Robust H_∞ Fuzzy Output-feedback Control With Both General Multiple Probabilistic Delays and Multiple Missing Measurements and Random Missing Control

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Abstract In this paper, the robust H_∞-control problem is reported for a class of uncertain discrete-time fuzzy systems with both multiple probabilistic delays and multiple missing measurements and random missing control from the fuzzy controllers to the actuator. A sequence of random variables including accounting for the probabilistic communication delays and the random missing control are thought as mutually independent and obey the Bernoulli distribution. The measurement-missing phenomenon can be assumed to occur stochastically. Assumption that the missing probability for each sensor satisfies a certain probabilistic distribution in the interval [0 1] is given. Much attention is focused on design of H_∞ the fuzzy output feedback controllers to ensure that the resulting close-loop Takagi-Sugeno (T-S) system is exponentially stable in the mean square. The developed method makes disturbance rejection attenuation satisfy a given level by means of the H_∞-performance index. Intensive analysis is employed to reach the sufficient conditions about the existence of admissible output feedback controllers which satisfies the exponential stability as well as the prescribed H_∞ performance. In addition, the cone-complementarity linearization procedure is utilized to transform the controller-design problem into a sequential minimization one which can be solved by the semi-definite program method. Simulation results conform the feasibility as weil as the effectiveness of the proposed design method.

Key words Discrete-time fuzzy systems, fuzzy control, multiple missing control, multiple missing measurements, multiple probabilistic time delays, networked-control systems (NCSs), robust H_∞ control, stochastic systems

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1 Introduction

Since recent few decades, some researchers focus their energy on the robust stability and controller design problems about the networked-control systems (NCSs) with some uncertain parameters because some networked-control systems have been succeeded in applications in modern complicated industry processes, e.g., aircraft and space shuttle, nuclear power stations, high-performance automobiles, etc. The fuzzy-logic control based on the Takagi-Sugeno (T-S) is widely used to dealing with complex nonlinear systems because it has simple dynamic structure and highly accurate approximation to any smooth nonlinear function in any compact set. One can consult [1]−[8] and the other cited literature therein [9]−[31]. Data-packet dropout is an important issue to be addressed in the networked-control systems [6], [32]. Zhang [33] solves the problem of H_∞ estimation for a class of Markov jump linear systems but he neglect possible dropout in practice. Reference [34] reports the problem of H_∞ stability of discrete-time switched linear system with average dwell time and with no dropout. In [6], piecewise Lyapunov function is proposed to analyze robust of the nonlinear NCSs without time-delay issue. Random data-packet dropout and time delay are well considered but the controlled NCSs are linear systems in [32]. Reference [8] discusses the problem of robust H_∞ output feedback control for a class of continuous-time Takagi-Sugeno (T-S) fuzzy affine dynamic systems with parametric uncertainties and input constraints on ignoring some nonlinearities induced by system with data-packet dropout and random time delay. Reference [5] investigates the robust H_∞ stability of a class of half nonlinear NCSs with multiple probabilistic delays and multiple missing measurements regardless of the dropout in the forward path. According to above consideration, we investigate a class of new nonlinear NCSs, in which not only sensors communicate with controllers by network but also controllers do with actuator in the same manner.

The highlights of this paper, which lie primarily on the new research problems and new system models, are summarized as follows:

1) A new model is established, in which the controllers communicate with the actuator by a wireless network and the random missing control from the controller to the actuator occurs and the sensors do with the controllers in the same manner.

2) The investigation on the T-S fuzzy model is used for a class of complex systems that describe the modeling errors, disturbance rejection attenuation, probabilistic delay, missing measurements and missing control within the same framework.

The rest of this paper is organized as follows. The problem under consideration is formulated in Section 2. Development of robust H_∞ fuzzy control performance on the exponentially stability the closed-loop fuzzy system are placed in Section 3. Section 4 gives design of robust H_∞ fuzzy controller. An illustrative example is given in Section 5, and we conclude the paper in Section 6.

Notation 1: The notation used in the paper is fairly standard. $\mathbb{R}^n$ denotes the n-dimensional real vectors; $\mathbb{R}^{m\times n}$ denotes the n-dimensional matrix; and I and 0 represent the identity matrix and zero matrix, respectively. The notation $P > 0$ ($P \geq 0$) means that P is real symmetric and positive definite (semi-definite), $\mathrm{tr}(M)$ refers to the trace of the matrix M, and $\|\cdot\|_2$ stands for the usual l_2 norm. In symmetric block matrices or complex matrix expressions,

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we use an "$\star$" to represent a term that is induced by symmetry, and diag$\{\cdots\}$ stands for a block-diagonal matrix. In addition, $E\{x\}$ and $E\{x|y\}$ will, respectively, mean expectation of x and expectation of x conditional on y.

2 Problem Formulation

In this note, the output feedback control problem for discrete-time fuzzy systems in NCSs is taken in our consideration, where the frame-work is depicted in Fig. 1.

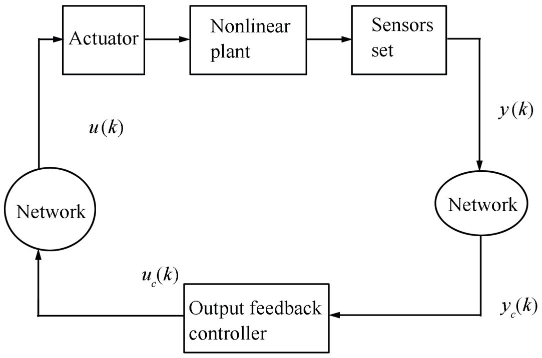

Fig. 1. Framework of output feedback control systems over network environment.

The sensors are connected to a network, which are shared by other NCSs and susceptible to communication delays and missing measurements or pack dropouts). As Fig. 1 depicts, pack dropouts from the controller to actuator can take place stochastically. The fuzzy systems with multiple stochastic communication delays and uncertain parameters can be read as follows:

Plant Rule i: If $\theta_1(k)$ is M_{i1}, and $\theta_2(k)$ is M_{i2}, and, ..., and $\theta_p(k)$ is M_{ip}, then

$$\begin{aligned} x(k+1) &= A_i(k)x(k) + A_{di}\sum_{m=1}^{h}\alpha_m(k)x(k-\tau_m(k)) \\ &\quad + B_{1i}u(k) + D_{1i}v(k) \\ \tilde{y}(k) &= C_i x(k) + D_{1i}v(k) \\ z(k) &= C_{zi}(k) + B_{2i}u(k) + D_{3i}v(k) \\ x(k) &= \phi(k) \quad \forall\, k \in \mathbb{Z}^-,\ \ i = 1,\dots,r \end{aligned} \tag{1}$$

where M_{ij} is the fuzzy set, r stands for the number of If-then rules, and $\theta(k) = [\theta_1(k), \theta_2(k), \dots, \theta_p(k)]$ is the premise variable vector, which is independent of the input variable $u(k)$. $x(k) \in \mathbb{R}^n$ is the state vector, $u(k) \in \mathbb{R}^m$, $\tilde{y} \in \mathbb{R}^s$ is the process output, $z(k) \in \mathbb{R}^q$ is the controlled output, $v(k) \in \mathbb{R}^p$ presents a vector of exogenous inputs, which belongs to $l_2[0,\infty)$, $\tau_m(k)$ $(m = 1, 2, \dots, h)$ are the communication delays that vary with the stochastic variables $\alpha_m(k)$, and $\phi(k)$ $(\forall\, k \in \mathbb{Z}^-)$ is the initial state.

The stochastic variables $\alpha_m(k) \in \mathbb{R}$ $(m = 1, 2, \dots, h)$ in (1) are assumed to satisfy mutually uncorrelated Bernoulli-distributed-white sequences described as follows:

$$\begin{aligned} \mathrm{Prob}\{\alpha_m(k) = 1\} &= E\{\alpha_m(k)\} = \bar{\alpha}_m \\ \mathrm{Prob}\{\alpha_m(k) = 0\} &= 1 - \bar{\alpha}_m. \end{aligned}$$

In this note, one can make the random communication-time delays satisfy the following assumption that the time-varying $\tau_m(k)$ $(m = 1, 2, \dots, h)$ are subject to $d_t \le \tau_m(k) \le d_T$. The matrices $A_i(k) = A_i + \Delta A_i(k)$, $C_{zi}(k) = C_{zi} + \Delta C_{zi}(k)$, where $A_i, A_{di}, B_{1i}, B_{2i}, C_i, C_{zi}, D_{1i}, D_{2i}$, and D_{3i} are known constant matrices with compatible dimensions. $\Delta A_i(k)$ and $\Delta C_{zi}(k)$ with the time-varying norm-bounded uncertainties satisfy

$$\begin{bmatrix} \Delta A_i(k) \\ \Delta C_{zi}(k) \end{bmatrix} = \begin{bmatrix} H_{ai} \\ H_{ci} \end{bmatrix} F(k)E \tag{2}$$

with H_{ai}, H_{ci} being constant matrices and $F^T(k)F(k) \le I$, $\forall\, k$.

In this note, the packet dropout (the miss-measurement) read as

$$\begin{aligned} y_c(k) &= \Xi C_i x(k) + D_{2i}(k) \\ &= \sum_{l=1}^{s}\beta_l C_{il}x(k) + D_{2i}v(k) \\ u(k) &= W(k)u_c(k) = W(k)C_{ki}x_c(k) \end{aligned} \tag{3}$$

where $\Xi = \mathrm{diag}\{\beta_1, \dots, \beta_s\}$ with β_l $(l = 1, 2, \dots, s)$ being s unrelated random variables, which are also unrelated with $\alpha_m(k)$ and $W(k)$ denoting the random packet missing from the controllers to the actuator. One can assume that β_l has the probabilistic-density function $q_l(s)$ $(l = 1, 2, \dots, s)$ on the interval $[0, 1]$ with mathematical expectation μ_l and variance σ_l^2. $C_{il} = \mathrm{diag}\{\underbrace{0,\dots,0}_{l-1}, 1, \underbrace{0,\dots,0}_{s-l}\}C_i$. We denote the stochastic pack dropouts from the controller to the actuator by $W(k) = \mathrm{diag}\{\omega_1(k), \dots, \omega_m(k)\}$, where ω_l $(l = 1, 2, \dots, m)$ are mutually unrelated random variables and obey Bernoulli distribution with mathematical expectation $\bar{\omega}_l$ and variance$\rho_l$and assumed to be unrelated with $\alpha_m(k)$. For a given pair of $(x(k), u(k))$, the final output of the fuzzy system is read as

$$\begin{aligned} x(k+1) &= \sum_{i=1}^{r} h_i(\theta(k))[A_i(k)x(k) + B_{1,i}u(k) \\ &\quad + A_{di}\sum_{m=1}^{h} x(k-\tau_m(k)) + D_{1i}v(k)] \\ y_c(k) &= \sum_{i=1}^{r} h_i(\theta(k))[\Xi C_i x(k) + D_{2i}v(k)] \\ z(k) &= \sum_{i=1}^{r} h_i(\theta(k))[C_{zi}(k)x(k) + B_{2i}u(k) + D_{3i}v(k)] \end{aligned} \tag{4}$$

where the fuzzy-basis functions are described as

$$\begin{aligned} h_i(\theta(k)) &= \frac{\vartheta_i(\theta(k))}{\sum\limits_{i=1}^{r}\vartheta_i(\theta(k))} \\ \vartheta_i(\theta(k)) &= \prod_{j=1}^{p} M_{ij}(\theta_j(k)) \end{aligned}$$

with $M_{ij}(\theta_j(k))$ being the grade of membership of $\theta_j(k)$ in M_{ij}. It is clear that $\vartheta_i(\theta(k)) \ge 0$, $i = 1, 2, \dots, r$, $\sum_{i=1}^{r}\vartheta_i(\theta(k)) > 0$, $\forall\, k$, and $h_i(\theta(k)) \ge 0$, $i = 1, 2, \dots, r$, $\sum_{i=1}^{r} h_i(\theta(k)) = 1$, $\forall\, k$. In the sequel, we denote $h_i = h_i(\theta(k))$ for brevity.

In the note, the fuzzy dynamic output-feedback controller for the fuzzy system (4) is given as

Controller Rule i: If $\theta_1(k)$ is M_{il} and $\theta_2(k)$ is M_{i2} and, ..., and $\theta_p(k)$ is M_{ip} then

$$\begin{cases} x_c(k+1) = A_{ki}x_c(k) + B_{ki}y_c(k) \\ u(k) = W(k)C_{ki}x_c(k) \end{cases} \tag{5}$$

with $x_c(k) \in \mathbb{R}^n$ being the controller state along with the controller parameters A_{ki}, B_{ki} and C_{ki} to be determined. Naturally, the overall fuzzy output-feedback controller is read as

$$\begin{cases} x_c(k+1) = \sum\limits_{i=1}^{r} h_i[A_{ki}x_c(k) + B_{ki}y(k)] \\ u(k) = \sum\limits_{i=1}^{r} h_i W(k)C_{ki}x_c(k), \quad i = 1, 2, \dots, r. \end{cases} \tag{6}$$

Combining (6) with (4), we can obtain the closed-loop system described as

$$\begin{cases} \bar{x}(k+1) = \sum\limits_{i-1}^{r}\sum\limits_{j=1}^{r} h_ih_j[(A_{ij} + B_{ij})\bar{x}(k) + D_{ij}v(k) \\ \qquad + \sum\limits_{m=1}^{h}(\bar{A}_{dmi} + \tilde{A}_{dmi})\bar{x}(k-\tau_m(k))] \\ z(k) = \sum\limits_{i=1}^{r}\sum\limits_{j=1}^{r} h_ih_j[\bar{C}_{ij}(k) + \bar{\bar{C}}_{ij}]\bar{x}(k) + D_{3i}v(k) \end{cases} \tag{7}$$

where

$$\bar{x}(k) = \begin{bmatrix} x(k) \\ x_c(k) \end{bmatrix}, \quad A_{ij} = \begin{bmatrix} A_i(k) & B_{1i}\bar{W}C_{kj} \\ B_{ki}\bar{\Xi}C_j & A_{ki} \end{bmatrix}$$

$$B_{ij} = \begin{bmatrix} 0 & B_{1i}\tilde{W}(k)C_{kj} \\ B_{ki}\tilde{\Xi}C_j & 0 \end{bmatrix}$$

$$\bar{A}_{dmi} = \begin{bmatrix} \bar{\alpha}_m A_{di} & 0 \\ 0 & 0 \end{bmatrix}, \quad \tilde{A}_{dmi} = \begin{bmatrix} \tilde{\alpha}_m A_{di} & 0 \\ 0 & 0 \end{bmatrix}$$

$$D_{ij} = \begin{bmatrix} D_{1i} \\ B_{ki}D_{2j} \end{bmatrix}, \quad \bar{C}_{ij}(k) = \begin{bmatrix} C_{zi}(k) & B_{2i}\bar{W}C_{kj} \end{bmatrix}$$

$$\bar{\bar{C}}_{ij}(k) = \begin{bmatrix} 0 & B_{2i}\tilde{W}(k)C_{kj} \end{bmatrix}$$

with $\tilde{\alpha}_m(k) = \alpha_m(k) - \bar{\alpha}_m(k)$ and $\tilde{\omega}_j(k) = \omega_j(k) - \bar{\omega}_j(k)$. It is evident that $E\{\tilde{\alpha}_m(k)\} = 0$ and that $E\{\tilde{\omega}_j(k)\} = 0$ and that $E\{\tilde{\alpha}_m^2(k)\} = \bar{\alpha}_m(1-\bar{\alpha}_m) = \sigma_m^2$ and that $E\{\tilde{\omega}_j^2(k)\} = \bar{\omega}_j(1-\bar{\omega}_j) = \rho_j^2$.

Denote

$$\begin{aligned} &\bar{x}(k-\tau) \\ &= \begin{bmatrix} \bar{x}^T(k-\tau_1(k)) & \bar{x}^T(k-\tau_2(k)) & \cdots & \bar{x}^T(k-\tau_h(k)) \end{bmatrix}^T \\ &\xi(k) = \begin{bmatrix} \bar{x}^T(k) & \bar{x}^T(k-\tau) & v^T(k) \end{bmatrix}^T \end{aligned}$$

then (7) can also be rewritten as

$$\begin{cases} \bar{x}(k+1) = \sum\limits_{i=1}^{r}\sum\limits_{j=1}^{r} h_ih_j \left[A_{ij} + B_{ij}, \hat{Z}_{mi} + \Delta\hat{Z}_{mi}, D_{ij}\right]\xi(k) \\ z(k) = \sum\limits_{i=1}^{r}\sum\limits_{j=1}^{r} h_ih_j\left[\bar{C}_{ij} + \bar{\bar{C}}_{ij}, 0, D_{3i}\right]\xi(k) \end{cases} \tag{8}$$

where $\hat{Z}_{mi} = [\bar{A}_{d1i}, \dots, \bar{A}_{dhi}]$ and $\Delta\hat{Z}_{mi} = [\tilde{A}_{d1i}, \dots, \tilde{A}_{dhi}]$. In order to smoothly formulate the problem in the note, we introduce the following definition.

Definition 1: For the system (7) and every initial conditions ϕ, the trivial solution is said to be exponentially mean square stable if, in the case of $v(k) = 0$, there exist constants $\delta > 0$ and $0 < \kappa < 1$ such that $E\{\|\bar{x}(k)\|^2\} \le \delta\kappa^k \sup_{-d_M \le i \le 0} E\{\|\phi(i)\|^2\}$, $\forall\, k \ge 0$.

We will develop techniques to settle the robust H_∞ dynamic output feedback problem for the discrete-time fuzzy system (7) subject to the following conditions:

1) The fuzzy system (7) is exponentially stable in the mean square.

2) Under zero-initial condition, the controlled output $z(k)$ satisfies

$$\sum_{k=0}^{\infty} E\left\{\|z(k)\|^2\right\} \le \gamma^2 \sum_{k=0}^{\infty} E\left\{\|v(k)\|^2\right\} \tag{9}$$

for all nonzero $v(k)$, where $\gamma > 0$ is a prescribed scalar.

Remark 1: The proposed new model has the function that not only the controllers communicate with the actuator by wireless but also the sensors do with the controllers by the same manner.

3 Development of Robust H_∞ Fuzzy Control Performance

At first, we give the following lemma, which will be adopted in obtaining our main results.

Lemma 1 (Schur complement): Given constant matrices S_1, S_2, S_3, where $S_1 = S_1^T$ and $0 < S_2 = S_2^T$, then $S_1 + S_3^T S_2^{-1} S_3 < 0$ if and only if

$$\begin{bmatrix} S_1 & S_3^T \\ S_3 & -S_2 \end{bmatrix} < 0 \;\; \text{or} \;\; \begin{bmatrix} -S_2 & S_3 \\ S_3^T & S_1 \end{bmatrix} < 0.$$

Lemma 2 (S-procedure) [5]: Letting $L = L^T$ and H and E be real matrices of appropriate dimensions with F satisfying $FF^T \le I$, then $L + HFE + E^TF^TH^T < 0$ if and only if there exists a positive scalar $\varepsilon > 0$ such that $L + \varepsilon^{-1}HH^T + \varepsilon E^TE < 0$, or equivalently

$$\begin{bmatrix} L & H & \varepsilon E^T \\ H^T & -\varepsilon I & 0 \\ \varepsilon E & 0 & -\varepsilon I \end{bmatrix} < 0.$$

Lemma 3: For any real matrices X_{ij} for $i, j = 1, 2, \dots, r$ and $n > 0$ with appropriate dimensions, we have [35]

$$\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{r}\sum_{l=1}^{r} h_ih_jh_kh_l X_{ij}^T \Lambda X_{kl} \le \sum_{i=1}^{r}\sum_{j=1}^{r} h_ih_j X_{ij}^T \Lambda X_{ij}.$$

Theorem 1: For given controller parameters and a prescribed H_∞ performance $\gamma > 0$, the nominal fuzzy system (7) is exponentially stable if there exist matrices $P > 0$ and $Q_k > 0$, $k = 1, 2, \dots, h$, satisfying

$$\begin{bmatrix} \Pi_i & \star \\ 0.5\Sigma_{ii} & \bigwedge \end{bmatrix} < 0 \tag{10}$$

$$\begin{bmatrix} 4\Pi_i & \star \\ \Sigma_{ij} & \bigwedge \end{bmatrix} < 0, \quad 1 \le i < j \le r \tag{11}$$

where

$$\begin{aligned} \Pi_i = \mathrm{diag}\Big\{ &-P + \sum_{k=1}^{h}(d_T - d_t + 1)Q_k, \hat{\alpha}\check{A}_{di}^T\check{P}\check{A}_{di} \\ &- \mathrm{diag}\{Q_1, Q_2, \dots, Q_h\}, -\gamma^2 I\Big\} \end{aligned} \tag{12}$$

$$\hat{\alpha} = \mathrm{diag}\left\{\bar{\alpha}_1(1-\bar{\alpha}_1), \dots, \bar{\alpha}_h(1-\bar{\alpha}_h)\right\}$$

$$\check{A}_{di} = \mathrm{diag}\{\underbrace{\hat{A}_{di}, \dots, \hat{A}_{di}}_{h}\}$$

$$\check{C}_{ij} = \left[\sigma_1\hat{C}_{11ij}^TP, \dots, \sigma_s\hat{C}_{1sij}^TP, \rho_1\hat{C}_{k1ij}^TP, \dots, \rho_m\hat{C}_{kmij}^TP\right]^T$$

$$\check{P} = \mathrm{diag}\{\underbrace{P, \dots, P}_{s+m}\}$$

$$\bigwedge = \mathrm{diag}\{-\check{P}, -P, -I, \mathrm{diag}\{\underbrace{-I, \dots, -I}_{m}\}\}$$

$$\breve{P} = \mathrm{diag}\{\underbrace{P, \dots, P}_{h}\}$$

$$\hat{A}_{di} = \begin{bmatrix} A_{di} & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Sigma_{ij} = \begin{bmatrix} \check{C}_{ij} + \check{C}_{ji} & 0 & 0 \\ PA_{ij} + PA_{ji} & P\hat{Z}_{mi} + P\hat{Z}_{mj} & PD_{ij} + PD_{ji} \\ \bar{C}_{ij} + \bar{C}_{ji} & 0 & D_{3i} + D_{3j} \\ [0 \quad \rho_1 B_{2i} C_{kj1} + \rho_1 B_{2j} C_{ki1}] & 0 & 0 \\ \vdots & \vdots & \vdots \\ [0 \quad \rho_m B_{2i} C_{kjm} + \rho_m B_{2j} C_{kim}] & 0 & 0 \end{bmatrix}.$$

Proof:

Let

$$\Theta_j(k) = \{x(k - \tau_j(k), x(k - \tau_j(k) + 1, \dots, x(k)\}$$

$$\chi(k) = \{\Theta_1(k) \bigcup \Theta_2(k) \bigcup \dots \bigcup \Theta_h(k)\} = \bigcup_{j=1}^{h} \Theta_j(k)$$

where $j = 1, 2, \dots, h$. We consider the following Lyapunov functional for the system of (7): $V(\chi(k)) = \sum_{i=1}^{3} V_i(k)$, where

$$V_1(k) = \bar{x}^T(k) P \bar{x}$$

$$V_2(k) = \sum_{j=1}^{h} \sum_{i=k-\tau_j(k)}^{k-1} \bar{x}^T(i) Q_j \bar{x}(i)$$

$$V_3(k) = \sum_{j=1}^{h} \sum_{m=-d_M+1}^{-d_m} \sum_{i=k+m}^{k-1} \bar{x}^T(i) Q_j \bar{x}(i)$$

with $P > 0$, $Q_j > 0$ $(j = 1, 2, \dots, h)$ being matrices to be determined.

$$\begin{aligned} E[\Delta V | x(k)] &= E[V(\chi(k+1)) | \chi(k)] - V(\chi(k)) \\ &= E[(V(\chi(k+1)) - V(\chi(k))) | \chi(k)] \\ &= \sum_{i=1}^{3} E[\Delta V_i | \chi(k)]. \end{aligned} \tag{13}$$

According to (7), we have

$$\begin{aligned} &E\{\Delta V_1 | \chi(k)\} \\ &\quad = E\left[(\bar{x}^T(k+1) P \bar{x}(k+1) - \bar{x}^T(k) P \bar{x}(k)) | \chi(k)\right] \\ &\quad \le \xi^T(k) \sum_{i=1}^{r} \sum_{j=1}^{r} \Omega_{ij} \xi(k) \end{aligned}$$

where

$$\Omega_{ij} = E\left\{\begin{bmatrix} A_{ij}^T P A_{ij} + B_{ij}^T P B_{ij} - P & A_{ij}^T P \hat{Z}_{mi} & A_{ij}^T P D_{ij} \\ \star & \hat{Z}_{mi}^T P \hat{Z}_{mi} + \Delta \hat{Z}_{mi}^T P \Delta \hat{Z}_{mi} & \hat{Z}_{mi}^T P D_{ij} \\ \star & \star & D_{ij}^T P D_{ij} \end{bmatrix}\right\}$$

$$B_{ij} = \begin{bmatrix} 0 & 0 \\ B_{ki} \tilde{\Xi} C_j & 0 \end{bmatrix} + \begin{bmatrix} 0 & B_{1i} \tilde{\omega}(k) C_{kj} \\ 0 & 0 \end{bmatrix}$$

$$\begin{aligned} &E\{B_{ij}^T P B_{ij}\} \\ &\quad = \sum_{l=1}^{s} \sigma_l^2 \begin{bmatrix} 0 & 0 \\ B_{ki} C_{jl} & 0 \end{bmatrix}^T P \begin{bmatrix} 0 & 0 \\ B_{ki} C_{jl} & 0 \end{bmatrix} \\ &\qquad + \sum_{l=1}^{m} \rho_l^2 \begin{bmatrix} 0 & B_{1i} C_{kjl} \\ 0 & 0 \end{bmatrix}^T P \begin{bmatrix} 0 & B_{1i} C_{kjl} \\ 0 & 0 \end{bmatrix} \\ &\quad = (\check{P}^{-1} \check{C}_{lij})^T \check{P} (\check{P}^{-1} \check{C}_{lij}) \end{aligned}$$

$$\check{P} = \mathrm{diag}\{\underbrace{P, \dots, P}_{s+m}\}$$

$$\hat{C}_{1lij} = \begin{bmatrix} 0 & 0 \\ B_{ki} C_{jl} & 0 \end{bmatrix}$$

$$\hat{C}_{klij} = \begin{bmatrix} 0 & B_{1i} C_{kjl} \\ 0 & 0 \end{bmatrix}$$

$$\check{C}_{ij} = \left[\sigma_1 \hat{C}_{11ij}^T P, \dots, \sigma_s \hat{C}_{1sij}^T P, \rho_1 \hat{C}_{k1ij}^T P, \dots, \rho_m \hat{C}_{kmij}^T P\right]^T$$

$$\begin{aligned} &E\left\{\Delta \hat{Z}_{mi}^T P \Delta \hat{Z}_{mi}\right\} \\ &\quad = \sum_{m=1}^{h} \bar{\alpha}_m (1 - \bar{\alpha}_m) \begin{bmatrix} A_{di} & 0 \\ 0 & 0 \end{bmatrix}^T P \begin{bmatrix} A_{di} & 0 \\ 0 & 0 \end{bmatrix} \\ &\quad = \sum_{m=1}^{h} \hat{A}_{di}^T P \hat{A}_{di} = \hat{\alpha} \breve{A}_{di}^T \breve{P} \breve{A}_{di} \end{aligned}$$

$$\hat{\alpha} = \mathrm{diag}\{\bar{\alpha}_1(1 - \bar{\alpha}_1), \dots, \bar{\alpha}_h(1 - \bar{\alpha}_h)\}$$

$$\breve{A}_{di} = \mathrm{diag}\{\underbrace{\hat{A}_{di}, \dots, \hat{A}_{di}}_{h}\}$$

$$\begin{aligned} E\{\Delta V_2 | \chi(k)\} \le E\Bigg\{ &\sum_{j=1}^{h} (\bar{x}^T(k) Q_j \bar{x}(k) \\ &- \bar{x}^T(k - \tau_j(k)) Q_j \bar{x}(k - \tau_j(k)) \\ &+ \sum_{i=k-d_M+1}^{k-d_m} \bar{x}^T(i) Q_j \bar{x}(i)) | \chi(k) \Bigg\} \end{aligned}$$

$$\begin{aligned} E\{\Delta V_3 | \chi(k)\} = E\Bigg\{ &\sum_{j=1}^{h} ((d_T - d_t) \bar{x}^T(k) Q_j \bar{x}(k) \\ &- \sum_{i=k-d_m+1}^{k-d_m} \bar{x}^T(i) Q_j \bar{x}(i)) | \chi(k) \Bigg\}. \end{aligned}$$

It is clear that

$$E\{\Delta V_2 | \chi(k)\} + E\{\Delta V_3 | \chi(k)\} \le \xi^T(k) T_{ij} \xi(k)$$

with

$$\begin{aligned} T_{ij} = \mathrm{diag}\Bigg\{ &\sum_{k=1}^{h} (d_T - d_t + 1) Q_k, \\ &- \mathrm{diag}\{Q_1, Q_2, \dots, Q_h\}, 0 \Bigg\}. \end{aligned}$$

Therefore, we have $E\{\Delta V|\chi(k)\} \leq \xi^T(k)\Gamma_{ij}\xi(k)$, where $\Gamma_{ij} = \Omega_{ij} + T_{ij}$. Due to

$$\begin{aligned}
&E\left\{z^T(k)z(k) - \gamma^2 v^T(k)v(k)\right\}\\
&\quad\leq \xi(k)\sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j E\left\{[\bar{C}_{ij} + \bar{\bar{C}}_{ij}, 0, D_{3i}]^T\right.\\
&\qquad\left.\times[\bar{C}_{ij} + \bar{\bar{C}}_{ij}, 0, D_{3i}] - \mathrm{diag}\{0, 0, \gamma^2 I\}\right\}\xi(k)
\end{aligned}$$

we can obtain

$$\begin{aligned}
&E\left\{z^T(k)z(k) - \gamma^2 v^T(k)v(k) + \Delta V(k)\right\}\\
&\quad\leq \xi^T(k)(\Omega_{ij}^T \mathrm{diag}\{P, I\}\Omega_{ij}\\
&\qquad + \mathcal{Z}_{ij}^T \mathrm{diag}\{\check{P}, I\}\mathcal{Z}_{ij} + \bar{P})\xi(k)
\end{aligned}$$

where

$$\Omega_{ij} = \begin{bmatrix} A_{ij} & \hat{Z}_{mi} & D_{ij}\\ \bar{C}_{ij} & 0 & D_{3i}\end{bmatrix}$$

$$\eth_{kijt} = \begin{bmatrix} \begin{bmatrix} 0 & \rho_t B_{2i} C_{kjt}\end{bmatrix} & 0 & 0\end{bmatrix}^T$$

$$\mathfrak{D}_{ij} = \begin{bmatrix} \eth_{kij1} & \dots & \eth_{kijm}\end{bmatrix}^T$$

$$\mathcal{Z}_{ij} = \begin{bmatrix} [\check{P}^{-1}\check{C}_{ij}, 0, 0]\\ \mathfrak{D}_{ij}\end{bmatrix}$$

$$\begin{aligned}
\bar{P} = \mathrm{diag}\Bigg\{ &- P + \sum_{k=1}^{h}(d_T - d_t + 1)Q_k, \hat{\alpha}\breve{A}_{di}^T\check{P}\breve{A}_{di}\\
&- \mathrm{diag}\{Q_1, Q_2, \dots, Q_h\}, -\gamma^2 I\Bigg\}.
\end{aligned}$$

Define $J(n) = E\sum_{k=0}^{n}[z^T(k)z(k) - \gamma^2 v^T(k)v(k)]$, we have

$$\begin{aligned}
J(n) = {} & E\sum_{k=0}^{n}\left[z^T(k)z(k) - \gamma^2 v^T(k)v(k) + \Delta V(\chi(k))\right]\\
&- EV(\chi(n+1))\\
\leq {} & E\sum_{k=0}^{n}\left[z^T(k)z(k) - \gamma^2 v^T(k)v(k) + \Delta V(\chi(k))\right]\\
\leq {} & \sum_{k=0}^{n}\sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j \xi^T(k)(\Omega_{ij}^T \mathrm{diag}\{P, I\}\Omega_{ij}\\
&+ \mathcal{Z}_{ij}^T \mathrm{diag}\{\check{P}, I\}\mathcal{Z}_{ij} + \bar{P})\xi(k)\\
= {} & \sum_{k=0}^{n}\sum_{i=1}^{r} h_i^2 \xi^T(k)(\Omega_{ii}^T \mathrm{diag}\{P, I\}\Omega_{ii}\\
&+ \mathcal{Z}_{ii}^T \mathrm{diag}\{\check{P}, I\}\mathcal{Z}_{ii} + \bar{P})\xi(k)\\
&+ \frac{1}{2}\sum_{k=0}^{n}\sum_{j=1, i<j}^{r} h_i h_j \xi^T(k)\\
&\times\Big[(\Omega_{ij} + \Omega_{ji})^T \mathrm{diag}\{P, I\}(\Omega_{ij} + \Omega_{ji})\\
&+ (\mathcal{Z}_{ij} + \mathcal{Z}_{ji})^T \mathrm{diag}\{\check{P}, I\}(\mathcal{Z}_{ij} + \mathcal{Z}_{ji}) + 4\bar{P}\Big]\xi(k).
\end{aligned}$$

According to Schur complement, we can conclude from (10) and (11) that $J(n) < 0$. Letting $n \to \infty$, we have

$$\sum_{n}^{\infty} E\left\{\|z(k)\|^2\right\} \leq \gamma^2 \sum_{n}^{\infty} E\left\{\|v(k)\|^2\right\}.$$

According to Schur complement again, we know that $E\{\Delta V|x(k)\} < 0$ if and only if (10) and (11) hold true. Furthermore, one can easily verify the fact that the discrete-time nominal (7) with $v(k) = 0$ is exponentially stable. ■

4 Design of Robust H_∞ Fuzzy Controller

In this section, we are devoted to how to determine the controller parameters in (6) such that the closed-loop system (7) is exponentially stable with H_∞ performace.

By Theorem 1, one can easily draw the conclusion as follow:

Theorem 2: For a prescribed constant $\gamma > 0$, the nominal fuzzy system (7) is exponentially stable if there exist positive definite matrices $P > 0$, $L > 0$, $Q_k > 0$ $(k = 1, 2, \dots, h)$, and K_i and $\bar{C}_{ki}$ such that

$$\Gamma_1 = \begin{bmatrix} \Pi_i & \star\\ 0.5\bar{\Sigma}_{ii} & \bar{\Lambda}\end{bmatrix} < 0,\quad i = 1, 2, \dots, r \tag{14}$$

$$\Gamma_2 = \begin{bmatrix} 4\Pi_i & \star\\ \bar{\Sigma}_{ij} & \bar{\Lambda}\end{bmatrix} < 0,\quad 1 \leq i < j \leq r \tag{15}$$

$$PL = I \tag{16}$$

hold, then the nominal system (7) is exponentially stable with disturbance attenuation γ, where $\bar{\bigwedge} = \mathrm{diag}\{-\bar{L}, -L, -I, \mathrm{diag}\{\underbrace{-I, \dots, -I}_{m}\}\}$

$$\bar{\Sigma}_{ij} = \begin{bmatrix} \Phi_{11ij} + \Phi_{11ji} & 0 & 0\\ \Phi_{21ij} + \Phi_{21ji} & \Phi_{22ij} + \Phi_{22ji} & \Phi_{23ij} + \Phi_{23ji}\\ \Phi_{31ij} + \Phi_{31ji} & 0 & \Phi_{33ij} + \Phi_{33ji}\\ \Phi_{41ij} + \Phi_{41ji} & 0 & 0\end{bmatrix} \tag{17}$$

$$I_l = \mathrm{diag}\{\underbrace{0, \dots, 0}_{l-1}, 1, \underbrace{0, \dots, 0}_{m-l}\},\quad K_i = \begin{bmatrix} A_{ki} & B_{ki}\end{bmatrix}$$

$$\bar{C}_{ki} = \begin{bmatrix} 0 & C_{ki}\end{bmatrix},\quad \bar{E} = \begin{bmatrix} 0\\ I\end{bmatrix},\quad \bar{\bar{E}} = \begin{bmatrix} I\\ 0\end{bmatrix}$$

$$\bar{A}_i = \begin{bmatrix} A_i & 0\\ 0 & 0\end{bmatrix},\quad \bar{B}_{1i} = \begin{bmatrix} B_{1i}\\ 0\end{bmatrix},\quad R_{il} = \begin{bmatrix} 0 & 0\\ C_{il} & 0\end{bmatrix}$$

$$\bar{D}_{1i} = \begin{bmatrix} D_{1i}\\ 0\end{bmatrix},\quad \bar{D}_{2i} = \begin{bmatrix} 0\\ D_{2i}\end{bmatrix}$$

$$\Phi_{11ij} = \begin{bmatrix} \sigma_1\bar{E}K_iR_{j1}\\ \vdots\\ \sigma_s\bar{E}K_iR_{js}\\ \rho_1\bar{E}\beta_{1i}I_1\bar{C}_{kj}\\ \vdots\\ \rho_m\bar{E}\beta_{1i}I_m\bar{C}_{kj}\end{bmatrix},\quad \Phi_{41ij} = \begin{bmatrix} \rho_1 B_{2i}I_1\bar{C}_{kj}\\ \vdots\\ \rho_m B_{2i}I_m\bar{C}_{kj}\end{bmatrix}$$

$$\Phi_{21ij} = \bar{A}_i + \bar{E}K_i\bar{R}_j + \bar{B}_{1i}\mathrm{diag}\{w_1, \dots, w_m\}\bar{C}_{kj}$$

$$\Phi_{31ij} = \bar{C}_{zi} + B_{2i}\mathrm{diag}\{w_1, \dots, w_m\}\bar{C}_{kj}$$

$$\bar{C}_{zi} = \begin{bmatrix} C_{zi} & 0\end{bmatrix},\quad \bar{L} = \mathrm{diag}\{\underbrace{L, \dots, L}_{s+m}\}$$

$$\Phi_{22ij} = \hat{Z}_{mi},\quad \Phi_{23ij} = D_{ij},\quad \Phi_{33ij} = D_{3i}.$$

Proof: We rewrite the parameters in Theorem 1 in the following form:

$$\begin{aligned}
&A_{ij} = \bar{A}_i + \bar{E}K_i\bar{R}_j + \bar{B}_{1i}\text{diag}\{w_1,\dots,w_m\}\bar{C}_{kj}\\
&\hat{C}_{lij} = \bar{E}K_iR_{jl}\\
&\bar{C}_{ij} = \bar{C}_{zi} + B_{2i}\text{diag}\{w_1,\dots,w_m\}\bar{C}_{kj}\\
&D_{ij} = \bar{D}_{1i} + \bar{D}_{1i}K_i\bar{D}_{2j}.
\end{aligned}$$

Pre- and post-multiplying the (10) and (11) by diag$\{I, I, I, \check{P}^{-1}, P^{-1}, \underbrace{I,\dots,I}_{m}\}$ and Letting $L = P^{-1}$, we have (14)−(16) and complete the proof easily. Now we will point out that the robust H_∞ controller parameters can be determined in light of Theorem 2. ■

Theorem 3: For given scalar $\gamma > 0$, if there exist positive define matrices $P > 0$, $L > 0$, $Q_k > 0$ $(k = 1, 2, \dots, h)$, and matrices K_i, $\bar{C}_{ki}$ of proper dimensions and a constant $\varepsilon > 0$ such that

$$\begin{bmatrix} \Gamma_1 & \star \\ \Xi_{ii} & \text{diag}\{-\varepsilon I, -\varepsilon I\} \end{bmatrix} < 0,\quad i = 1, 2, \dots, r \tag{18}$$

$$\begin{bmatrix} \Gamma_2 & \star \\ \Xi_{ij} & \text{diag}\{-\varepsilon I, -\varepsilon I\} \end{bmatrix} < 0,\quad 1 \le i < j \le r \tag{19}$$

$$PL = I \tag{20}$$

hold, where

$$\Xi_{ii} = \begin{bmatrix} 0 & 0 & 0 & 0 & [H_{ai}^T\ \ 0] & H_{ci}^T & 0 \\ \varepsilon[E\ \ 0] & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\Xi_{ij} = \begin{bmatrix} 0 & 0 & 0 & 0 & [H_{ai}^T + H_{aj}^T\ \ 0] & H_{ci}^T + H_{cj}^T & 0 \\ \varepsilon[E\ \ 0] & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

then the uncertain fuzzy system (7) is exponentially stable and the controller parameters K_i and $\bar{C}_{ki}$ can be obtained naturally.

Proof: Replace $\bar{A}_i$, $\bar{A}_j$, $\bar{C}_{zi}$, and $\bar{C}_{zj}$ in Theorem 2 by $\bar{A}_i + \triangle\bar{A}_i(k)$, $\bar{A}_j\triangle\bar{A}_j(k)$, $\bar{C}_{zi} + \triangle\bar{C}_{zi}(k)$, and $\bar{C}_{zj} + \triangle\bar{C}_{zj}(k)$, respectively, where

$$\triangle\bar{A}_i(k) = \begin{bmatrix} \triangle A_i(k) & 0 \\ 0 & 0 \end{bmatrix},\quad \triangle\bar{C}_{zi}(k) = [\triangle C_{zi}(k)\ \ 0].$$

According to Lemma 1, (18) and (19) can be rewritten as follows:

$$\begin{aligned}
&\Gamma_1 + H_1F(k)E + E^TF(k)^TH_1^T < 0\\
&\Gamma_2 + H_2F(k)E + E^TF(k)^TH_2^T < 0
\end{aligned}$$

where

$$\begin{aligned}
&E = [E\ \ 0]\\
&H_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & [H_{ai}^T\ \ 0] & H_{ci}^T & 0 \end{bmatrix}\\
&H_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & [H_{ai}^T + H_{aj}^T\ \ 0] & H_{ci}^T + H_{cj}^T & 0 \end{bmatrix}.
\end{aligned}$$

According to Lemma 1 along with Schur complement, we can easily obtain (18) and (19). ■

In order to solve (18), (19) and (20), the cone-complementarity linearization (CCL) algorithm proposed in [36] and [37] is used in this note.

The nonlinear minimization problem: $\min \text{tr}(PL)$ subject to (18) and (19) and

$$\begin{bmatrix} P & I \\ I & L \end{bmatrix} \ge 0. \tag{21}$$

The following algorithm [5] is borrowed to solve the above problem.

Algorithm 1:

Step 1: Find a feasible set $(P_0, L_0, Q_{k(0)}, K_{i(0)}, \bar{C}_{ki(0)})$ satisfying (18), (19) and (21). Set $q = 0$.

Step 2: Solving the linear matrix inequality (LMI) problem, $\min \text{tr}(PL_{(0)} + P_{(0)}L)$ subject to (18), (19) and (21).

Step 3: Substitute the obtained matrix variables (P, L, $Q_k, K_{i(0)}, \bar{C}_{ki}$) into (14) and (15). If conditions(14) and (15) are satisfied with $|\text{tr}(PL) - n| < \delta$ for some sufficiently small scalar $\delta > 0$, then output the feasible solutions. Exit.

Step 4: If $q > N$, where N is the maximum number of iterations allowed, then output the feasible solutions (P, L, Q_k, K_i, $\bar{C}_{ki}$), and exit. Else, set $q = q + 1$, and goto Step 2.

5 An Illustrative Example

we give an illustrative examples to explain the proposed model is effective and feasible in this section.

Example 1: Consider a T-S fuzzy model (1). The rules are given as follows:

Plant Rule 1: If $x_1(k)$ is $h_1(x_1(k))$ then

$$\begin{cases} x(k+1) = A_1(k)x(k) + A_{d1}\sum\limits_{m=1}^{h}\alpha_m(k)x(k-\tau_m(k)) \\ \qquad + B_{11}u(k) + D_{11}v(k) \\ y(k) = \Xi C_1x(k) + D_{21}v(k) \\ z(k) = C_{z1}(k)x(k) + B_{21}u(k) + D_{31}v(k) \end{cases} \tag{21}$$

Plant Rule 2: If $x_1(k)$ is $h_2(x_1(k))$ then

$$\begin{cases} x(k+1) = A_2(k)x(k) + A_{d2}\sum\limits_{m=1}^{h}\alpha_m(k)x(k-\tau_m(k)) \\ \qquad + B_{12}u(k) + D_{12}v(k) \\ y(k) = \Xi C_2x(k) + D_{22}v(k) \\ z(k) = C_{z2}(k)x(k) + B_{22}u(k) + D_{32}v(k) \end{cases} \tag{22}$$

The given model parameters are written as follows:

$$A_1 = \begin{bmatrix} 1 & 0.2 & 0 \\ 0.1 & 0.1 & 0.1 \\ 0.1 & 0.2 & 0.2 \end{bmatrix},\quad D_{11} = \begin{bmatrix} 0.1 \\ 0 \\ 0 \end{bmatrix}$$

$$A_{d1} = \begin{bmatrix} 0.03 & 0 & -0.01 \\ 0.02 & 0.03 & 0 \\ 0.04 & 0.05 & -0.1 \end{bmatrix},\quad B_{11} = \begin{bmatrix} 1 & 1 \\ 0.4 & 1 \\ 0 & 1 \end{bmatrix}$$

$$D_{31} = \begin{bmatrix} -0.1 \\ 0 \\ 0.1 \end{bmatrix},\quad C_1 = \begin{bmatrix} 1 & 0.8 & 0.7 \\ -0.6 & 0.9 & 0.6 \end{bmatrix}$$

$$C_2 = \begin{bmatrix} 0.1 & 0.8 & 0.7 \\ -0.6 & 0.9 & 0.6 \end{bmatrix},\quad D_{21} = \begin{bmatrix} 0.15 \\ 0 \end{bmatrix}$$

$$D_{22} = \begin{bmatrix} 0.1 \\ 0 \end{bmatrix},\quad C_{z1} = \begin{bmatrix} 0.2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0.1 \end{bmatrix}$$

$$B_{21}=\begin{bmatrix}1&1\\0&1\\0&1\end{bmatrix},\quad H_{a1}=\begin{bmatrix}0.1\\0.1\\0.1\end{bmatrix},\quad H_{c1}=\begin{bmatrix}0.1\\0\\0.1\end{bmatrix}$$

$$H_{a2}=\begin{bmatrix}0.1\\0\\0.1\end{bmatrix},\quad H_{c2}=\begin{bmatrix}0.1\\0\\0.5\end{bmatrix},\quad D_{32}=\begin{bmatrix}0.1\\0\\0.1\end{bmatrix}$$

$$E=\begin{bmatrix}0.1\\0.1\\0.1\end{bmatrix}^T,\quad A_2=\begin{bmatrix}1&-0.38&0\\-0.2&0&0.21\\0.1&0&-0.55\end{bmatrix}$$

$$B_{12}=\begin{bmatrix}1&0\\1&1\\0&1\end{bmatrix},\quad A_{d2}=\begin{bmatrix}0&0.01&-0.01\\0.02&0.03&0\\0.04&0.05&-0.1\end{bmatrix}$$

$$D_{12}=\begin{bmatrix}0.1\\0\\0.1\end{bmatrix},\quad C_{z2}=\begin{bmatrix}0.1&0&0\\0.2&0&0.2\\0&0.1&0.2\end{bmatrix}$$

$$B_{22}=\begin{bmatrix}1&0\\0&1\\1&1\end{bmatrix}.$$

Assume that the time-varying communication delays satisfy $2\leq\tau_m\leq 6$ $(m=1,2)$ and

$$\bar\alpha_1=E\{\alpha_1(k)\}=0.8,\quad \bar\alpha_2=E\{\alpha_2(k)\}=0.6$$
$$\bar\omega_1=E\{\omega_1(k)\}=0.4,\quad \bar\omega_2=E\{\omega_2(k)\}=0.6.$$

Assume also that the probabilistic density functions of β_1 and β_2 in $[0 \quad 1]$ are read as

$$q_1(s_1)=\begin{cases}0, & s_1=0\\ 0.1, & s_2=0.5\\ 0.9, & s_3=1\end{cases},\quad q_2(s_2)=\begin{cases}0, & s_2=0\\ 0.2, & s_2=0.5\\ 0.8, & s_3=1\end{cases}. \tag{23}$$

The membership functions are described as

$$h_1=\begin{cases}1, & x_0(1)=0\\ \left|\dfrac{\sin(x_0(1))}{x_0(1)}\right|, & \text{else}\end{cases}$$
$$h_2=1-h_1. \tag{24}$$

Now, we are to design a dynamic-output feedback paralleled controller in the form of (6) such that (7) is exponentially stable with a given H_∞ norm bound γ. In the example, we assume $\gamma=0.9$ and obtain the desired H_∞ controller parameters as follows

$$A_{k1}=\begin{bmatrix}-0.0127&-0.0083&-0.0317\\0.0229&0.0149&0.0221\\-0.0588&-0.0429&-0.0654\end{bmatrix}$$

$$A_{k2}=\begin{bmatrix}-0.1365&-0.1296&-0.0570\\-0.0107&-0.0095&0.0239\\-0.0125&-0.0129&-0.0260\end{bmatrix}$$

$$B_{k1}=\begin{bmatrix}-0.3236&0.1389\\0.0291&-0.0043\\-0.3077&0.1867\end{bmatrix}$$

$$B_{k2}=\begin{bmatrix}0.1664&0.0834\\0.1374&-0.0712\\-0.4340&0.5688\end{bmatrix}$$

$$C_{k1}=\begin{bmatrix}0.1355&0.0856&0.1789\\0.0311&0.0209&0.0372\end{bmatrix}$$

$$C_{k2}=\begin{bmatrix}0.0110&0.0464&0.0731\\0.0832&0.0622&0.0502\end{bmatrix}.$$

We take the initial conditions $x_0=[1 \quad 0 \quad -1]^T$, $x_{c0}=[0 \quad 0 \quad 0]^T$ for the simulation purpose and let external disturbance $v(k)=0$. Fig. 2 depicts the state responses for the uncontrolled fuzzy systems, which are unstable. We can see the fact that the closed-loop fuzzy systems are exponentially stable from the Fig. 3.

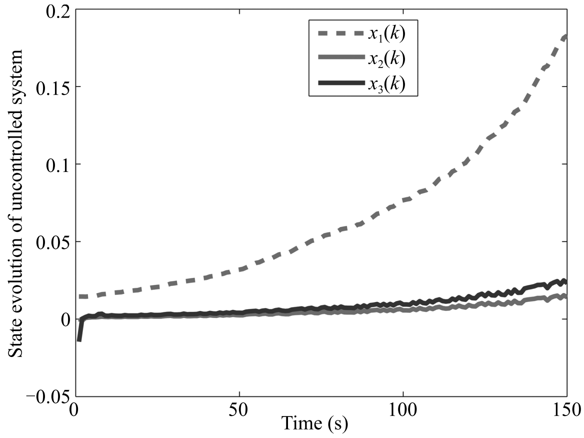

Fig. 2. State evolution $x(k)$ of uncontrolled systems.

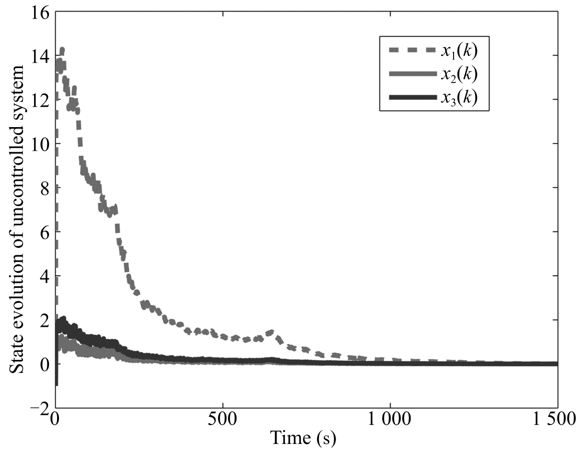

Fig. 3. State evolution $x(k)$ of controlled systems.

In order to illustrate the disturbance-attenuation performance,we take the external disturbance

$$v(k)=\begin{cases}0.3, & 20\leq k\leq 30\\ -0.2, & 50\leq k\leq 60\\ 0, & \text{else.}\end{cases}$$

Fig. 4 presents the controller-state evolution $x_c(k)$, Fig. 5 plots the state evolution of the controlled output $z(k)$, and Fig. 6 shows the output feedback controller. From Figs. 3−6, one can see that the convergence rate is rapid and effective. By the above simulation results, we can draw the conclusion that our theoretical analysis to the robust H_∞ fuzzy-control problem is right completely.

Remark 2: The above simulation is performed on the basis of the software MATLAB 7.0 and the cone-complementarity linearization algorithm may takes several minutes because of choosing initial feasible set.

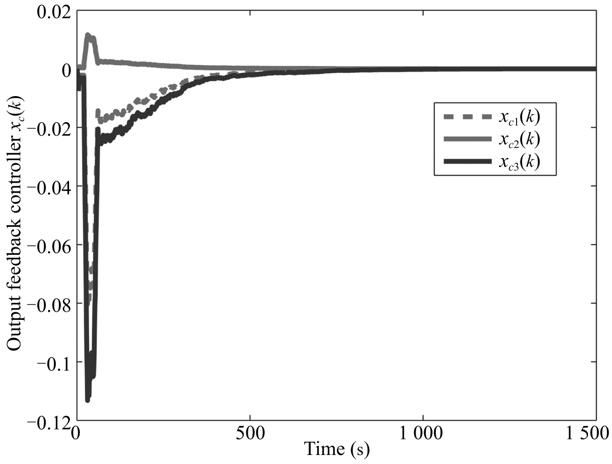

Fig. 4. Output feedback controller $x_c(k)$.

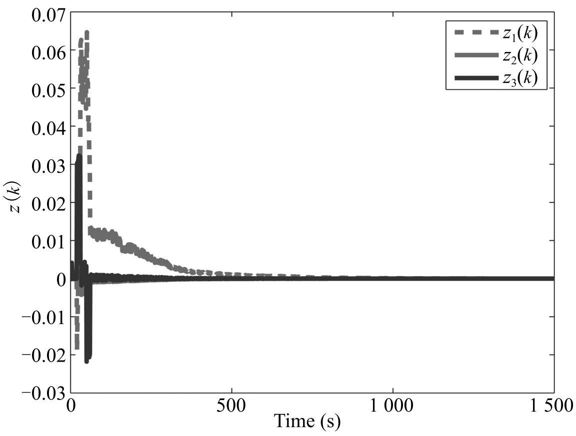

Fig. 5. Controlled output $z(k)$.

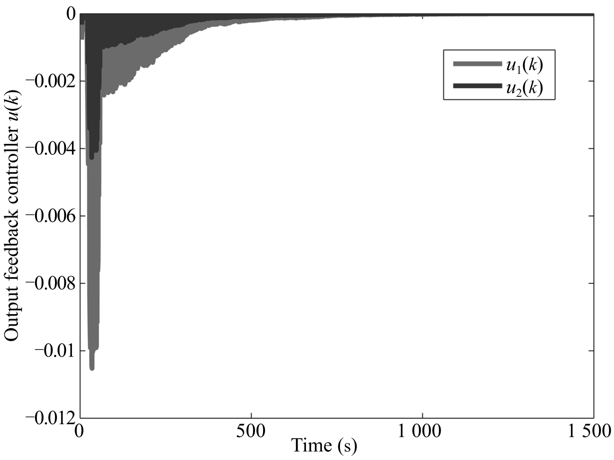

Fig. 6. Output feedback controller $u(k)$.

6 Conclusion

In this paper, we establish general networked systems model with multiple time-varying random communication delays and multiple missing measurements as weil as the random missing control and discuss its robust H_∞ fuzzy-output feedback-control problem. The proposed system model includes parameter uncertainties, multiple stochastic time-varying delays, multiple missing measurements, and stochastic control input missing. The control strategy adopts the parallel distributed compensation. We obtain the sufficient conditions on the robustly exponential stability of the closed-loop T-S fuzzy-control system by using the CCL algorithm and the explicit expression of the desired controller parameters. An illustrative simulation example further shows that the fuzzy-control method to the proposed new control model is feasible and the new control model can be used for future applications. Whether to construct piecewise Lyapunov functions [8] to solve the proposed control model or not is an interesting topic and in active thought.

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