i & C_i^T \Delta B_i^T \Delta B_i C_i \\ C_i^T \Delta A_i & C_i^T \Delta A_i \end{bmatrix}$$

(2)

However, since

$$\varepsilon_i^{-1}\Delta A_i^T \Delta A_i = \begin{bmatrix} \Delta A_i^T \Delta A_i + C_i^T \Delta B_i \Delta B_i C_i & \Delta A_i^T \Delta B_i C_i \\ C_i^T \Delta B_i^T \Delta A_i & C_i^T \Delta B_i^T \Delta B_i C_i \end{bmatrix}$$

(3)

it immediately follows from (2) and (3) that (1) is incorrect. Thus, the corresponding conclusion of Theorem 2 of [1] becomes untenable and, of course, should be corrected. In fact, from the above analysis it is easy to see that Theorem 2 of [1] is valid only when $\Delta C_i = 0$. It should also be pointed out that the simulation provided in [1] exactly satisfies this condition and thus does not reveal the limitation.

3 Correction of Theorem 2 of [1]

If we directly use the following estimate of $\Delta A_i^T P_i + P_i \Delta A_i$:

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\[ \Delta \hat{A}_t^T P_t + P_t \Delta A_t \leq \varepsilon_t P_t P_t + \varepsilon_t^{-1} \Delta \hat{A}_t^T \Delta A_t \] (4)

then the term \( \Delta C_t^T B_t^T B_t C_t \), which appears in (2) and leads to the incorrect conclusion in [1], will prevent us from further processing. Instead of using the method in [1], we can first decompose \( \Delta \hat{A}_t \) in (4) as follows:

\[
\Delta \hat{A}_t = \Delta \hat{A}_{t1} + \Delta \hat{A}_{t2} = \begin{bmatrix} \Delta A_t^T & \Delta B_t & C_t & 0 \\ 0 & 0 & \Delta C_t & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & C_t \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & C_t \\ 0 & 0 & 0 \end{bmatrix} \]

(5)

Then using lemma 2 of [1], we can estimate \( \Delta \hat{A}_{t1}^T P_t + P_t \Delta A_t \) and \( \Delta \hat{A}_{t2}^T P_t + P_t \Delta A_t \), respectively, without introducing the term \( \Delta C_t^T B_t^T B_t C_t \). Now we give a correction to Theorem 2 of [1] as follows.

Given a fuzzy dynamic system described as in (21) of [1], its feedback controller described as in (22) of [1], an approximate upper bound of the uncertainties of (21) of [1] defined by (29) of [1], and the \( H_\infty \) performance bound \( \gamma > 0 \) as a constant, if there exist a set of constants \( \varepsilon_{t1}, \varepsilon_{t2}, \varepsilon_{t3}, \varepsilon_{t4} > 0 \), and a set of variables \( \bar{A}_i, \bar{B}_i, \bar{C}_i, \bar{E}_{i0}, \bar{E}_{ii}, \bar{E}_{iz}, X_i, \) and \( Y_i \) satisfying the following inequalities:

\[
\begin{bmatrix} M_{ii} & M_{i2} \\ M_{i2}^T & M_{22} \end{bmatrix} < 0, \quad \begin{bmatrix} X_i & I \\ I & Y_i \end{bmatrix} > 0, \quad P(\tau_i^+) \leq P(\tau_i^{+}), \quad i = 1, 2, \ldots, T
\]

where

\[
M_{ii} = \begin{bmatrix} \Omega_{i1} & \Omega_{i2} \\ \Omega_{i1}^T & \Omega_{i2}^T \end{bmatrix}
\]

\[
M_{i2} = \text{diag}\left\{ -\varepsilon_{t1} \begin{bmatrix} \bar{E}_{i0} & X_i & I \\ X_i & I & \bar{E}_{ii} \end{bmatrix}, -\varepsilon_{t2} \begin{bmatrix} 1 & Y_i & 0 \\ 0 & Y_i & \bar{E}_{iz} \end{bmatrix}, -\varepsilon_{t3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}, \varepsilon_{t4} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \right\}
\]

\[
M_{22} = \begin{bmatrix} X_i & I & \bar{E}_{i0} & \bar{B}_i & \bar{E}_{iul} & (C_d X_i)^T & (E_d X_i)^T & 0 & (E_d X_i)^T \\ I & Y_i & E_{i0}^T (Y_i E_{i0}) & Y_i B_{ul} & Y_i E_{iul} & C_d^T & E_{iul}^T & Y_i & \bar{B}_i & \bar{E}_{iul} \end{bmatrix}
\]

\[
\Omega_{i1} = A_i X_i + X_i A_i^T + B_i \bar{C}_i + (B_i \bar{C}_i)^T, \quad \Omega_{i2} = A_i^T Y_i + Y_i A_i + B_i \bar{C}_i - (B_i \bar{C}_i)^T
\]

then the closed loop system (23) of [1] is asymptotically stable with \( H_\infty \) performance bound \( \gamma \). Furthermore, if the matrices \( \bar{M} \) and \( \bar{N} \) are in the form of full rank decomposition

\[ \bar{M}_i \bar{N}_i^T = I - X_i Y_i \]

then a parameterization of \( H_\infty \) output feedback controller can be determined as

\[ C_d - \bar{C}_d \bar{M}_i^T, \quad B_d - \bar{N}_i^+ \bar{B}_i, \quad A_d - \bar{N}_i^+ (A_i - (N_i B_d C_i X_i + Y_i B_d C_i \bar{M}_i^T + Y_i A_i X_i)) \bar{M}_i^T \]

where \( \bar{M}_i^T \) and \( \bar{N}_i^+ \) denote the Moore-Penrose general inverses of \( \bar{M}_i \) and \( \bar{N}_i \), respectively.

4 Conclusion

In this note a possible careless mistake which leads to the incorrect conclusion of [1] is pointed out, and its correction is given.

There is much work that needs to be discussed for the problem of stability and performance of the closed system by using switching control. Some classical results have been discussed in detail in [2] and the related results and techniques in [3] and [4] are beneficial to this issue.

References

1. Zhang Ning, Feng Gang. \( H_\infty \) output feedback control design of fuzzy dynamic systems via LMI. \textit{Acta Automatica Sinica}. 2001, \textbf{27}(1), 495~499


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与“基于 LMI 的模糊动态系统 $H_{\infty}$ 输出反馈控制设计”一文商榷

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摘要 本文提出了《自动化学报》2001年27卷1期上刊登的文章“基于 LMI 的模糊动态系统 $H_{\infty}$ 输出反馈控制设计”中某些结论的不足之处，并给出了修改。

关键词  模糊动态系统, $H_{\infty}$ 输出反馈控制, 线性矩阵不等式(LMI), 惯近稳定性
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