Asymptotic Stability of 2-D Positive Linear Systems with Orthogonal Initial States

ZHU Qiao[1] CUI Jia-Rui[1] HU Guang-Da[1]

Abstract This paper deals with the asymptotic stability of 2-D positive linear systems with orthogonal initial states. Different from the 1-D systems, the asymptotic stability of 2-D systems with orthogonal initial states $\boldsymbol{x}(i,0)$, $\boldsymbol{x}(0,j)$ (Fornasini-Marchesini (FM) model) or $\boldsymbol{x}^v(i,0)$, $\boldsymbol{x}^h(0,j)$ (Roesser model) is strictly dependent on proper boundary conditions. Firstly, an asymptotic stability criterion for 2-D positive FM first model is presented by making initial states $\boldsymbol{x}(i,0)$, $\boldsymbol{x}(0,j)$ absolutely convergent. Then, a similar result is also given for 2-D positive Roesser model with any absolutely convergent initial states $\boldsymbol{x}^v(i,0)$, $\boldsymbol{x}^h(0,j)$. Finally, two examples are given to show the effectiveness of these criteria and to demonstrate the convergence of the trajectories by making exponentially convergent initial states.

Key words 2-D positive linear system, asymptotic stability, orthogonal initial states, boundary conditions

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The theory (especially the stability theory) of 2-D discrete-time dynamic systems, has attracted great attention for more than two decades, due to its wide existence in many practical applications, such as image data processing and transmission, thermal processes, gas absorption and water stream heating, etc[1−2]. Especially, 2-D dynamical systems described by the Roesser model[3−4] and the FM model[5−10], have been deeply and widely investigated. Furthermore, the theory and application of 2-D positive systems also have been paid great attention[11]. For example, the asymptotic stability of 2-D positive Roesser model was investigated in [12−13]. A 2-D positive linear system with delay was analyzed in [14−15]. 2-D positive hybrid linear system and 2-D positive continuous-discrete linear system were also deeply investigated, such as in [16−17].

For 2-D linear systems with any diagonal initial states $\boldsymbol{x}(i,-i)$, $\forall\, i \in \mathbf{Z}$, sufficient and necessary conditions of asymptotic stability have been established with the form of 2-D characteristic polynomial (see e.g., Theorem 1 in [18] for Roesser model and Proposition 1 in [5] for FM model). However, these sufficient and necessary conditions do not hold for any orthogonal initial states $\boldsymbol{x}(i,0)$, $\boldsymbol{x}(0,j)$ (or $\boldsymbol{x}^v(i,0)$, $\boldsymbol{x}^h(0,j)$). For example, for the 2-D FM first model $\boldsymbol{x}(i+1,j+1)=A_0\boldsymbol{x}(i,j)+A_1\boldsymbol{x}(i+1,j)+A_2\boldsymbol{x}(i,j+1)$, we can see $\boldsymbol{x}(i+1,1)=A_0\boldsymbol{x}(i,0)+A_1\boldsymbol{x}(i+1,0)+A_2\boldsymbol{x}(i,1)$. Assume that $\boldsymbol{x}(i,0)$ and $\boldsymbol{x}(i,1)$ are convergent with respect to i. Then, we have $\boldsymbol{x}(\infty,1)=(I_n-A_2)^{-1}(A_0+A_1)\boldsymbol{x}(\infty,0)$. Clearly, we almost cannot get $\boldsymbol{x}(i,1)$ approaches zero for any initial states $\boldsymbol{x}(i,0)$ which implies the asymptotic stability almost cannot hold for any initial states $\boldsymbol{x}(i,0)$, $\boldsymbol{x}(0,j)$. This is very different with the asymptotic stability of 1-D systems which do not depend on the initial state. Furthermore, for many 2-D systems and their engineering applications (e.g., the 2-D iterative learning control model introduced in [4]), the orthogonal initial states $\boldsymbol{x}(i,0)$, $\boldsymbol{x}(0,j)$ (or $\boldsymbol{x}^v(i,0)$, $\boldsymbol{x}^h(0,j)$) are most widely used. Therefore, it is very interesting and necessary to research that under which initial conditions we can obtain the asymptotic stability of 2-D systems with orthogonal initial states.

The aim of this paper is to obtain asymptotic stability criteria for 2-D positive linear systems with orthogonal initial states. The organization of this paper is as follows. In Section 1, an asymptotic stability criterion is given for 2-D positive FM first model with absolutely convergent initial states $\boldsymbol{x}(i,0)$, $\boldsymbol{x}(0,j)$. In Section 2, for 2-D positive Roesser model with absolutely convergent initial states $\boldsymbol{x}^v(i,0)$, $\boldsymbol{x}^h(0,j)$, an asymptotic stability criterion is also established. In Section 3, two numerical examples are presented to validate the effectiveness of the proposed stability criteria and boundary conditions.

Notations. Let $\mathbf{R}^n$ denote the n-dimensional Euclidean space, $\mathbf{R}^n_+$ the set of all n-dimensional real vectors with nonnegative integers, $\mathbf{R}^{n\times m}$ the set of all $n\times m$ real matrices, $\mathbf{R}^{n\times m}_+$ the set of all $n\times m$ real matrices with nonnegative elements, $\mathbf{Z}$ the set of all integers, $\mathbf{Z}_+$ the set of all nonnegative integers, I_n the $n\times n$ identity matrix, $|\cdot|$ the usual Euclidean norm and $\rho(\cdot)$ the spectral radius.

1 2-D positive FM first model

Consider the following 2-D FM first model:

$$\boldsymbol{x}(i+1,j+1)=A_0\boldsymbol{x}(i,j)+A_1\boldsymbol{x}(i+1,j)+A_2\boldsymbol{x}(i,j+1) \tag{1}$$

where $\boldsymbol{x}\in\mathbf{R}^n$ is the state, and $A_0, A_1, A_2\in\mathbf{R}^{n\times n}$ are the system matrices.

Definition 1. The 2-D FM first model (1) is called positive if $\boldsymbol{x}(i,j)\in\mathbf{R}^n_+$, $\forall\, i,j\in\mathbf{Z}_+$ for any orthogonal initial states $\boldsymbol{x}(i,0)$, $\boldsymbol{x}(0,j)\in\mathbf{R}^n_+$, $\forall\, i,j\in\mathbf{Z}_+$.

Definition 2. The 2-D FM first model (1) is asymptotically stable if assuming the orthogonal initial states $\boldsymbol{x}(i,0)$, $\boldsymbol{x}(0,j)$, $\forall\, i,j\ge 0$ are finite, $\lim_{i+j\to\infty}|\boldsymbol{x}(i,j)|=0$.

Lemma 1 (see Theorem 5 in [14]). The 2-D FM first model (1) is positive if and only if

$$A_k\in\mathbf{R}^{n\times n}_+,\ \ k=0,1,2 \tag{2}$$

Theorem 1. The 2-D positive FM first model (1) is asymptotically stable if

$$\rho(A_1)<1,\ \ \rho[(I_n-A_1)^{-1}(A_0+A_2)]<1 \tag{3}$$

$$\rho(A_2)<1,\ \ \rho[(I_n-A_2)^{-1}(A_0+A_1)]<1 \tag{4}$$

and the orthogonal initial states are absolutely convergent, i.e.,

$$\sum_{i=0}^{\infty}\boldsymbol{x}(i,0)<\infty,\quad \sum_{j=0}^{\infty}\boldsymbol{x}(0,j)<\infty \tag{5}$$

Proof. First, we need to prove $\lim_{j\to\infty}\boldsymbol{x}(i,j)<\infty$, $\forall\, i\in\mathbf{Z}_+$. Note that $\boldsymbol{x}(i+1,j+1)=A_0x(i,j)+A_1\boldsymbol{x}(i+1,j)+A_2\boldsymbol{x}(i,j+1)$. Then, we have

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$$\boldsymbol{x}(1,j+1) = A_1\boldsymbol{x}(1,j) + [A_0\boldsymbol{x}(0,j) + A_2\boldsymbol{x}(0,j+1)] \quad (6)$$

It implies that $\lim_{j\to\infty}\boldsymbol{x}(1,j) < \infty$, if $\rho(A_1) < \infty$ and $\boldsymbol{x}(0,j) < \infty$, $\forall j \in \mathbf{Z}_+$. Continuing this procedure, we can obtain that $\lim_{j\to\infty}\boldsymbol{x}(i,j) < \infty$, $\forall i \in \mathbf{Z}_+$.

Furthermore, for any integer $q > 0$, we have

$$\sum_{j=0}^{q}\boldsymbol{x}(i+1,j+1) = A_0\sum_{j=0}^{q}\boldsymbol{x}(i,j) + A_1\sum_{j=0}^{q}\boldsymbol{x}(i+1,j) + A_2\sum_{j=0}^{q}\boldsymbol{x}(i,j+1)$$

which can be rewritten as

$$(I_n - A_1)\sum_{j=1}^{q+1}\boldsymbol{x}(i+1,j) = (A_0 + A_2)\times \sum_{j=1}^{q+1}\boldsymbol{x}(i,j) + A_1(\boldsymbol{x}(i+1,0) - \boldsymbol{x}(i+1,q+1)) + A_0((\boldsymbol{x}(i,0) - \boldsymbol{x}(i,q+1)) \quad (7)$$

Let $q\to\infty$, $G = (I_n - A_1)^{-1}(A_0 + A_2)$, $H = (I_n - A_1)^{-1}$, $Y(i) = \sum_{j=1}^{\infty} x(i,j)$, $U(i) = A_0\boldsymbol{x}(i,0) + A_1\boldsymbol{x}(i+1,0) + A_0\boldsymbol{x}(i,\infty) + A_1\boldsymbol{x}(i+1,\infty)$. Then, (7) can be rewritten as

$$Y(i+1) = GY(i) + HU(i) \quad (8)$$

Note that $U(i) < \infty$, $\forall i \in \mathbf{Z}_+$. Then, from 1-D BIBO stability theory, it follows that

$$\sum_{j=1}^{\infty}\boldsymbol{x}(i,j) < \infty,\ \ \forall i \in \mathbf{Z}_+ \quad (9)$$

if $\rho(G) < 1$ and $\sum_{j=0}^{\infty}\boldsymbol{x}(0,j) < \infty$. It implies

$$\lim_{j\to\infty}\boldsymbol{x}(i,j) = 0,\ \ \forall i \in \mathbf{Z}_+ \quad (10)$$

since $\boldsymbol{x}(i,j) \in \mathbf{R}_+^n$, $\forall i,j \in \mathbf{Z}_+$.

Similarly, we can also have

$$\lim_{i\to\infty}\boldsymbol{x}(i,j) = 0,\ \ \forall j \in \mathbf{Z}_+$$

if $\rho(A_2) < 1$, $\rho[(I_n - A_2)^{-1}(A_0 + A_1)] < 1$ and $\sum_{i=0}^{\infty}\boldsymbol{x}(i,0) < \infty$. □

Remark 1. In [19−20], sufficient and necessary conditions for asymptotic stability of 2-D positive FM model are investigated. For example, the sufficient and necessary condition $\rho(A_0 + A_1 + A_2) < 1$ was introduced for 2-D positive FM first model (1). However, these conditions are only available for any initial states $\boldsymbol{x}(i,-i)$ rather than $\boldsymbol{x}(i,0)$, $\boldsymbol{x}(0,j)$, since the conditions are essentially established on the basis of Proposition 1 in [5] which holds only for any initial states $\boldsymbol{x}(i,-i)$.

2 2-D positive Roesser model

Now, let us consider the following 2-D Roesser model

$$\begin{bmatrix} \boldsymbol{x}^h(i+1,j) \\ \boldsymbol{x}^v(i,j+1) \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} \boldsymbol{x}^h(i,j) \\ \boldsymbol{x}^v(i,j) \end{bmatrix} \quad (11)$$

where $\boldsymbol{x} = \begin{bmatrix} \boldsymbol{x}^h \\ \boldsymbol{x}^v \end{bmatrix} \in \mathbf{R}^n$ is the state, and $\boldsymbol{x}^h \in \mathbf{R}^{n_1}$, $\boldsymbol{x}^v \in \mathbf{R}^{n_2}$, $n_1 + n_2 = n$ represent the horizontal and vertical states, respectively; $A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$ is the system matrix with the submatrices A_{ij}, $i,j = 1,2$ of appropriate dimensions.

Definition 3. The 2-D Roesser model (11) is called positive if $\boldsymbol{x}(i,j) \in \mathbf{R}_+^n$, $\forall i,j \in \mathbf{Z}_+$ for any orthogonal initial states $\boldsymbol{x}^v(i,0)$, $\boldsymbol{x}^h(0,j) \in \mathbf{R}_+^n$, $\forall i,j \in \mathbf{Z}_+$.

Definition 4. The 2-D Roesser model (11) is asymptotically stable if assuming the orthogonal initial states $\boldsymbol{x}^v(i,0)$, $\boldsymbol{x}^h(0,j)$, $\forall i,j \geq 0$ are finite, $\lim_{i+j\to\infty}|\boldsymbol{x}(i,j)| = 0$.

Lemma 2 (see Theorem 6 in [14]). The 2-D Roesser model (11) is positive if and only if

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \in \mathbf{R}_+^{n\times n} \quad (12)$$

Theorem 2. The 2-D positive Roesser model (11) is asymptotically stable if

$$\rho(A_{22}) < 1,\ \ \rho[A_{11} + A_{12}(I_{n_2} - A_{22})^{-1}A_{21}] < 1 \quad (13)$$

$$\rho(A_{11}) < 1,\ \ \rho[A_{22} + A_{21}(I_{n_1} - A_{11})^{-1}A_{12}] < 1 \quad (14)$$

and the orthogonal initial states are absolutely convergent, i.e.,

$$\sum_{i=0}^{\infty}\boldsymbol{x}^v(i,0) < \infty,\ \ \sum_{j=0}^{\infty}\boldsymbol{x}^h(0,j) < \infty \quad (15)$$

Proof. Note that $\boldsymbol{x}^v(0,j+1) = A_{22}\boldsymbol{x}^v(0,j) + A_{21}\boldsymbol{x}^h(0,j)$. Then, we have that $\boldsymbol{x}^v(0,j) < \infty$, $\forall j \in \mathbf{Z}_+$ if $\rho(A_{22}) < 1$ and $\boldsymbol{x}^h(0,j) < \infty$, $\forall j \in \mathbf{Z}_+$. It implies $\boldsymbol{x}(0,j) < \infty$, $\forall j \in \mathbf{Z}_+$. Now, let us show $\boldsymbol{x}(1,j) < \infty$, $\forall j \in \mathbf{Z}_+$. Note that $\boldsymbol{x}^h(1,j+1) = A_{11}\boldsymbol{x}^h(0,j) + A_{12}\boldsymbol{x}^v(0,j)$. Then, we have $\boldsymbol{x}^h(1,j) < \infty$, $\forall j \in \mathbf{Z}_+$. Furthermore, since $\boldsymbol{x}^v(1,j+1) = A_{22}\boldsymbol{x}^v(1,j) + A_{21}(A_{11}\boldsymbol{x}^h(0,j) + A_{12}\boldsymbol{x}^v(0,j))$, we can get that $\boldsymbol{x}^v(1,j) < \infty$, $\forall j \in \mathbf{Z}_+$ if $\rho(A_{22}) < 1$ and $\boldsymbol{x}(0,j) < \infty$, $\forall j \in \mathbf{Z}_+$. It follows that $\boldsymbol{x}(1,j) < \infty$, $\forall j \in \mathbf{Z}_+$. Continuing this procedure, we can show that $\boldsymbol{x}(i,j) < \infty$, $\forall i,j \in \mathbf{Z}_+$.

For any integer $q > 0$, it is not hard to see from (11) that

$$\sum_{j=0}^{q}\boldsymbol{x}^h(i+1,j) = A_{11}\sum_{j=0}^{q}\boldsymbol{x}^h(i,j) + A_{12}\sum_{j=0}^{q}\boldsymbol{x}^v(i,j)$$

$$\sum_{j=0}^{q}\boldsymbol{x}^v(i,j+1) = A_{21}\sum_{j=0}^{q}\boldsymbol{x}^h(i,j) + A_{22}\sum_{j=0}^{q}\boldsymbol{x}^v(i,j)$$

which implies

$$\sum_{j=0}^{q}\boldsymbol{x}^h(i+1,j) = [A_{11} + A_{12}(I_{n_2} - A_{22})^{-1}A_{21}]\sum_{j=0}^{q}\boldsymbol{x}^h(i,j) + A_{12}(I_{n_2} - A_{22})^{-1}(\boldsymbol{x}^v(i,0) + \boldsymbol{x}^v(i,q+1)) \quad (16)$$

Let $q\to\infty$. Then, based on the 1-D BIBO stability theory, it follows that

$$\sum_{j=0}^{\infty}\boldsymbol{x}^h(i,j) < \infty,\ \ \forall i \in \mathbf{Z}_+ \quad (17)$$

if $\rho[A_{11} + A_{12}(I_{n_2} - A_{22})^{-1}A_{21}] < 1$ and $\sum_{j=0}^{\infty}\boldsymbol{x}^h(0,j) < \infty$.

Now, let us prove $\sum_{j=0}^{\infty} \boldsymbol{x}^v(i,j) < \infty$, $\forall\, i \in \mathbf{Z}_+$. From (11), we can also get

$$(I - A_{22}) \sum_{j=0}^{q} \boldsymbol{x}^v(i,j) =$$

$$A_{21} \sum_{j=0}^{\infty} \boldsymbol{x}^h(i,j) + \boldsymbol{x}^v(i,0) - \boldsymbol{x}^v(i,q+1)$$

Clearly, we have

$$\sum_{j=0}^{\infty} \boldsymbol{x}^v(i,j) < \infty, \ \ \forall\, i \in \mathbf{Z}_+ \tag{18}$$

if $\rho(A_{22}) < 1$ and $\sum_{j=0}^{\infty} \boldsymbol{x}^h(i,j) < \infty$, $\lim_{j\to\infty} \boldsymbol{x}^v(i,j) < \infty$. Therefore, from (17) and (18), we can easily obtain

$$\lim_{j\to\infty} \boldsymbol{x}(i,j) = 0, \ \ \forall\, i \in \mathbf{Z}_+ \tag{19}$$

if $\rho(A_{22}) < 1$, $\rho[A_{11} + A_{12}(I_{n_2} - A_{22})^{-1}A_{21}] < 1$ and $\sum_{j=0}^{\infty} \boldsymbol{x}^h(0,j) < \infty$.

Using a similar procedure, we can also get

$$\lim_{i\to\infty} \boldsymbol{x}(i,j) = 0, \ \ \forall\, j \in \mathbf{Z}_+ \tag{20}$$

if $\rho(A_{11}) < 1$, $\rho[A_{22} + A_{21}(I_{n_1} - A_{11})^{-1}A_{12}] < 1$ and $\sum_{i=0}^{\infty} \boldsymbol{x}^v(i,0) < \infty$. □

Remark 2. It was shown in [13] that the asymptotic stability of 2-D positive Roesser model (11) is equivalent to $\rho(A) < 1$. However, $\rho(A) < 1$ is the necessary and sufficient condition only for any initial states $\boldsymbol{x}(i,-i)$ rather than $\boldsymbol{x}^v(i,0)$, $\boldsymbol{x}^h(0,j)$, since the result is established on the basis of Theorem 1 in [18] whose initial states are $\boldsymbol{x}(i,-i)$.

3 Numerical examples

Example 1. Consider the 2-D positive FM first model (1) with

$$A_0 = \begin{bmatrix} 0.5 & 0.1 \\ 0 & 0.5 \end{bmatrix}, \ \ A_1 = \begin{bmatrix} 0.2 & 0 \\ 0.1 & 0.2 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix}$$

It is not hard to see that Example 1 satisfies the condition given by (3). Thus, from Theorem 1, the above system with any orthogonal initial states satisfying (5) is asymptotically stable. Assume that the orthogonal initial states are given as $\boldsymbol{x}(i,0) = [1\ 1]^{\rm T}10(0.9)^i(\sin(i)+1)$, $\boldsymbol{x}(0,j) = [1\ 1]^{\rm T}10(0.9)^j(\cos(j)+1)$, $\forall\, i \geq 1$, $j \geq 0$. Then, the simulation result of the trajectory is given in Fig. 1.

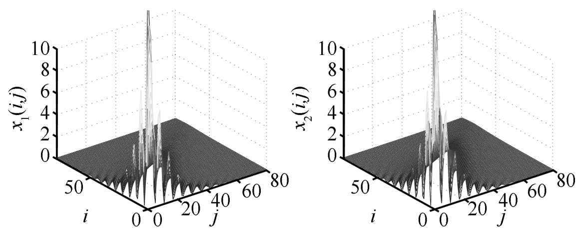

Fig. 1 The solution of Example 1 at times $i, j \in [0, 80]$

Example 2. Consider the 2-D positive Roesser model (11) with

$$A_{11} = \begin{bmatrix} 0.5 & 0.1 \\ 0.2 & 0.5 \end{bmatrix}, \ \ A_{12} = \begin{bmatrix} 0 & 0.2 \\ 0.1 & 0.3 \end{bmatrix}$$

$$A_{21} = \begin{bmatrix} 0 & 0.5 \\ 0.5 & 0 \end{bmatrix}, \ \ A_{22} = \begin{bmatrix} 0.6 & 0.1 \\ 0.1 & 0.4 \end{bmatrix}$$

Clearly, the above system satisfies the condition (13). Thus, from Theorem 2, the above system is asymptotically stable for any orthogonal initial states satisfying (15). Assume that the orthogonal initial states $\boldsymbol{x}^v(i,0) = [1\ 1]^{\rm T}0.5^i$, $\boldsymbol{x}^h(0,j) = [1\ 1]^{\rm T}0.5^j$, $\forall\, i \in \mathbf{Z}_+$. Then, the simulation result of the trajectory is given in Fig. 2.

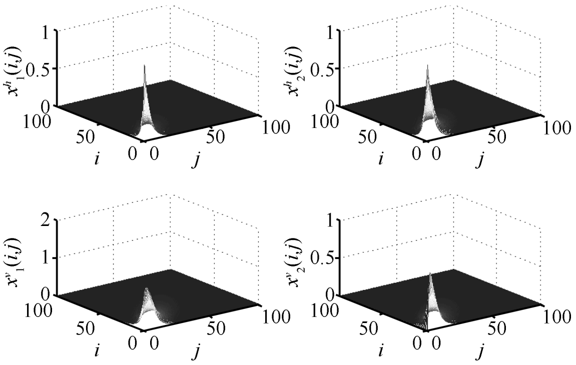

Fig. 2 The solution of Example 2 at times $i, j \in [0, 100]$

From Figs. 1 and 2, we can see that Theorems 1 and 2 are effective and applicable for exponentially convergent initial states.

4 Conclusions

The main contribution of this study is to recognize the impact of the orthogonal initial states $\boldsymbol{x}(i,0)$, $\boldsymbol{x}(0,j)$ (or $\boldsymbol{x}^v(i,0)$, $\boldsymbol{x}^h(0,j)$) on the asymptotic stability of 2-D systems. Note that 2-D systems originally come from the state-space realization of digital filters. Therefore, we do not need to consider boundary conditions when designing 2-D digital filters, because the initial states of digital filters can be artificially chosen, and necessary and sufficient stability conditions of asymptotical stability of 2-D linear systems were presented for any diagonal initial states. However, when we investigate the engineering applications of 2-D systems (e.g., the 2-D system based iterative learning control), it is inevitable to consider the impact of the orthogonal initial states on the asymptotic stability of 2-D systems, because generally, most of the engineering applications have orthogonal initial states. Thus, it is very meaningful to study the asymptotic stability of 2-D system with orthogonal initial states.

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