Delay-distribution-dependent Robust Stability Analysis of Uncertain Lurie Systems with Time-varying Delay

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Abstract In this paper, the problem of robust absolute stability of Lurie system with probabilistic time-varying delay and norm-bounded parametric uncertainty is considered. The time delay variation range is divided into two sub-intervals. By considering the probability distribution of the time-varying delay between the two sub-intervals and the knowledge of the delay variation range, a novel linear matrix inequalities (LMIs) based stability condition is derived by defining a Lyapunov Krasovskii functional. It is illustrated with the help of numerical examples that the derived stability criteria can lead to less conservative results as compared to the results obtained by the traditional method of using the delay variation range information only.

Key words Lurie system, time-delay, probability distribution, linear matrix inequality (LMI)

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The problem of absolute stability of Lurie system has been widely studied by many researchers due to its theoretical as well as practical significance[1−3]. Most of the physical systems possess time delay. Hence, the problem of absolute stability of time delay Lurie control system has been the topic of considerable research[4−16]. It is known that the delay-dependent stability conditions are generally less conservative than delay-independent stability conditions[4−5], especially when the size of delay is small. Therefore, more efforts have been made to derive the delay-dependent stability conditions for the absolute stability of Lurie system[6−16]. In [6], model transformation along with Moon′s inequality was used for deriving the absolute stability condition. In [7], a new stability condition was proposed using Jensen′s inequality. Neither model transformation nor bounding technique for cross terms was used therein. A less conservative stability criteria was devised in [8] by employing free-weighting matrices to express the relationship between the terms in Leibniz-Newton formula. However, [11] used a discretized Lyapunov functional approach to deal with the problem of absolute stability for a class of nonlinear neutral systems. Further, [12] presents a new absolute stability criteria by using a Lur′e-Postnikov function as the Lyapunov-like function. Nevertheless, in all these results, the time delay was assumed to be constant.

In the recent years, some results have been reported for the absolute stability of Lurie system with time varying delay[13−16]. The results derived in [13] by using integral inequality approach were further improved in [14] by including all the useful terms in the derivative of the Lyapunov-Krasovskii functional. In [15], the problem of robust absolute stability was studied by defining a Lyapunov functional which divided the delay interval into two subintervals. The delay interval was divided into multiple segments and a different Lyapunov matrix was defined for each segment in [16] to reduce the conservatism in the stability analysis of neutral Lurie system. However, it may be noted that in all the above results, only the information about the delay variation range was considered to derive the stability conditions. In some applications, for instance networked control systems, the probability of delay taking a large value is small. In such cases, the knowledge of delay distribution is an important parameter to describe the network system condition. As shown in [17], this additional information in the form of probability distribution of delay along with the knowledge of delay variation range can be utilized for the stability analysis to give less conservative results.

In this paper, we are concerned with stability analysis of Lurie system by utilizing the information about both the delay variation range and the delay distribution probability. To the best of our knowledge, the problem of stability analysis of Lurie system by considering the information of the probability distribution of time varying delay has not been considered so far. In this paper, by utilizing the information of probability distribution of the stochastic time delay, a mean square stability (MSS) criteria is derived in terms of LMIs for the time-delayed Lurie system, which depends on both delay variation range and the delay distribution information. Physical systems usually suffer from uncertainties that arise due to unmodeled dynamics and variation in system parameters. These parametric uncertainties may also result in instability of the system. Therefore, to analyze the effect of these uncertainties on the stability analysis, we extend the derived condition to the above systems with norm-bounded parametric uncertainties.

The rest of this paper is organized as follows. Section 1 describes the problem statement. In Section 2, a delay dependent criteria for the stability of time-delayed Lurie system is derived in terms of LMIs. The derived condition is extended to the above system with norm bounded parametric uncertainties. Further, we prove that the result presented in Theorem 1 of [18], for the absolute stability of Lurie system, is a special case of our results. The effectiveness of the proposed approach is illustrated in Section 3 with the help of numerical examples. Finally, Section 4 gives the concluding remarks.

Notations. The following notations are used throughout the paper. The superscript "T" denotes the transpose of a matrix. $\mathbf{R}^n$ and $\mathbf{R}^{n\times m}$ stand for the set of real vectors with dimension n and real matrix of size $n \times m$, respectively. $P > 0$ denotes a symmetric positive definite matrix. I and 0 mean the identity matrix and zero matrix, respectively with compatible dimensions. In a symmetric matrix, the symbol "$*$" is used to denote the term that is induced by symmetry. Wherever the dimensions of the matrices are not mentioned, they are assumed to be of compatible dimensions. $\mathrm{E}\{\cdot\}$ denotes the mathematical expectation.

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1 Problem statement

Consider the following Lurie system:

$$\begin{aligned}\dot{x}(t) &= Ax(t) + Bx(t-h(t)) + D\omega(t)\\ z(t) &= Mx(t) + Nx(t-h(t))\\ \omega(t) &= -\varphi(t, z(t))\\ x(t) &= \psi(t),\quad t\in[-h_m,\,0]\end{aligned} \tag{1}$$

where $x(t)\in\mathbf{R}^n$, $\omega(t)\in\mathbf{R}^m$, and $z(t)\in\mathbf{R}^m$ are the state vector, input vector, and output vector of the system, respectively. A, B, D, M, and N are real constant matrices with appropriate dimensions. $\varphi(t, z(t))$ is a class of memoryless, time-varying, nonlinear vector-valued functions that are piecewise continuous in t and globally Lipschitz in $z(t)$. $\varphi(t, 0) = 0$ and satisfies the following sector condition for $\forall\, t\geq 0$, $\forall\, z(t)\in\mathbf{R}^m$:

$$\varphi^{\mathrm{T}}(t, z(t))[\varphi(t, z(t) - Kz(t)] \leq 0 \tag{2}$$

$$[\varphi(t, z(t)) - K_1 z(t)]^{\mathrm{T}}[\varphi(t, z(t) - K_2 z(t)] \leq 0 \tag{3}$$

where K_1 and K_2 are constant real matrices of appropriate dimensions and $K = K_2 - K_1$ is a symmetric positive definite matrix. The nonlinear function $\varphi(t, z(t))$ satisfying (2) is said to belong to the sector $[0, K]$, whereas when $\varphi(t, z(t))$ satisfies (3), it is said to belong to the sector $[K_1, K_2]$. The time varying delay satisfies the following:

$$0\leq h(t)\leq h_m < \infty,\quad \forall\, t\geq 0 \tag{4}$$

The initial condition, $\psi(t)$, is a differentiable vector-valued function of $t\in[-h_m, 0]$.

The information of the probabilistic distribution of the time-varying delay is incorporated in (1) by defining a Bernoulli distributed random variable $\rho(t)$ with the expectation ρ_0 $(0\leq\rho_0\leq 1)$ as follows:

$$\rho(t) = \begin{cases} 1, & h(t)\in[0,\,h_0]\\ 0, & h(t)\in(h_0,\,h_m]\end{cases} \tag{5}$$

where $h_0\in[0, h_m]$ is a constant.

From (5) we get $\mathrm{Prob}\{\rho(t) = 1\} = \mathrm{E}(\rho(t)) = \rho_0$ and $\mathrm{Prob}\{\rho(t) = 0\} = 1 - \mathrm{E}(\rho(t)) = 1 - \rho_0$.

By using (5), system (1) can be re-expressed as follows:

$$\begin{aligned}\dot{x}(t) &= Ax(t) + \rho(t)Bx(t-h_1(t)) +\\ &\quad (1-\rho(t))Bx(t-h_2(t)) + D\omega(t)\\ z(t) &= Mx(t) + \rho(t)Nx(t-h_1(t)) +\\ &\quad (1-\rho(t))Nx(t-h_2(t))\\ x(t) &= \psi(t)\,,\qquad t\in[-h_m,\;0]\end{aligned} \tag{6}$$

where $h_1\in[0, h_0]$ and $h_2\in[h_0, h_m]$ are defined as

$$h_1(t) = \begin{cases} h(t), & h(t)\in[0\,,\,h_0]\\ h_0, & h(t)\in(h_0\,,\,h_m]\end{cases} \tag{7}$$

$$h_2(t) = \begin{cases} h_0, & h(t)\in[0\,,\,h_0]\\ h(t), & h(t)\in(h_0\,,\,h_m]\end{cases} \tag{8}$$

Remark 1. The random variable $\rho(t)$ describes the distribution information of the delay. It is clear from the definition of $\rho(t)$ that $\mathrm{E}(\rho(t)-\rho_0) = 0$, $\mathrm{E}(\rho^2(t)) = \rho_0$ and $\mathrm{E}(\rho(t)-\rho_0)^2 = \rho_0(1-\rho_0)$.

Remark 2. It can be observed that the system representations (1) and (6) are equivalent. Therefore, the stability of (1) can be deduced from the stability analysis of (6).

Further, the stability analysis for system (1) is extended to the uncertain systems represented by

$$\begin{aligned}\dot{x}(t) &= \bar{A}x(t) + \bar{B}x(t-h(t)) + \bar{D}\omega(t)\\ z(t) &= Mx(t) + Nx(t-h(t))\end{aligned} \tag{9}$$

where $\bar{A} = A + \Delta A$, $\bar{B} = B + \Delta B$ and $\bar{D} = D + \Delta D$. ΔA, ΔB and ΔD are real valued matrix functions representing parameter uncertainties, which are assumed to have the following form:

$$[\Delta A\ \ \Delta B\ \ \Delta D] = HF(t)[E_a\ \ E_b\ \ E_d] \tag{10}$$

where H, E_a, E_b and E_d are known constant matrices of appropriate dimensions, and $F(t)$ is an unknown real and possibly time-varying matrix with Lebesgue-measurable elements satisfying:

$$F^{\mathrm{T}}F(t)\leq I \tag{11}$$

By using (5), the uncertain Lurie system (9) can be rewritten as

$$\begin{aligned}\dot{x}(t) &= \bar{A}x(t) + \rho(t)\bar{B}x(t-h_1(t)) +\\ &\quad (1-\rho(t))\bar{B}x(t-h_2(t)) + \bar{D}\omega(t)\\ z(t) &= Mx(t) + \rho(t)Nx(t-h_1(t)) +\\ &\quad (1-\rho(t))Nx(t-h_2(t))\end{aligned} \tag{12}$$

The following definitions and lemmas will be used to derive the main results of this paper.

Definition 1[19]**.** Denote $x_t(s) = x(t+s)$, $-h_m\leq s\leq 0$. For a given function $V(x_t)$, its infinitesimal operator L is defined as

$$LV(x_t) = \lim_{\Delta\to 0^+}\frac{1}{\Delta}[\mathrm{E}(V(x_{t+\Delta})|x_t) - V(x_t)] \tag{13}$$

Definition 2[20]**.** System (6) is said to be mean square stable if for any $\epsilon > 0$ there is a $\delta(\epsilon)$ such that

$$\mathrm{E}\|\ x(t)\ \|^2 < \epsilon,\ \forall\, t > 0$$

when

$$\sup_{-h_m\leq s\leq 0}\mathrm{E}\|\ \psi(s)\ \|^2 < \delta(\epsilon)$$

Lemma 1 (Schur complement lemma[21]**).** Given constant matrices Ω_1, Ω_2 and Ω_3 of appropriate dimensions, with Ω_1 and Ω_2 symmetric, then $\Omega_1 - \Omega_3\Omega_2^{-1}\Omega_3^{\mathrm{T}} > 0$ and $\Omega_2 > 0$ if and only if

$$\begin{bmatrix}\Omega_1 & \Omega_3\\ \Omega_3^{\mathrm{T}} & \Omega_2\end{bmatrix} > 0$$

Lemma 2[22]**.** Given matrices Σ, Ξ, and Ω with $\Omega = \Omega^{\mathrm{T}}$, then

$$\Omega + \Sigma F(\sigma)\Xi + \Xi^{\mathrm{T}}F^{\mathrm{T}}(\sigma)\Sigma^{\mathrm{T}} < 0$$

holds for any $F(\sigma)$ satisfying $F^{\mathrm{T}}(\sigma)F(\sigma)\leq I$ if and only if there exists a scalar ε_s such that

$$\Omega + \varepsilon_s^{-1}\Sigma\Sigma^{\mathrm{T}} + \varepsilon_s\Xi^{\mathrm{T}}\Xi < 0 \tag{14}$$

2 Main results

In this section, we will first derive stability criteria for the Lurie time delay system represented by (6). The derived result will be extended to robust stability analysis of the uncertain system represented by (9). Further, it will be shown that the result presented in Theorem 1 of [18] is a special case of our result.

Theorem 1. For given scalars h_m (>0) and ρ_0 ($0 \leq \rho_0 \leq 1$), the Lurie system represented by (6), with the nonlinear function $\varphi(t, z(t))$ satisfying (9), is absolutely stable in the mean square sense if there exist matrices $P > 0$, $Q_i > 0$, $Z_i > 0$ ($i = 1, 2$), S, T, W, U of appropriate dimensions and a scalar $\varepsilon > 0$ such that the following LMIs hold:

$$\Omega(l) = \begin{bmatrix} \Xi_{11} & \Sigma_l & \Xi_{13} & \Xi_{14} & \Xi_{15} & \Xi_{16} \\ * & \Upsilon_l & 0 & 0 & 0 & 0 \\ * & * & -Z_1 & 0 & 0 & 0 \\ * & * & * & -Z_1 & 0 & 0 \\ * & * & * & * & -Z_2 & 0 \\ * & * & * & * & * & -Z_2 \end{bmatrix} < 0 \tag{15}$$

where

$$l = 1, 2, 3, 4$$

$$\Xi_{11} = \begin{bmatrix} \Pi_{11} & \Pi_{12} & 0 & \Pi_{14} & 0 & \Pi_{16} \\ * & 0 & 0 & 0 & 0 & \Pi_{26} \\ * & * & -Q_1 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & \Pi_{46} \\ * & * & * & * & -Q_2 & 0 \\ * & * & * & * & * & \Pi_{66} \end{bmatrix} + \Gamma + \Gamma^{\mathrm{T}}$$

$$\Gamma = [S \quad -S+T \quad -T+U \quad -U+W \quad -W \quad 0]$$

$$\Pi_{11} = A^{\mathrm{T}}P + PA + Q_1 + Q_2,\ \Pi_{12} = \rho_0 PB$$
$$\Pi_{14} = (1-\rho_0)PB,\ \Pi_{16} = PD - \varepsilon M^{\mathrm{T}}K^{\mathrm{T}}$$
$$\Pi_{26} = \varepsilon\rho_0 N^{\mathrm{T}}K^{\mathrm{T}},\ \Pi_{46} = \varepsilon(1-\rho_0)N^{\mathrm{T}}K^{\mathrm{T}}$$
$$\Pi_{66} = -2\varepsilon I$$

$$\Xi_{13} = \Big[\sqrt{\rho_0 h_0}Z_1A \quad \sqrt{\rho_0 h_0}Z_1B \quad 0 \quad 0 \quad 0 \quad \sqrt{\rho_0 h_0}Z_1D\Big]^{\mathrm{T}}$$
$$\Xi_{14} = \Big[\sqrt{(1-\rho_0)h_0}Z_1A \quad 0 \quad 0 \quad \sqrt{(1-\rho_0)h_0}Z_1B \quad 0 \quad \sqrt{(1-\rho_0)h_0}Z_1D\Big]^{\mathrm{T}}$$
$$\Xi_{15} = \Big[\sqrt{\rho_0 h_{0m}}Z_2A \quad \sqrt{\rho_0 h_{0m}}Z_2B \quad 0 \quad 0 \quad 0 \quad \sqrt{\rho_0 h_{0m}}Z_2D\Big]^{\mathrm{T}}$$
$$\Xi_{16} = \Big[\sqrt{(1-\rho_0)h_{0m}}Z_2A \quad 0 \quad 0 \quad \sqrt{(1-\rho_0)h_{0m}}Z_2B \quad 0 \quad \sqrt{(1-\rho_0)h_{0m}}Z_2D\Big]^{\mathrm{T}}$$

$$\Sigma_1 = -2h_0S \ \Sigma_2 = -2h_0T \ \Sigma_3 = -2h_0U,\ \Sigma_4 = -2h_0W$$
$$\Upsilon_1 = \Upsilon_2 = -2h_0Z_1,\ \Upsilon_3 = \Upsilon_4 = -2h_{0m}Z_2$$
$$h_{0m} = h_m - h_0$$

Proof. Choose the Lyapunov functional candidate as

$$V(x_t) = \sum_{i=1}^{3} V_i(x_t) \tag{16}$$

where

$$V_1(x_t) = x^{\mathrm{T}}(t)Px(t)$$
$$V_2(x_t) = \int_{t-h_0}^{t} x^{\mathrm{T}}(s)Q_1x(s)\mathrm{d}s + \int_{t-h_m}^{t} x^{\mathrm{T}}(s)Q_2x(s)\mathrm{d}s$$
$$V_3(x_t) = \int_{t-h_0}^{t}\int_{s}^{t} \dot{x}^{\mathrm{T}}(v)Z_1\dot{x}(v)\mathrm{d}v\mathrm{d}s + \int_{t-h_m}^{t-h_0}\int_{s}^{t} \dot{x}^{\mathrm{T}}(v)Z_2\dot{x}(v)\mathrm{d}v\mathrm{d}s$$

Using the infinitesimal operator (13) for $V(x_t)$ and taking the expectation on it, we can obtain

$$\mathrm{E}\{LV_1(x_t)\} = \mathrm{E}\{2x^{\mathrm{T}}(t)P(Ax(t) + \rho_0Bx(t-h_1(t)) + (1-\rho_0)Bx(t-h_2(t)) + D\omega(t))\} \tag{17}$$

$$\mathrm{E}\{LV_2(x_t)\} = \mathrm{E}\{x^{\mathrm{T}}(t)(Q_1+Q_2)x(t) - x^{\mathrm{T}}(t-h_0)Q_1x(t-h_0) - x^{\mathrm{T}}(t-h_m)Q_1x(t-h_m)\} \tag{18}$$

$$\mathrm{E}\{LV_3(x_t)\} = \mathrm{E}\Big\{\dot{x}^{\mathrm{T}}(t)Z_{12}\dot{x}(t) - \int_{t-h_0}^{t}\dot{x}^{\mathrm{T}}(s)Z_1\dot{x}(s)\mathrm{d}s - \int_{t-h_m}^{t-h_0}\dot{x}^{\mathrm{T}}(s)Z_2\dot{x}(s)\mathrm{d}s\Big\} \tag{19}$$

where $Z_{12} = h_0Z_1 + (h_m - h_0)Z_2$.

From (6), we get

$$\begin{aligned} \mathrm{E}\Big(\dot{x}^{\mathrm{T}}(t)Z_{12}\dot{x}(t)\Big) = {} & x^{\mathrm{T}}(t)A^{\mathrm{T}}Z_{12}Ax(t) + \\ & \rho_0x^{\mathrm{T}}(t)A^{\mathrm{T}}Z_{12}Bx(t-h_1(t)) + \\ & (1-\rho_0)x^{\mathrm{T}}(t)A^{\mathrm{T}}Z_{12}Bx(t-h_2(t)) + \\ & x^{\mathrm{T}}(t)A^{\mathrm{T}}Z_{12}D\omega(t) + \\ & \rho_0x^{\mathrm{T}}(t-h_1(t))B^{\mathrm{T}}Z_{12}Bx(t-h_1(t)) + \\ & \rho_0x^{\mathrm{T}}(t-h_1(t))B^{\mathrm{T}}Z_{12}D\omega(t) + \\ & (1-\rho_0)x^{\mathrm{T}}(t-h_2(t)) \times \\ & B^{\mathrm{T}}Z_{12}Bx(t-h_2(t)) + \\ & (1-\rho_0)x^{\mathrm{T}}(t-h_2(t))B^{\mathrm{T}}Z_{12}D\omega(t) + \\ & \omega^{\mathrm{T}}(t)D^{\mathrm{T}}Z_{12}D\omega(t) \end{aligned} \tag{20}$$

Further, the following holds:

$$\begin{aligned} & 2\zeta^{\mathrm{T}}(t)S\left[x(t) - x(t-h_1(t)) - \int_{t-h_1(t)}^{t}\dot{x}(s)\mathrm{d}s\right] = 0 \\ & 2\zeta^{\mathrm{T}}(t)T\left[x(t-h_1(t)) - x(t-h_0) - \int_{t-h_0}^{t-h_1(t)}\dot{x}(s)\mathrm{d}s\right] = 0 \\ & 2\zeta^{\mathrm{T}}(t)U\left[x(t-h_0) - x(t-h_2(t)) - \int_{t-h_2(t)}^{t-h_0}\dot{x}(s)\mathrm{d}s\right] = 0 \\ & 2\zeta^{\mathrm{T}}(t)W\left[x(t-h_2(t)) - x(t-h_m) - \int_{t-h_m}^{t-h_2(t)}\dot{x}(s)\mathrm{d}s\right] = 0 \end{aligned} \tag{21}$$

where

$$\zeta^{\mathrm{T}}(t) = [x^{\mathrm{T}}(t) \quad x^{\mathrm{T}}(t-h_1(t)) \quad x^{\mathrm{T}}(t-h_0) \quad x^{\mathrm{T}}(t-h_2(t)) \quad x^{\mathrm{T}}(t-h_m) \quad \omega^{\mathrm{T}}(t)]$$

It follows from (1), (2) and (6) that

$$\mathrm{E}\{\omega^{\mathrm{T}}(t)\omega(t) + x^{\mathrm{T}}(t)M^{\mathrm{T}}K^{\mathrm{T}}\omega(t) + \rho(t)x^{\mathrm{T}}(t-h_1(t))N^{\mathrm{T}}K^{\mathrm{T}}\omega(t) +$$

$$(1-\rho(t))x^{\mathrm{T}}(t-h_2(t))N^{\mathrm{T}}K^{\mathrm{T}}\omega(t)\} \le 0$$

which results in the following:

$$\begin{aligned}&\omega^{\mathrm{T}}(t)\omega(t)+x^{\mathrm{T}}(t)M^{\mathrm{T}}KT\omega(t)+\\&\quad\rho_0x^{\mathrm{T}}(t-h_1(t))N^{\mathrm{T}}K^{\mathrm{T}}\omega(t)+\\&\quad(1-\rho_0)x^{\mathrm{T}}(t-h_2(t))N^{\mathrm{T}}K^{\mathrm{T}}\omega(t)\le 0\end{aligned}\tag{22}$$

From (16) ∼ (22), on applying S-procedure[21], we obtain

$$\begin{aligned}&\mathrm{E}\{LV(x_t)\}=\\&\quad\mathrm{E}\Big\{\zeta^{\mathrm{T}}(t)\tilde{\Xi}_{11}\zeta(t)-\\&\quad2\zeta^{\mathrm{T}}(t)S\int_{t-h_1(t)}^{t}\dot{x}(s)\mathrm{d}s-2\zeta^{\mathrm{T}}(t)T\int_{t-h_0}^{t-h_1(t)}\dot{x}(s)\mathrm{d}s-\\&\quad2\zeta^{\mathrm{T}}(t)U\int_{t-h_2(t)}^{t-h_0}\dot{x}(s)\mathrm{d}s-2\zeta^{\mathrm{T}}(t)W\int_{t-h_m}^{t-h_2(t)}\dot{x}(s)\mathrm{d}s-\\&\quad\int_{t-h_0}^{t}\dot{x}^{\mathrm{T}}(s)Z_1\dot{x}(s)\mathrm{d}s-\int_{t-h_m}^{t-h_0}\dot{x}^{\mathrm{T}}(s)Z_2\dot{x}(s)\mathrm{d}s\Big\}\end{aligned}\tag{23}$$

where

$$\tilde{\Xi}_{11}=\left[\begin{array}{cccccc}\tilde{\Pi}_{11}&\tilde{\Pi}_{12}&0&\tilde{\Pi}_{14}&0&\tilde{\Pi}_{16}\\ *&\tilde{\Pi}_{22}&0&0&0&\tilde{\Pi}_{26}\\ *&*&-Q_1&0&0&0\\ *&*&*&\tilde{\Pi}_{44}&0&\tilde{\Pi}_{46}\\ *&*&*&*&-Q_2&0\\ *&*&*&*&*&\tilde{\Pi}_{66}\end{array}\right]+\Gamma+\Gamma^{\mathrm{T}}$$

$$\begin{aligned}&\tilde{\Pi}_{11}=A^{\mathrm{T}}P+PA+Q_1+Q_2+A^{\mathrm{T}}Z_{12}A\\&\tilde{\Pi}_{12}=\rho_0PB+\rho_0A^{\mathrm{T}}Z_{12}B,\ \tilde{\Pi}_{22}=\rho_0B^{\mathrm{T}}Z_{12}B\\&\tilde{\Pi}_{14}=(1-\rho_0)PB+(1-\rho_0)A^{\mathrm{T}}Z_{12}B\\&\tilde{\Pi}_{16}=PD+A^{\mathrm{T}}Z_{12}D-\varepsilon M^{\mathrm{T}}K^{\mathrm{T}}\\&\tilde{\Pi}_{26}=\rho_0B^{\mathrm{T}}Z_{12}D-\rho_0\varepsilon N^{\mathrm{T}}K^{\mathrm{T}}\\&\tilde{\Pi}_{44}=(1-\rho_0)B^{\mathrm{T}}Z_{12}B,\quad\tilde{\Pi}_{66}=D^{\mathrm{T}}Z_{12}D-2\varepsilon I\\&\tilde{\Pi}_{46}=(1-\rho_0)B^{\mathrm{T}}Z_{12}D-\varepsilon(1-\rho_0)N^{\mathrm{T}}K^{\mathrm{T}}\end{aligned}$$

ε is a positive scalar and Γ is being defined in (15). Now, (23) can be written as

$$\begin{aligned}&\mathrm{E}\{LV(x_t)\}=\\&\quad\mathrm{E}\Big\{\frac{1}{2h_0}\Big(\int_{t-h_1(t)}^{t}\tilde{\zeta}^{\mathrm{T}}(t,s)\tilde{\Omega}_1\tilde{\zeta}(t,s)\mathrm{d}s+\\&\quad\int_{t-h_0}^{t-h_1(t)}\tilde{\zeta}^{\mathrm{T}}(t,s)\tilde{\Omega}_2\tilde{\zeta}(t,s)\mathrm{d}s\Big)+\\&\quad\frac{1}{2(h_m-h_0)}\Big(\int_{t-h_2(t)}^{t-h_0}\tilde{\zeta}^{\mathrm{T}}(t,s)\tilde{\Omega}_3\tilde{\zeta}(t,s)\mathrm{d}s+\\&\quad\int_{t-h_m}^{t-h_2(t)}\tilde{\zeta}^{\mathrm{T}}(t,s)\tilde{\Omega}_4\tilde{\zeta}(t,s)\mathrm{d}s\Big)\Big\}\end{aligned}\tag{24}$$

where

$$\begin{aligned}\tilde{\zeta}^{\mathrm{T}}(t)=[&x^{\mathrm{T}}(t)\ \ x^{\mathrm{T}}(t-h_1(t))\ \ x^{\mathrm{T}}(t-h_0)\ \ x^{\mathrm{T}}(t-h_2(t))\\&x^{\mathrm{T}}(t-h_m)\ \ \omega^{\mathrm{T}}(t)\ \ \dot{x}^{\mathrm{T}}(s)]\end{aligned}$$

$$\tilde{\Omega}_1=\left[\begin{array}{cc}\tilde{\Xi}_{11}&-2h_0S\\ *&-2h_0Z_1\end{array}\right],\ \tilde{\Omega}_2=\left[\begin{array}{cc}\tilde{\Xi}_{11}&-2h_0T\\ *&-2h_0Z_1\end{array}\right]$$
$$\tilde{\Omega}_3=\left[\begin{array}{cc}\tilde{\Xi}_{11}&-2h_{0m}U\\ *&-2h_0Z_2\end{array}\right],\ \tilde{\Omega}_4=\left[\begin{array}{cc}\tilde{\Xi}_{11}&-2h_0W\\ *&-2h_0Z_2\end{array}\right]\tag{25}$$

It can be observed from (24) and (25) that $\mathrm{E}\{LV(x_t)\}$ holds true when the following is satisfied:

$$\tilde{\Omega}_1<0,\ \ \tilde{\Omega}_2<0,\ \ \tilde{\Omega}_3<0,\ \ \tilde{\Omega}_4<0$$

By applying Lemma 1, it can be easily shown that the inequalities $\tilde{\Omega}_1<0$, $\tilde{\Omega}_2<0$, $\tilde{\Omega}_3<0$, $\tilde{\Omega}_4<0$ are equivalent to inequalities $\Omega(1)<0$, $\Omega(2)<0$, $\Omega(3)<0$, $\Omega(4)<0$, respectively in (15). This completes the proof. □

Remark 3. It may be observed from (16) that knowledge of an internal point (h_0) in the delay range is utilized to define the Lypunov functional. Unlike [18], where the same Lyapunov matrices were used over the complete delay range, different Lyapunov matrices are used for the two sub-intervals to define the Lyapunov functional. As shown in Section 3, the defining of different Lyapunov matrices in the two sub-intervals along with the probability distribution of the delay may reduce the conservatism of the results.

Remark 4. By using the loop transformation[23] as in [7], it can be easily shown that the stability analysis of system (6) with the nonlinearity $\varphi(t,z(t))$ satisfying the sector condition (3) is equivalent to the stability analysis for the following system in the sector $[0,K_2-K_1]$.

$$\begin{aligned}\dot{x}(t)&=(A-DK_1M)x(t)+\rho(t)(B-DK_1N)x(t-h_1(t))+\\&\quad(1-\rho(t))(B-DK_1N)x(t-h_2(t))+D\omega(t)\\z(t)&=Mx(t)+\rho(t)Nx(t-h_1(t))+\\&\quad(1-\rho(t))Nx(t-h_2(t))\\x(t)&=\psi(t),\quad t\in[-h_m,0]\end{aligned}\tag{26}$$

The stability criteria for the Lurie system (6) with the nonlinear function $\varphi(t,z(t))$ in the sector $[K_1,K_2]$ is given below in Corollary 1. It can be easily obtained by following the similar procedure as in Theorem 1.

Corollary 1. For given scalars h_m (>0) and ρ_0 ($0\le\rho_0\le 1$), the Lurie system represented by (6) with the nonlinear function $\varphi(t,z(t))$ satisfying (3) is absolutely stable in the mean square sense, if there exists matrices $P>0$, $Q_i>0$, $Z_i>0$ ($i=1,2$), S, T, W, U, of appropriate dimensions and a scalar $\varepsilon>0$ such that the following LMIs hold:

$$\hat{\Omega}(l)=\left[\begin{array}{cccccc}\hat{\Xi}_{11}&\Sigma_l&\hat{\Xi}_{13}&\hat{\Xi}_{14}&\hat{\Xi}_{15}&\hat{\Xi}_{16}\\ *&\Upsilon_l&0&0&0&0\\ *&*&-Z_1&0&0&0\\ *&*&*&-Z_1&0&0\\ *&*&*&*&-Z_2&0\\ *&*&*&*&*&-Z_2\end{array}\right]<0\tag{27}$$

where

$$l=1,2,3,4$$

$$\hat{\Xi}_{11}=\left[\begin{array}{cccccc}\hat{\Pi}_{11}&\hat{\Pi}_{12}&0&\hat{\Pi}_{14}&0&\hat{\Pi}_{16}\\ *&0&0&0&0&\hat{\Pi}_{26}\\ *&*&-Q_1&0&0&0\\ *&*&*&0&0&\hat{\Pi}_{46}\\ *&*&*&*&-Q_2&0\\ *&*&*&*&*&\Pi_{66}\end{array}\right]+\Gamma+\Gamma^{\mathrm{T}}$$

$$\begin{aligned}&\hat{\Pi}_{11}=\hat{A}^{\mathrm{T}}P+P\hat{A}+Q_1+Q_2,\quad\hat{\Pi}_{12}=\rho_0P\hat{B}\\&\hat{\Pi}_{14}=(1-\rho_0)P\hat{B},\ \hat{\Pi}_{16}=PD-\varepsilon M^{\mathrm{T}}\hat{K}^{\mathrm{T}}\\&\hat{\Pi}_{26}=\varepsilon\rho_0N^{\mathrm{T}}\hat{K}^{\mathrm{T}},\ \hat{\Pi}_{46}=\varepsilon(1-\rho_0)N^{\mathrm{T}}\hat{K}^{\mathrm{T}}\end{aligned}$$

$$
\begin{aligned}
\hat{\Xi}_{13} &= \Big[\sqrt{\rho_0 h_0} Z_1 \hat{A} \ \ \sqrt{\rho_0 h_0} Z_1 \hat{B} \\
&\qquad 0 \ \ 0 \ \ 0 \ \ \sqrt{\rho_0 h_0} Z_1 D\Big]^{\mathrm{T}} \\
\hat{\Xi}_{14} &= \Big[\sqrt{(1-\rho_0) h_0} Z_1 \hat{A} \ \ 0 \ \ 0 \ \ \sqrt{(1-\rho_0) h_0} Z_1 \hat{B} \\
&\qquad 0 \ \ \sqrt{(1-\rho_0) h_0} Z_1 D\Big]^{\mathrm{T}} \\
\hat{\Xi}_{15} &= \Big[\sqrt{\rho_0 h_{0m}} Z_2 \hat{A} \ \ \sqrt{\rho_0 h_{0m}} Z_2 \hat{B} \ \ 0 \ \ 0 \ \ 0 \\
&\qquad \sqrt{\rho_0 h_{0m}} Z_2 D\Big]^{\mathrm{T}} \\
\hat{\Xi}_{16} &= \Big[\sqrt{(1-\rho_0) h_{0m}} Z_2 \hat{A} \ \ 0 \ \ 0 \ \ \sqrt{(1-\rho_0) h_{0m}} Z_2 \hat{B} \\
&\qquad 0 \ \ \sqrt{(1-\rho_0) h_{0m}} Z_2 D\Big]^{\mathrm{T}} \\
\hat{A} &= A - DK_1M, \quad \hat{B} = B - DK_1N, \quad \hat{K} = K_2 - K_1
\end{aligned}
$$

with Σ_l, Υ_l, Π_{66}, Γ being the same as defined in (15).

As shown in Theorem 2 below, the approach of Theorem 1 can be extended to the robust absolute stability analysis of the time-delayed Lurie system with time-varying parametric uncertainties. For this case the system is represented by (10) $\sim$ (12).

Theorem 2. For given scalars h_m (>0) and ρ_0 $(0 \le \rho_0 \le 1)$, the uncertain Lurie system represented by (12) with the nonlinear function $\varphi(t, z(t))$ satisfying (2) is absolutely stable in the mean square sense, if there exist matrices $P > 0$, $Q_i > 0$, $Z_i > 0$ $(i = 1, 2)$, S, T, W, U of appropriate dimensions and scalars $\varepsilon > 0$ and $\mu > 0$ such that the following LMIs hold:

$$
\bar{\Omega}(l) = \left[\begin{array}{cccccccccccc}
\Xi_{11} & \Sigma_l & \Xi_{13} & \Xi_{14} & \Xi_{15} & \Xi_{16} & \Xi_{17} & \mu\Xi_{18} & 0 & \mu\Xi_{110} & 0 & \mu\Xi_{112} \\
* & \Upsilon_l & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & -Z_1 & 0 & 0 & 0 & 0 & 0 & \Xi_{39} & 0 & 0 & 0 \\
* & * & * & -Z_1 & 0 & 0 & 0 & 0 & 0 & 0 & \Xi_{411} & 0 \\
* & * & * & * & -Z_2 & 0 & 0 & 0 & \Xi_{59} & 0 & 0 & 0 \\
* & * & * & * & * & -Z_2 & 0 & 0 & 0 & 0 & \Xi_{611} & 0 \\
* & * & * & * & * & * & -\mu I & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & -\mu I & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & -\mu I & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & * & -\mu I & 0 & 0 \\
* & * & * & * & * & * & * & * & * & * & -\mu I & 0 \\
* & * & * & * & * & * & * & * & * & * & * & -\mu I
\end{array}\right] < 0 \tag{28}
$$

where

$$l = 1, 2, 3, 4$$

$$
\begin{aligned}
&\Xi_{17} = [H^{\mathrm{T}}P \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ 0]^{\mathrm{T}}, \ \Xi_{39} = \sqrt{\rho_0 h_0} Z_1 H \\
&\Xi_{18} = [E_a \ \ \rho_0 E_b \ \ 0 \ \ (1-\rho_0)E_b \ \ 0 \ \ E_d]^{\mathrm{T}} \\
&\Xi_{110} = [E_a \ \ E_b \ \ 0 \ \ 0 \ \ 0 \ \ E_d]^{\mathrm{T}}, \ \Xi_{59} = \sqrt{\rho_0 h_{0m}} Z_2 H \\
&\Xi_{112} = [E_a \ \ 0 \ \ 0 \ \ E_b \ \ 0 \ \ E_d]^{\mathrm{T}} \\
&\Xi_{411} = \sqrt{(1-\rho_0 h_0)} Z_1 H, \ \ \Xi_{611} = \sqrt{(1-\rho_0 h_{0m})} Z_2 H
\end{aligned}
$$

Ξ_{11}, Ξ_{13}, Ξ_{14}, Ξ_{15}, Ξ_{16}, Σ_l, Υ_l being the same as defined in (15).

Proof. On replacing A, B and D in (15) by $A + HF(t)E_a$, $B + HF(t)E_b$, and $D + HF(t)E_d$, respectively, we get the following:

$$
\Omega(l) + \bar{H}F(t)\bar{E} + \bar{E}^{\mathrm{T}}F(t)\bar{H}^{\mathrm{T}} < 0, \quad l = 1, 2, 3, 4 \tag{29}
$$

where $\Omega(l)$ is defined in (15) and

$$
\begin{aligned}
&\bar{H} = [\bar{H}_1 \ \bar{H}_2 \ \bar{H}_3], \ \bar{E} = [\bar{E}_1^{\mathrm{T}} \ \bar{E}_2^{\mathrm{T}} \ \bar{E}_3^{\mathrm{T}}]^{\mathrm{T}} \\
&\bar{H}_1 = [H^{\mathrm{T}}P \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ 0]^{\mathrm{T}} \\
&\bar{H}_2 = [0 \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ \sqrt{\rho_0 h_0} H^{\mathrm{T}} Z_1 \ \ 0 \\
&\qquad \sqrt{\rho_0 h_{0m}} H^{\mathrm{T}} Z_2 \ \ 0]^{\mathrm{T}} \\
&\bar{H}_3 = [0 \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ \sqrt{(1-\rho_0 h_0)} H^{\mathrm{T}} Z_1 \\
&\qquad 0 \ \ \sqrt{(1-\rho_0 h_{0m})} H^{\mathrm{T}} Z_2]^{\mathrm{T}} \\
&\bar{E}_1 = [E_a \ \ \rho_0 E_b \ \ 0 \ \ (1-\rho_0)E_b \ \ 0 \ \ E_d \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ 0] \\
&\bar{E}_2 = [E_a \ \ E_b \ \ 0 \ \ 0 \ \ 0 \ \ E_d \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ 0] \\
&\bar{E}_3 = [E_a \ \ 0 \ \ 0 \ \ E_b \ \ 0 \ \ E_d \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ 0]
\end{aligned}
$$

On applying Lemma 2, it can be shown that (29) is equivalent to the following

$$
\Omega(l) + \mu^{-1}\bar{H}\bar{H}^{\mathrm{T}} + \mu\bar{E}^{\mathrm{T}}\bar{E} < 0 \tag{30}
$$

On applying Lemma 1 to (30), it can be shown that (30) is equivalent to (28). □

The stability criteria for the Lurie system (12) with the nonlinear function $\varphi(t, z(t))$ satisfying (3) is given below in Corollary 2. It can be obtained from Corollary 1 by following the similar procedure as in Theorem 2.

Corollary 2. For given scalars h_m (>0) and ρ_0 $(0 \le \rho_0 \le 1)$, the uncertain Lurie system represented by (12) with the nonlinear function $\varphi(t, z(t))$ satisfying (3) is absolutely stable in the mean square sense, if there exist matrices $P > 0$, $Q_i > 0$, $Z_i > 0$ $(i = 1, 2)$, S, T, W, U of appropriate dimensions and scalars $\varepsilon > 0$ and $\mu > 0$ such that the following LMIs hold:

$$
\hat{\Omega}_2(l) = \left[\begin{array}{ccccccc}
\hat{\Xi}_{11} & \Sigma_l & \hat{\Xi}_{13} & \hat{\Xi}_{14} & \hat{\Xi}_{15} & \hat{\Xi}_{16} & \Xi_{17} \\
* & \Upsilon_l & 0 & 0 & 0 & 0 & 0 \\
* & * & -Z_1 & 0 & 0 & 0 & 0 \\
* & * & * & -Z_1 & 0 & 0 & 0 \\
* & * & * & * & -Z_2 & 0 & 0 \\
* & * & * & * & * & -Z_2 & 0 \\
* & * & * & * & * & * & -\mu I \\
* & * & * & * & * & * & * \\
* & * & * & * & * & * & * \\
* & * & * & * & * & * & * \\
* & * & * & * & * & * & * \\
* & * & * & * & * & * & *
\end{array}\right.
$$

$$\left.\begin{array}{ccccc}
\mu\hat{\Xi}_{18} & 0 & \mu\hat{\Xi}_{110} & 0 & \mu\hat{\Xi}_{112} \\
0 & 0 & 0 & 0 & 0 \\
0 & \Xi_{39} & 0 & 0 & 0 \\
0 & 0 & 0 & \Xi_{411} & 0 \\
0 & \Xi_{59} & 0 & 0 & 0 \\
0 & 0 & 0 & \Xi_{611} & 0 \\
0 & 0 & 0 & 0 & 0 \\
-\mu I & 0 & 0 & 0 & 0 \\
* & -\mu I & 0 & 0 & 0 \\
* & * & -\mu I & 0 & 0 \\
* & * & * & -\mu I & 0 \\
* & * & * & * & -\mu I
\end{array}\right] < 0 \tag{31}$$

where

$$l = 1, 2, 3, 4$$

$$\hat{\Xi}_{18} = [E_a - E_d K_1 M \;\; \rho_0(E_b - E_d K_1 N) \;\; 0$$
$$0 \;\; (1-\rho_0)(E_b - E_d K_1 N) \;\; 0 \;\; E_d]^{\mathrm{T}}$$
$$\hat{\Xi}_{110} = [E_a - E_d K_1 M \;\; E_b - E_d K_1 N \;\; 0 \;\; 0 \;\; 0 \;\; E_d]^{\mathrm{T}}$$
$$\hat{\Xi}_{112} = [E_a - E_d K_1 M \;\; 0 \;\; 0 \;\; E_b - E_d K_1 N \;\; 0 \;\; E_d]^{\mathrm{T}}$$

with $\hat{\Xi}_{11}$, $\hat{\Xi}_{13}$, $\hat{\Xi}_{14}$, $\hat{\Xi}_{15}$, $\hat{\Xi}_{16}$, Σ_l, Υ_l being the same as defined in (27) and Ξ_{17}, Ξ_{39}, Ξ_{59}, Ξ_{411}, Ξ_{611} being defined in (28).

If we set $\rho(t) = 1$ and $N = 0$ in (6), then system (6) reduces to the following time-delay Lurie system (32) with time-varying delay $h_1(t)$ such that $h_1(t) \in [0, h_0]$, where h_0 is now the upper delay bound.

$$\begin{aligned}
\dot{x}(t) &= Ax(t) + Bx(t - h_1(t)) + D\omega(t) \\
z(t) &= Mx(t) \\
\omega(t) &= -\varphi(t, z(t)) \\
x(t) &= \psi(t), \quad t \in [-h_0, 0]
\end{aligned} \tag{32}$$

Using the result of Theorem 1, the following Corollary is obtained for the stability analysis of system (32).

Corollary 3. For a given upper delay bound h_0 (> 0), the Lurie system represented by (32) with the nonlinear function $\varphi(t, z(t))$ satisfying (2) is stable in the mean square sense, if there exist matrices $P > 0$, $Q_1 > 0$, $Z_1 > 0$, $S = [S_1^{\mathrm{T}} \;\; S_2^{\mathrm{T}} \;\; 0 \;\; 0 \;\; 0 \;\; 0]^{\mathrm{T}}$, $T = [T_1^{\mathrm{T}} \;\; T_2^{\mathrm{T}} \;\; 0 \;\; 0 \;\; 0 \;\; 0]^{\mathrm{T}}$ of appropriate dimensions such that the following LMIs hold:

$$\Lambda(i) = \begin{bmatrix}
\lambda_{11} & \lambda_{12} & -T_1 & \lambda_{14} & \lambda_{15}^i & \lambda_{16} \\
* & \lambda_{22} & -T_2 & 0 & \lambda_{25}^i & \lambda_{26} \\
* & * & -Q_1 & 0 & 0 & 0 \\
* & * & * & -2I & 0 & \lambda_{46} \\
* & * & * & * & -h_0 Z_1 & 0 \\
* & * & * & * & * & -Z_1
\end{bmatrix} < 0 \tag{33}$$

where

$$i = 1, 2$$

$$\begin{aligned}
&\lambda_{11} = A^{\mathrm{T}}P + PA + Q_1 + S_1 + S_1^{\mathrm{T}}, \quad \lambda_{14} = PD - M^{\mathrm{T}}K^{\mathrm{T}} \\
&\lambda_{12} = PB - S_1 + S_2^{\mathrm{T}} + T_1, \quad \lambda_{15}^1 = -h_0 S_1, \quad \lambda_{15}^2 = -h_0 T_1 \\
&\lambda_{16} = \sqrt{h_0} A^{\mathrm{T}} Z_1, \; \lambda_{22} = -S_2 - S_2^{\mathrm{T}} + T_2 + T_2^{\mathrm{T}} \\
&\lambda_{25}^1 = -h_0 S_2, \; \lambda_{25}^2 = -h_0 T_2, \; \lambda_{26} = \sqrt{h_0} B^{\mathrm{T}} Z_1 \\
&\lambda_{46} = \sqrt{h_0} D^{\mathrm{T}} Z_1
\end{aligned}$$

Proof. The above condition can be derived from (15) by equating $\rho_0 = 1$, $\varepsilon = 1$, $N = 0$, $U = 0$, $W = 0$, $Z_2 = 0$, $Q_2 = 0$ and replacing $-2h_0S_1$, $-2h_0S_2$, $-2h_0T_1$, $-2h_0T_2$ and $-2h_0Z_1$ by $-h_0S_1$, $-h_0S_2$, $-h_0T_1$, $-h_0T_2$ and $-h_0Z_1$, respectively. □

Remark 5. The stability condition (33) derived in Corollary 3 is the same as the stability condition presented in Theorem 1 of [18] for the absolute stability of time delay Lurie system. This shows that the stability condition in Theorem 1 of [18] is a special case of our result.

3 Numerical examples

Example 1. Consider system (1) with the following system matrices[18]:

$$A = \begin{bmatrix} 0 & 1 \\ 1 & -2 \end{bmatrix}, \; B = \begin{bmatrix} -0.7539 & -0.3319 \\ 0 & 0 \end{bmatrix}, \; D = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$M = [\; 1 \quad -0.5 \;], \quad N = [\; 0 \quad 0 \;], \quad K = 1$$

Table 1 shows a comparison of the maximum allowable delay (h_m) obtained by Theorem 1 for $h_0 = 0.5$ and different values of ρ_0 with some of the earlier results.

Table 1 Comparison of the delay bound

Approach	Delay bound
Reference [24]	1.2562
Reference [18]	1.2890
Theorem 1 ($\rho_0 = 0.4$)	1.5344
Theorem 1 ($\rho_0 = 0.7$)	2.0606
Theorem 1 ($\rho_0 = 0.9$)	4.3815
Theorem 1 ($\rho_0 = 0.99$)	15.8235

Remark 6. It can be observed from Table 1 that when the probability distribution of the delay is known a priori, using the results of this paper can lead to a larger upper delay bound than that obtained by using the conventional approach of using the delay range information only.

Example 2. Consider the time delay Chua′s oscillator[12] with the following system matrices:

$$A = \begin{bmatrix} 1.3018 & -1.3018 & 0 \\ 1 & -1 & 1 \\ 0 & 0.0136 & 0.0297 \end{bmatrix}$$

$$B = \begin{bmatrix} -1 & 1 & 1 \\ 0 & -1 & -1 \\ -0.368 & 0 & -1 \end{bmatrix}, \quad D = \begin{bmatrix} 1.3018 \\ 0 \\ 0 \end{bmatrix}$$

$$M = [\; 1 \quad 0 \quad 0 \;], \quad N = [\; 0 \quad 0 \quad 0 \;]$$

$$K_1 = -0.1, \quad K_2 = 0.3$$

Table 2 shows the maximum allowable delay obtained by Corollary 1 for $h_0 = 0.2$ and different values of ρ_0.

Table 2 Comparison of the delay bounds

Approach	Delay bound
Corollary 1 ($\rho_0 = 0.4$)	0.3757
Corollary 1 ($\rho_0 = 0.7$)	0.5190
Corollary 1 ($\rho_0 = 0.9$)	0.9344
Corollary 1 ($\rho_0 = 0.99$)	4.0160

4 Conclusion

In this paper, the problem of robust absolute stability for Lurie system with stochastic time delay and norm-bounded parametric uncertainties has been investigated. By using the information of probability distribution of the time-delay, a new model of the time delayed Lurie system was proposed. In addition to the delay bound, the additional information about the delay in terms of the probability distribution was used to derive the stability condition. The sufficient conditions for the MSS were obtained in terms of LMIs. Furthermore, it has been shown with the help of examples that when the probability distribution of the delay is known a priori, a larger upper delay bound can be obtained than that obtained by the existing methods.

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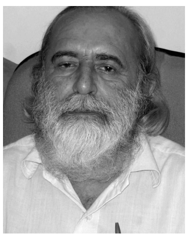

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