Iterative Learning Control for a Class of Linear Discrete-time Switched Systems

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Abstract In this paper, the iterative learning control (ILC) is considered for a class of linear discrete-time switched systems with arbitrary switching signals. It is assumed that the switched system is operated during a finite time interval repetitively, and then the first order P-type ILC scheme can be used to achieve perfect tracking over the whole time interval. By the super vector approach, a convergence condition for such ILC systems in the iteration domain can be given. The theoretical analysis is supported by simulations.

Key words Iterative learning control (ILC), switched systems, tracking, arbitrary switching signals, convergence

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In recent years, the study of switched systems has received a growing attention. The primary motivation for studying such switched systems comes partly from the fact that switched systems and switched multi-controller systems have numerous applications in the control of mechanical systems, process control, automotive industry, power systems, aircraft and traffic control, and many other fields[1−4]. A switched system belongs to a hybrid system, which consists of a finite number of subsystems and an associated switching signal governing the switching among them. The switching signal may belong to a certain set and the set may be diverse, and it can be state-driven or time-driven.

Switched systems with all subsystems described by linear differential or difference equations are called switched linear systems, and have attracted most of the attention[5−7]. Recent researches in switched linear system typically focus on the analysis of dynamic behaviors, such as stability[1, 8], controllability, reachability[9−10] and observability[11] etc., and aim to design controllers with guaranteed stability and performance[12−14]. Compared with the fruitful results for stability problem, relatively few efforts are made for designing a controller to achieve tracking of switched systems. Fortunately, for repeated systems, iterative learning control (ILC) offers a systematic design that can improve the tracking performance by iterations in a fixed time interval[15−21]. It has been shown to be one of the most effective methodologies for repeated tracking control tasks for deterministic systems. Control objectives can be achieved iteratively through updating the control input in the iteration domain. However, to the best of our knowledge, no one has studied the iterative learning control for switched systems. This observation motivates the present study.

In this paper, we present the problem of iterative learning control for a class of linear discrete-time switched systems with arbitrary switching rules, which are assumed operating during a finite time interval repetitively. A convergence condition is given by using the super vector approach for such ILC systems. It is shown that P-type ILC scheme can guarantee the convergence of the output tracking error between the given desired output and the actual output through the iterative learning process.

It is worth pointing out that the problem considered in this paper is similar to the ILC for time varying linear system. However, there are some crucial differences. The time varying system admits a family of solutions that can be parameterized solely by the initial condition, whereas the switched system admits a family of solutions that is parameterized both by the initial condition and the switching signal. In the super vector formulation, the time varying linear system has a fixed lower-triangular Markov parameter matrix during the whole time interval, and the initial condition is identical for the whole system. While the lifting matrix of a linear switched system is determined by both subsystems and the switching signal. Besides, the initial condition is only identical for the first subsystem, not for all the subsystems, which results in the convergence analysis to be different from time-varying systems.

The rest of this paper is organized as follows. In Section 1, the problem formulation is described. In Section 2, a sufficient condition which guarantees the stability of the ILC system is given. In Section 3, a simulation example is presented to validate the theoretical result. Some conclusions are given in Section 4.

1 Problem formulation

Let us consider a class of single input single output linear switched systems given by

$$\begin{cases} x(t+1) = A_{\alpha(t)}x(t) + B_{\alpha(t)}u(t) \\ y(t) = C_{\alpha(t)}x(t) \end{cases} \tag{1}$$

where $x(t) \in \mathbf{R}^n$, $u(t) \in \mathbf{R}$, $y(t) \in \mathbf{R}$ are the state, input, and output variables, respectively. t denotes the time variable. $\alpha(t)$ is a switching rule defined by $\alpha(t) : N \rightarrow I$ with $I = \{1, 2, \cdots, l\}$. This means that the matrices ($A_{\alpha(t)}$, $B_{\alpha(t)}$, $C_{\alpha(t)}$) are allowed to take values, at an arbitrary discrete time, in the finite set

$$\{(A_1, B_1, C_1), \cdots, (A_l, B_l, C_l)\}$$

Such systems are said to be switched and belong to the class of hybrid. When the system (1) is operated during a finite time interval repetitively, the switched system can be described as

$$\begin{cases} x_k(t+1) = A_{\alpha(t)}x_k(t) + B_{\alpha(t)}u_k(t) \\ y_k(t) = C_{\alpha(t)}x_k(t) \end{cases} \tag{2}$$

where k denotes the iteration number.

Basic assumptions for the system are given as follows:

Assumption 1. 1) every operation begins at an identical initial condition, i.e., $x_k(0) = 0$ for all k; 2) the desired trajectory $y_d(t)$ is iteration invariant.

Assumption 2. For a given desired trajectory $y_d(t)$,

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there exists a desired control input $u_d(t)$ such that

$$\begin{cases} x_d(t+1) = A_{\alpha(t)}x_d(t) + B_{\alpha(t)}u_d(t) \\ y_d(t) = C_{\alpha(t)}x_d(t) \end{cases} \tag{3}$$

Note that $\alpha(t)$ is an arbitrary switching rule during the finite time interval $[0, N]$, which is different from the aforementioned studies[3−7]. Without loss of generality, we can assume the arbitrary switching rule $\alpha(t)$ is given as

$$\alpha(t) = i = \begin{cases} 1, & t \in [0, m_1] \\ 2, & t \in [m_1+1, m_2] \\ \vdots & \vdots \\ l, & t \in [m_{l-1}+1, N] \end{cases} \tag{4}$$

where $0 = m_1 < m_2 < \cdots < m_{l-1} < N$. Then, the switched system (2) can be represented as

$$\begin{cases} x_k(t+1) = A_i x_k(t) + B_i u_k(t) \\ y_k(t) = C_i x_k(t) \end{cases} \tag{5}$$

and $i \in \{1, 2, \cdots, l\}$.

In this paper, the control target is to find a control input sequence $u_k(t) = u_d(t)$, such that $y_k(t)$ converges to $y_d(t)$ as $k \to \infty$. We consider the first order ILC algorithm as follows:

$$u_{k+1}(t) = u_k(t) + \gamma e_k(t+1) \tag{6}$$

where $e_k(t+1) = y_d(t+1) - y_k(t+1)$ is output tracking error, and γ is a constant learning gain. In the following, we consider how to choose the learning gain γ, then the convergence of the output tracking error can be guaranteed.

2 Main results

In this section, we firstly analyze the stability of switched system with two subsystems for the sake of convenience, and then the result is extended to the multi-subsystems case.

2.1 Two-subsystem switched process

In this case, the switched system (5) only contains two subsystems $\{(A_1, B_1, C_1), (A_2, B_2, C_2)\}$ and the switching law is given as follows:

$$i = \begin{cases} 1, & t \in [0, m] \\ 2, & t \in [m+1, N] \end{cases} \tag{7}$$

By the lifting approach, the switched system (5) with the switching sequence (7) can be represented as

$$\begin{bmatrix} Y_{1,k} \\ Y_{2,k} \end{bmatrix} = \begin{bmatrix} H_1 & \\ & H_2 \end{bmatrix} \begin{bmatrix} U_{1,k} \\ U_{2,k} \end{bmatrix} + \begin{bmatrix} D_1 & \\ & D_2 \end{bmatrix} \begin{bmatrix} x_k(0) \\ x_k(m) \end{bmatrix} \tag{8}$$

where

$$U_{1,k} = [u_k(0), u_k(1), \cdots, u_k(m-1)]^{\mathrm{T}}$$
$$Y_{1,k} = [y_k(1), y_k(2), \cdots, y_k(m)]^{\mathrm{T}}$$
$$U_{2,k} = [u_k(m), u_k(m+1), \cdots, u_k(N-1)]^{\mathrm{T}}$$
$$Y_{2,k} = [y_k(m+1), y_k(m+2), \cdots, y_k(N)]^{\mathrm{T}}$$

and

$$H_1 = \begin{bmatrix} C_1B_1 & 0 & 0 & \cdots & 0 \\ C_1A_1B_1 & C_1B_1 & 0 & \cdots & 0 \\ C_1A_1^2B_1 & C_1A_1B_1 & C_1B_1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ C_1A_1^{m-1}B_1 & C_1A_1^{m-2}B_1 & C_1A_1^{m-3}B_1 & \cdots & C_1B_1 \end{bmatrix}$$

$$H_2 = \begin{bmatrix} C_2B_2 & 0 & 0 & \cdots & 0 \\ C_2A_2B_2 & C_2B_2 & 0 & \cdots & 0 \\ C_2A_2^2B_2 & C_2A_2B_2 & C_2B_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ C_2A_2^{N-m-1}B_2 & C_2A_2^{N-m-2}B_2 & C_2A_2^{N-m-3}B_2 & \cdots & C_2B_2 \end{bmatrix}$$

$$D_1 = [{C_1}^{\mathrm{T}}, (C_1A_1)^{\mathrm{T}}, \cdots, (C_1{A_1}^{m-1})^{\mathrm{T}}]$$

$$D_2 = [{C_2}^{\mathrm{T}}, (C_2A_2)^{\mathrm{T}}, \cdots, (C_2{A_2}^{N-m-1})^{\mathrm{T}}]$$

Now, we can give the following theorem.

Theorem 1. For the linear switched system (5) with the switching sequence (7), when the ILC algorithm (6) is used, then the output tracking error is convergent if

$$\|I - \gamma H_i\| < 1, \ \ i = 1, 2 \tag{9}$$

where $\|\cdot\|$ denotes the matrix norm.

Proof. Define

$$Y_{1,d} = [y_d(1), y_d(2), \cdots, y_d(m)]^{\mathrm{T}}$$
$$Y_{2,d} = [y_d(m+1), y_d(m+2), \cdots, y_d(N)]^{\mathrm{T}}$$
$$E_{1,k} = [e_k(1), e_k(2), \cdots, e_k(m)]^{\mathrm{T}}$$
$$E_{2,k} = [e_k(m+1), e_k(m+2), \cdots, e_k(N)]^{\mathrm{T}}$$

then

$$E_{1,k} = Y_{1,d} - Y_{1,k}, \ \ E_{2,k} = Y_{2,d} - Y_{2,k}$$

The super vector formulation of ILC algorithm (6) is

$$U_{k+1} = U_k + \gamma E_k \tag{10}$$

where $U_k = [U_{1,k}, U_{2,k}]^{\mathrm{T}}, E_k = [E_{1,k}, E_{2,k}]^{\mathrm{T}}$. From (8) and (10), we can obtain

$$E_{1,k+1} = (1 - \gamma H_1) E_{1,k} \tag{11}$$

$$E_{2,k+1} = (1 - \gamma H_2) E_{2,k} + D_1 \Delta x_{k+1}(m) \tag{12}$$

where $\Delta x_{k+1}(m) = x_{k+1}(m) - x_k(m)$. From (11), we have $\|E_{1,k+1}\| \le \|1 - \gamma H_1\| \|E_{1,k}\|$. Thus, since $\|1 - \gamma H_1\| < 1$, $\lim_{k\to\infty} \|E_{1,k}\| = 0$ can be obtained.

Now, we prove the convergence of $E_{2,k}$. $\lim_{k\to\infty} \|E_{1,k}\| = 0$ implies $\lim_{k\to\infty} e_k(m) = 0$, which leads to $\lim_{k\to\infty} \Delta x_{k+1}(m) = 0$. From (12), we get

$$E_{2,k+1} =$$
$$(I - \gamma H_2)E_{2,k} + D_2\Delta x_{k+1}(m) =$$
$$(I - \gamma H_2)^2 E_{2,k-1} + (I - \gamma H_2)D_2\Delta x_k(m) +$$

$$
\begin{aligned}
&D_2\Delta x_{k+1}(m) = \cdots = \\
&(I-\gamma H_2)^k E_{2,1} + (I-\gamma H_2)^{k-1} D_2\Delta x_2(m) + \cdots + \\
&(I-\gamma H_2) D_2 \Delta x_k(m) + D_2\Delta x_{k+1}(m)
\end{aligned}
$$

then, the following inequality can be obtained:

$$
\begin{aligned}
&\|E_{2,k+1}\| \le \\
&\|(I-\gamma H_2)^k\| \, \|E_{2,1}\| + \|(I-\gamma H_2)^{k-1}\| \, \|D_2\Delta x_2(m)\| + \\
&\cdots + \|(I-\gamma H_2)\| \, \|D_2\Delta x_k(m)\| + \|D_2\Delta x_{k+1}(m)\| \le \\
&\|I-\gamma H_2\|^k \, \|E_{2,1}\| + \|D_2\| \, \|I-\gamma H_2\|^{k-1} \, |\Delta x_2(m)| + \\
&\cdots + \|D_2\| \, \|(I-\gamma H_2)\| \, |\Delta x_k(m)| + \|D_2\| \, |\Delta x_{k+1}(m)|
\end{aligned} \tag{13}
$$

Define $\Delta\tilde{x}_k = \sup\{|\Delta x_k(m)|, |\Delta x_{k+1}(m)|, \cdots\}$. Hence, $\Delta\tilde{x}_k \ge \Delta\tilde{x}_{k+1} \ge 0$ and $\Delta\tilde{x}_k \ge |\Delta x_k|$. From (13), if k is even, then

$$
\begin{aligned}
&\|E_{2,k+1}\| \le \|I-\gamma H_2\|^k \, \|E_{2,1}\| + \\
&\|D_2\| \, \|I-\gamma H_2\|^{k-1} \, |\Delta x_2(m)| + \cdots + \\
&\|D_2\| \, \|(I-\gamma H_2)\| \, |\Delta x_k(m)| + \|D_2\| \, |\Delta x_{k+1}(m)| \le \\
&\|I-\gamma H_2\|^k \, \|E_{2,1}\| + \|D_2\| \, \|I-\gamma H_2\|^{k-1} \Delta\tilde{x}_2 + \cdots + \\
&\|D_2\| \, \|(I-\gamma H_2)\|^{\frac{k}{2}} \Delta\tilde{x}_{\frac{k}{2}+1} + \\
&\|D_2\| \, \|(I-\gamma H_2)\|^{\frac{k}{2}-1} \Delta\tilde{x}_{\frac{k}{2}+2} + \cdots + \\
&\|D_2\| \, \|(I-\gamma H_2)\| \, \Delta\tilde{x}_k + \|D_2\| \, \Delta\tilde{x}_{k+1} \le \\
&\|I-\gamma H_2\|^k \, \|E_{2,1}\| + \|D_2\| \, \|I-\gamma H_2\|^{k-1} \Delta\tilde{x}_2 + \cdots + \\
&\|D_2\| \, \|(I-\gamma H_2)\|^{\frac{k}{2}} \Delta\tilde{x}_2 + \\
&\|D_2\| \, \|(I-\gamma H_2)\|^{\frac{k}{2}-1} \Delta\tilde{x}_{\frac{k}{2}+2} + \cdots + \\
&\|D_2\| \, \|(I-\gamma H_2)\| \, \Delta\tilde{x}_{\frac{k}{2}+2} + \|D_2\| \, \Delta\tilde{x}_{\frac{k}{2}+2} \le \\
&\|(I-\gamma H_2)\|^{\frac{k}{2}} \left(\|E_{2,1}\| + \frac{k}{2}\|D_2\| \, \Delta\tilde{x}_2\right) + \\
&\Delta\tilde{x}_{\frac{k}{2}+2} \|D_2\| \times \\
&\left(\|(I-\gamma H_2)\|^{\frac{k}{2}-1} + \cdots + \|(I-\gamma H_2)\| + 1\right) = \\
&\|(I-\gamma H_2)\|^{\frac{k}{2}} \left(\|E_{2,1}\| + \frac{k}{2}\|D_2\| \, \Delta\tilde{x}_2\right) + \\
&\Delta\tilde{x}_{\frac{k}{2}+2} \|D_2\| \frac{1-\|(I-\gamma H_2)\|^{\frac{k}{2}-1}}{1-\|(I-\gamma H_2)\|}
\end{aligned}
$$

Therefore, $\lim_{i\to\infty}\|E_{2,k+1}\| = 0$. Similarly, when k is odd, $\lim_{i\to\infty}\|E_{2,k+1}\| = 0$ also can be derived. □

Remark 1. The condition $\|I-\gamma H_i\| < 1$ $(i=1,2)$ means that the gain γ can guarantee convergence for all H_i. Alternatively, we also can choose the switched ILC schemes as $u_{k+1}(t) = u_k(t) + \gamma_i e_k(t+1)$. In this case, the convergence condition in Theorem 1 becomes $\|I-\gamma_i H_i\| < 1$.

Remark 2. Switched sequence (7) implies the switching only occurs once during the whole interval $[0, N]$, and the dwell time of two subsystems are $[0, m]$ and $[m+1, N]$ respectively. From the proof of Theorem 1, we know that the result can also be extended to the arbitrary switching times of the two subsystems with random small dwell time.

2.2 Multi-subsystem switched process

In this case, the switched system (5) contains multi subsystems $\{(A_1, B_1, C_1), \cdots, (A_l, B_l, C_l)\}$, and the switch variable i is given as follows:

$$
i = \begin{cases} 1, & t\in[0, m_1] \\ 2, & t\in[m_1+1, m_2] \\ \vdots & \vdots \\ l, & t\in[m_{l-1}+1, N] \end{cases} \tag{14}
$$

where $0 = m_1 < m_2 < \cdots < m_{l-1} < N$. Define $m_0 = 0$, $m_l = N$ and

$$
U_{i,k} = [u_k(m_{i-1}), u_k(m_{i-1}+1), \cdots, u_k(m_i - 1)]^{\rm T}
$$

$$
Y_{i,k} = [y_k(m_{i-1}+1), y_k(m_{i-1}+2), \cdots, y_k(m_i)]^{\rm T}
$$

Using the lifting approach for ILC, the system (5) can be described as follows:

$$
Y_{i,k} = H_i U_{i,k} + D_i x_k(m_{i-1}) \tag{15}
$$

where

$$
H_i = \begin{bmatrix}
C_iB_i & 0 & 0 & \cdots & 0 \\
C_iA_iB_i & C_iB_i & 0 & \cdots & 0 \\
C_iA_i^2B_i & C_iA_iB_i & C_iB_i & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
C_iA_i^{m_i-m_{i-1}-1}B_i & C_iA_i^{m_i-m_{i-1}-2}B_i & C_iA_i^{m_i-m_{i-1}-3}B_i & \cdots & C_iB_i
\end{bmatrix}
$$

Now, the following theorem can be given.

Theorem 2. For the linear switched system (5) with the switching sequence (14), when the ILC algorithm (6) is used, then the output tracking error is convergent if

$$
\|I-\gamma H_i\| < 1, \quad i = 1, 2, \cdots, l \tag{16}
$$

Proof. From (15), we can give the following formulation

$$
\begin{bmatrix} Y_{1,k} \\ Y_{2,k} \\ \vdots \\ Y_{l,k} \end{bmatrix} = \begin{bmatrix} H_1 & & & \\ & H_2 & & \\ & & \ddots & \\ & & & H_l \end{bmatrix} \begin{bmatrix} U_{1,k} \\ U_{2,k} \\ \vdots \\ U_{l,k} \end{bmatrix} +
$$

$$
\begin{bmatrix} D_1 & & & \\ & D_2 & & \\ & & \ddots & \\ & & & D_l \end{bmatrix} \begin{bmatrix} x_k(0) \\ x_k(m_1) \\ \vdots \\ x_k(m_{l-1}) \end{bmatrix} \tag{17}
$$

Define

$$Y_{i,d} = [y_d(m_{i-1}+1), y_d(m_{i-1}+2), \cdots, y_d(m_i)]^{\mathrm{T}}$$

$$E_{i,k} = [e_k(m_{i-1}+1), e_k(m_{i-1}+2), \cdots, e_k(m_i)]^{\mathrm{T}}$$

$$E_{i,k} = Y_{i,d} - Y_{i,k}, \quad i = 1, 2, \cdots, l$$

From (8) and (17), we can obtain

$$E_{i,k+1} = (I - \gamma H_i)E_{i,k} + D_2 \Delta x_{k+1}(m_{i-1}) \tag{18}$$

For $i = 1$, note that $\Delta x_{k+1}(0) = x_{k+1}(0) - x_k(0) = 0$, (18) gives

$$E_{1,k+1} = (I - \gamma H_1)E_{1,k}$$

Hence, since $\|I - \gamma H_1\| < 1$, we can obtain $\lim_{k\to\infty} \|E_{1,k}\| = 0$, which also leads to $\lim_{k\to\infty} e_k(m_1) = 0$ and $\lim_{k\to\infty} \Delta x_{k+1}(m_1) = 0$.

For $i = 2$, (18) gives

$$E_{2,k+1} = (I - \gamma H_2)E_{2,k} + D_2 \Delta x_{k+1}(m_1)$$

according to the analysis in Theorem 1, we can obtain $\lim_{k\to\infty} \|E_{2,k}\| = 0$ according to the condition $\|I - \gamma H_2\| < 1$ and $\lim_{k\to\infty} \Delta x_{k+1}(m_1) = 0$. In this analogy, $\lim_{k\to\infty} \|E_{i,k}\| = 0$ $(i > 2)$ can also be obtained. □

Remark 3. The condition $\|I - \gamma H_i\| < 1$ $(i = 1, 2, \cdots, l)$ means the learning gain γ can guarantee convergence for all H_i. Similarly, we also can choose the switched ILC schemes as $u_{k+1}(t) = u_k(t) + \gamma_i e_k(t+1)$ with the convergence condition $\|I - \gamma_i H_i\| < 1$. On the other hand, switching sequence (14) implies the switching only occurs $l - 1$ times during the whole interval $[0, N]$. From the proof of Theorem 2, we know that the result can also be extended to the arbitrary switch times of the multi-subsystems with random small dwell time.

3 Numerical example

In this section, an example is used to verify our conclusions. Let us consider the linear discrete-time switched system as follows, which contains three subsystems:

$$\begin{cases} x_k(t+1) = A_i x_k(t) + B_i u_k(t) \\ y_k(t) = C_i x_k(t) \end{cases}, \quad i = 1, 2, 3 \tag{19}$$

where

$$A_1 = \begin{bmatrix} 0 & 1 \\ 0.125 & -0.2 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 0.1 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -0.25 & 1 \\ 0 & -0.3 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C_2 = \begin{bmatrix} -0.2 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 1 & 0 \\ 0.2 & -0.1 \end{bmatrix}, \quad B_3 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C_3 = \begin{bmatrix} 0.25 & 1 \end{bmatrix}$$

The desired repetitive reference trajectory is $y_d(t) = \sin(8.0(t-1)/25)$ and $t \in [0, 60]$. The initial conditions are given as $x_k(0) = 0$ for all k and $u_0(t) = 0$ for all t. The ILC scheme is constructed as $u_{k+1}(t) = u_k(t) + 0.5 e_k(t+1)$.

3.1 Case 1

In this case, we assume the switching sequence is

$$i = \begin{cases} 1, & t \in [0, 15] \\ 2, & t \in [16, 45] \\ 3, & t \in [46, 60] \end{cases}$$

Using the super vector formulation, the switched ILC system can be represented as

$$\begin{bmatrix} Y_{1,k} \\ Y_{2,k} \\ Y_{3,k} \end{bmatrix} = \begin{bmatrix} H_1 & & \\ & H_2 & \\ & & H_3 \end{bmatrix} \begin{bmatrix} U_{1,k} \\ U_{2,k} \\ U_{3,k} \end{bmatrix} + \begin{bmatrix} D_1 & & \\ & D_2 & \\ & & D_3 \end{bmatrix} \begin{bmatrix} x_k(0) \\ x_k(15) \\ x_k(45) \end{bmatrix}$$

where

$$U_{1,k} = [u_k(0), \cdots, u_k(14)]^{\mathrm{T}}, \quad Y_{1,k} = [y_k(1), \cdots, y_k(15)]^{\mathrm{T}}$$

$$U_{2,k} = [u_k(15), \cdots, u_k(44)]^{\mathrm{T}}, \; Y_{2,k} = [y_k(16), \cdots, y_k(45)]^{\mathrm{T}}$$

$$U_{3,k} = [u_k(45), \cdots, u_k(59)]^{\mathrm{T}}, \; Y_{3,k} = [y_k(46), \cdots, y_k(60)]^{\mathrm{T}}$$

and

$$H_1 = \begin{bmatrix} C_1 B_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ C_1 A_1^{14} B_1 & \cdots & C_1 B_1 \end{bmatrix}$$

$$H_2 = \begin{bmatrix} C_2 B_2 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ C_2 A_2^{29} B_2 & \cdots & C_2 B_2 \end{bmatrix}$$

$$H_3 = \begin{bmatrix} C_3 B_3 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ C_3 A_3^{14} B_3 & \cdots & C_3 B_3 \end{bmatrix}$$

Using Theorem 2, we have $\|I - \gamma H_1\|_\infty = 0.6667 < 1$, $\|I - \gamma H_2\|_\infty = 0.7421 < 1$, $\|I - \gamma H_3\|_\infty = 0.6972 < 1$. The convergence condition in Theorem 2 is satisfied. Fig. 1 shows that $\|E_k\|_\infty$ is convergent on the iteration domain. It also demonstrates the output tracking error is convergent to 0 as iteration number increased. Fig. 2 and Fig. 3 show the output and input variables at the different iteration number. Obviously, system output profiles and control input profiles are also convergent on iteration domain.

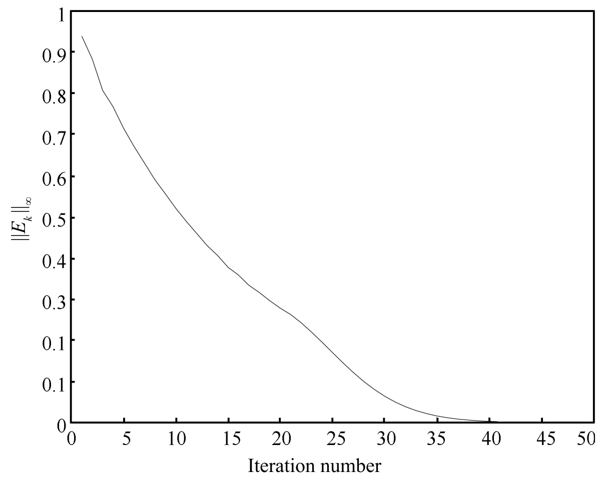

Fig. 1 The $\|E_k\|_\infty$ on the iteration domain

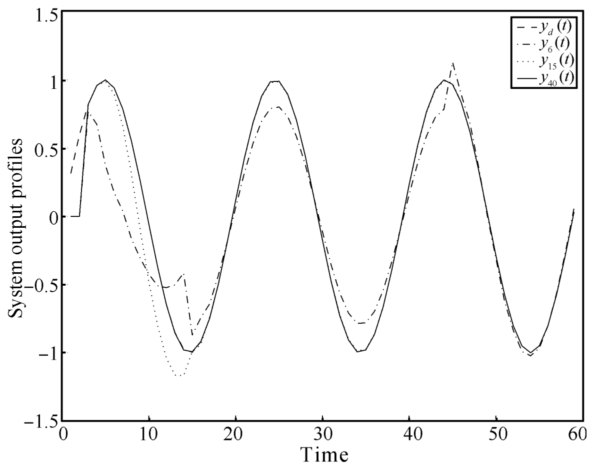

Fig. 2 The system output profiles at 6th, 15th, and 40th iteration

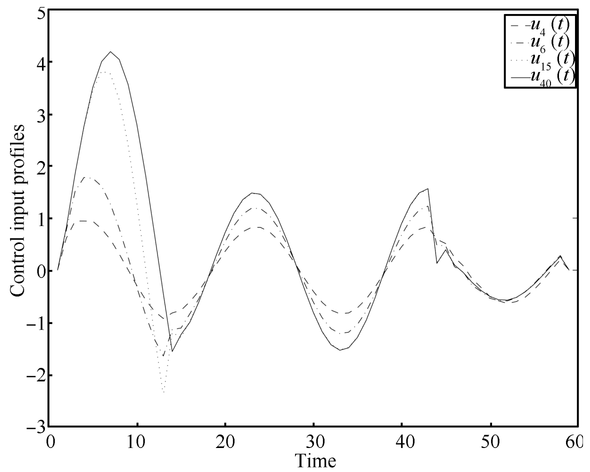

Fig. 3 The control input profiles at 4th, 6th, 15th, and 40th iteration

3.2 Case 2

In this case, we assume the switching law $\alpha(t)$ is a random sequence, which is produced by a random variable with the value 1, 2, and 3, as shown in Fig. 4. If $\alpha(t) = 1$, the system (19) is (A_1, B_1, C_1), and if $\alpha(t) = 2$, then system is (A_2, B_2, C_2), otherwise, the system is (A_3, B_3, C_3).

The results of simulation are shown in Figs. 5 ~ 7. It can be observed that by using the proposed iterative learning control law, the asymptotic convergence of the output error can also be guaranteed. Fig. 6 shows that the output tracking is perfect after 40 iterations. To understand the effectiveness of iterative learning control, we also give the control input profiles in Fig. 7. The input profile at 40th iteration is close to the desired input profile, which is difficult to be obtained by other control approaches.

4 Conclusions

The problem of iterative learning control for a class of linear switched systems has been discussed. Here, the considered switched system has been assumed to be with arbitrary switching signals in time domain. It has also been shown that under some conditions, the P-type ILC algorithm can guarantee the asymptotic convergence of the output tracking error between the given desired output and the actual output through the iterative learning process. The theoretical analysis is supported by simulations. In the future work, we will consider switched systems with arbitrary switching signals in iteration domain.

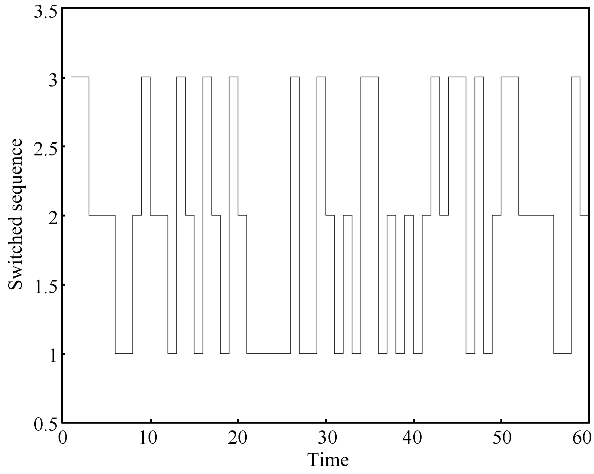

Fig. 4 The random switching sequence of $\alpha(t)$

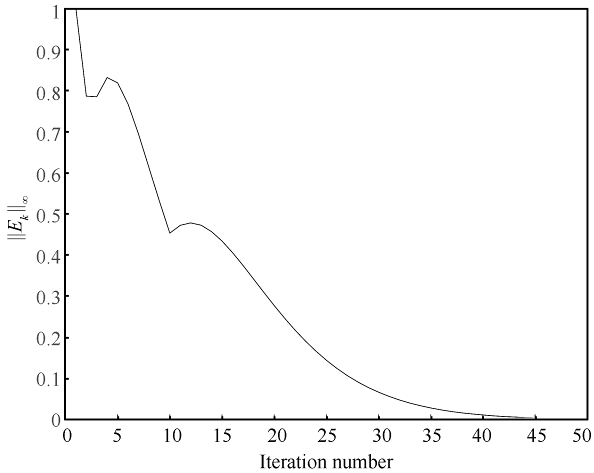

Fig. 5 The $\|E_k\|_\infty$ on the iteration domain for random switching sequence

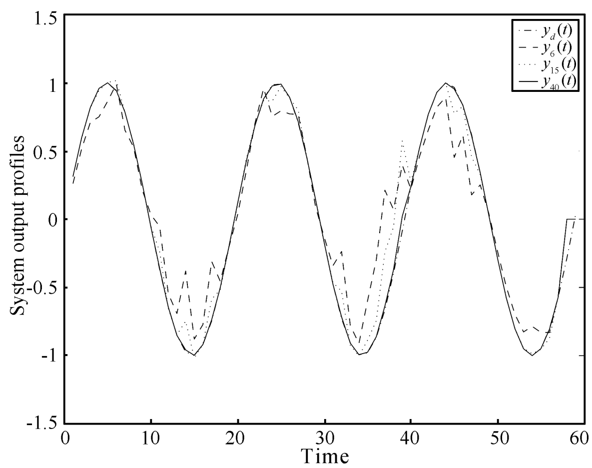

Fig. 6 The system output profiles for random switching sequence

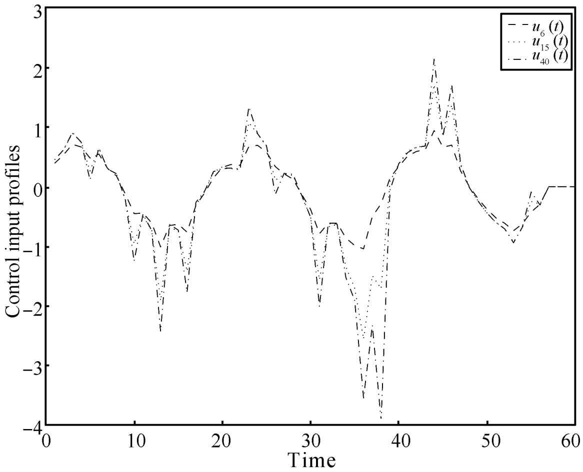

Fig. 7 The control input profiles for random switching sequence

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