一类含有时变时滞的不确定中立型 Hopfield 神经网络的鲁棒稳定性判据

刘国权 [1] 周书民 [1]

摘 要 针对一类不确定中立型时变时滞 Hopfield 神经网络的鲁棒稳定性问题, 构造了一个新 Lyapunov-Krasovskii 泛函, 并结合自由矩阵方法和牛顿–莱布尼茨公式, 得到了新的时滞相关稳定性判据. 该判据考虑了中立型时变时滞 Hopfield 神经网络中的参数不确定性, 所得结果以线性矩阵不等式 (Linear matrix inequality, LMI) 的形式给出, 容易验证. 最后, 通过两个数值算例验证了该结果的有效性及可行性. 该判据对丰富与完善中立型神经网络的稳定性理论体系, 具有积极的意义.

关键词 鲁棒稳定性, 中立型 Hopfield 神经网络, 线性矩阵不等式, 时变时滞, Lyapunov-Krasovskii 泛函

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Robust Stability for a Class of Uncertain Hopfield Neural Networks of Neutral-type with Time-varying Delays

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Abstract This paper investigates the problem of robust stability analysis for a class of uncertain Hopfield neutral neural networks with time-varying delays. By constructing a new Lyapunov-Krasovskii functional, together with the Newton-Leibniz formula and some free-weighting matrix, new delay-dependent stability criteria are obtained. The proposed results are given in terms of linear matrix inequalities (LMI), which can be easily verified. Finally, two examples show the practicability and validity of the novel criteria. The proposed criteria have a vital significance for enriching and improving the stability theory of neural networks.

Key words Robust stability, Hopfield neural networks of neutral-type, linear matrix inequality (LMI), time-varying delays, Lyapunov-Krasovskii functional

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Hopfield 神经网络是一种固定权值的神经网络, 具有并行处理、联想记忆和可训练性等特点, 该模型的动力学分析受到人们的重视和关注[1]. 在众多的动力学分析中, 尤其在神经网络的构造和分析中, 稳定性对网络的性能有着重要的影响. 关于神经网络的稳定性分析便成了一个热点研究方向. 最近, 学者们在 Hopfield 神经网络的渐近 (指数) 稳定性问题上, 取得了一系列的成果[2−8]. 另外, 在 Hopfield 神经网络系统的实现过程中, 基于放大器切换速度的影响, 常常引入时滞, 而时滞的存在可能引起振荡、分岔以及系统失稳[3]. 因此, 对含有时滞的 Hopfield 神经网络稳定性的探讨, 有着实际工程背景和研究价值.

目前来讲, 在许多工程设计分析阶段中常采用中立型的时滞微分方程 (泛函微分方程) 来建立模型, 易于分析, 具有灵活性和经济性. 可见, 开展对中立型时滞系统的分析十分重要. 近来, 一些学者研究了一类含中立型时滞的神经网络的各种稳定性问题, 尤其在获得时滞相关的稳定性判据上做了很多工作[9−13]. 文献 [12] 讨论了一类中立型时滞神经网络指数稳定性问题, 取得了时滞相关的全局指数稳定性判定条件. 文献 [13] 进一步分析了这类模型的状态估计问题, 基于 Lyapunov 方法和矩阵不等式分析技术, 得到了新的稳定性判定条件. 另一方面, 对一个预先设计好的系统, 由于外部摄动、参数变化和模型本身的误差等常出现的不确定因素, 其稳定性不可避免地会遭到破坏. 可以预见, 我们在设计真

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实系统时, 须考虑系统的鲁棒稳定性. 近几年来, 参数不确定中立型神经网络的稳定性分析也取得了一些成果[14−18]. 例如, 在文献 [14, 17] 中, 在神经元激活函数不要求有界的条件下, 作者分别讨论了含有参数不确定性的中立型变时滞神经网络的鲁棒时滞稳定性以及指数稳定性, 给出了一些以线性矩阵不等式 (Linear matrix inequality, LMI) 形式的稳定性判据; 在文献 [18] 中, Mahmoud 等分析了不确定中立型 Hopfield 神经网络鲁棒指数稳定性问题, 基于 Lyapunov-Krasovskii 泛函和 LMI 方法, 得到了时滞相关全局指数稳定性条件. 目前来讲, 关于含有变时滞和不确定扰动的中立型 Hopfield 神经网络的鲁棒稳定性的结论较少, 还需进一步研究和探讨.

基于此, 本文考虑了不确定中立型 Hopfield 时滞神经网络的鲁棒稳定性问题, 其中, 不确定项为范数有界且时滞是可变的. 通过构造一个新的 Lyapunov-Krasovskii 泛函, 并结合牛顿–莱布尼茨公式, 获到了基于线性矩阵不等式表示的稳定性判据. 此外, 通过两个仿真算例阐述了所得充分判据是有效且可行的.

全文沿用如下记号: $\mathbf{R}^n$ 和 $\mathbf{R}^{n\times n}$ 分别表示 n 维欧几里德空间及空间内所有 $n\times n$ 维实数矩阵的集合; X^{-1}, X^{T} 分别表示一个方阵的逆矩阵及转置; $X>0$ 表示正定, $X<0$ 负定矩阵; $X\le Y$ 意味着 $X-Y\le 0$ 是半负定; I_n 表示 $n\times n$ 的单位矩阵; $\mathrm{diag}\{M_1, M_2, \cdots, M_n\}$ 表示由对角线上矩阵 $M_1, M_2, \cdots, M_n$ 组成的分块对角矩阵; $*$ 表示对称矩阵的对称部分.

1 系统的描述与准备

考虑如下含有时变时滞的不确定中立型 Hopfield 神经网络模型:

$$\begin{cases} \dot{\boldsymbol{y}}(t) = -\left(A+\Delta A(t)\right)\boldsymbol{y}(t) + \left(B+\Delta B(t)\right)\times \\ \quad \boldsymbol{g}(\boldsymbol{y}(t)) + \left(C+\Delta C(t)\right)\boldsymbol{g}(\boldsymbol{y}(t-\tau(t))) + \\ \quad \left(D+\Delta D(t)\right)\dot{\boldsymbol{y}}(t-h(t)) + \boldsymbol{I} \\ \boldsymbol{y}(t) = \boldsymbol{\phi}(t), \quad t\in[-\rho, 0], \quad \rho\in\max\{\tau_2, h\} \end{cases} \tag{1}$$

式中, $\boldsymbol{y}(t) = [y_1(t), y_2(t), \cdots, y_n(t)]^{\mathrm{T}} \in \mathbf{R}^n$ 是神经元状态向量, $\boldsymbol{g}(\boldsymbol{y}(t)) = [g_1(y_1(t)), g_2(y_2(t)), \cdots, g_n(y_n(t))]^{\mathrm{T}} \in \mathbf{R}^n$ 是神经元激活函数向量, $A = \mathrm{diag}\{a_1, a_2, \cdots, a_n\} > 0, B\in\mathbf{R}^{n\times n}$, $C\in\mathbf{R}^{n\times n}$ 和 $D\in\mathbf{R}^{n\times n}$ 是连接权矩阵, $\Delta A(t), \Delta B(t), \Delta C(t)$ 和 $\Delta D(t)$ 表示不确定参数; $\boldsymbol{\phi}(t)$ 为连续可微的初始函数; $\boldsymbol{I} = [I_1, I_2, \cdots, I_n]^{\mathrm{T}} \in \mathbf{R}^n$ 为外部输入向量; $\tau(t)$ 和 $h(t)$ 分别表示离散和中立型时滞.

为了得到本文主要结果, 一般作如下假设:

假设 1[15]**.** 神经元激活函数 $g_i(\cdot)$ $(i = 1, 2, \cdots, n)$ 在实数域内有界, 并且存在常数 l_i^-, l_i^+, 使得:

$$l_i^- \le \frac{g_i(\xi_1) - g_i(\xi_2)}{\xi_1 - \xi_2} \le l_i^+, \quad \forall \xi_1, \xi_2 \in \mathbf{R},\ \xi_1 \ne \xi_2 \tag{2}$$

注 1. 在本文中, l_i^-, l_i^+ 可取正数、零和负数, 这意味着我们的结果将具有更少的保守性和限制性.

假设 2. 时滞 $\tau(t)$ 和 $h(t)$ 分别满足如下:

$$\begin{aligned} &\tau_1 \le \tau(t) \le \tau_2,\ \dot{\tau}(t) \le \tau_d, \\ &0 < h(t) \le h, \quad \dot{h}(t) \le h_d < 1 \end{aligned} \tag{3}$$

其中, $\tau_1, \tau_2, \tau_d, h$, 和 h_d 为正常量.

注 2. 显然, 当 $\tau_d = 0$ 即 $\tau_1 \equiv \tau_2$ 时, 这意味着 $\tau(t)$ 为常时滞, 这种情况在文献 [12−13] 中已经被研究.

假设 3. 不确定参数矩阵 $\Delta A(t), \Delta B(t), \Delta C(t)$ 和 $\Delta D(t)$ 满足:

$$\begin{aligned} \Delta A(t) = H_1F_1(t)T_1, \Delta B(t) = H_2F_2(t)T_2 \\ \Delta C(t) = H_3F_3(t)T_3, \Delta D(t) = H_4F_4(t)T_4 \end{aligned} \tag{4}$$

其中, T_1, T_2, T_3, T_4 和 H_1, H_2, H_3, H_4 为已知具有适当维数的常矩阵. 不确定时变矩阵 $F_i(t)$ $(i = 1, 2, 3, 4)$ 满足:

$$F_i^{\mathrm{T}}(t)F_i(t) \le I, \ \ \forall t\in\mathbf{R} \tag{5}$$

假设 $\boldsymbol{y}^* = [y_1^*, y_2^*, \cdots, y_n^*]^{\mathrm{T}}$ 是系统 (1) 的一个平衡点. 通常, 由坐标转换 $x_i(t) = y_i(t) - \boldsymbol{y}^*$ 将平衡点转移到原点, 这时系统 (1) 可变成如下形式:

$$\begin{cases} \dot{\boldsymbol{x}}(t) = -\left(A+\Delta A(t)\right)\boldsymbol{x}(t) + \left(B+\Delta B(t)\right)\times \\ \quad \boldsymbol{f}(x(t)) + \left(C+\Delta C(t)\right)\boldsymbol{f}(x(t-\tau(t))) + \\ \quad \left(D+\Delta D(t)\right)\dot{\boldsymbol{x}}(t-h(t)) \\ \boldsymbol{x}(t) = \boldsymbol{\varphi}(t),\ t\in[-\rho, 0], \rho\in\max\{\tau_2, h\} \end{cases} \tag{6}$$

式中, $\boldsymbol{x}(t) = [x_1(t), x_2(t), \cdots, x_n(t)]^{\mathrm{T}} \in \mathbf{R}^n$ 是神经元状态向量 $\boldsymbol{f}(\boldsymbol{x}(t)) = [f_1(x_1(t)),\ f_2(x_2(t)), \cdots, f_n(x_n(t))]^{\mathrm{T}} \in \mathbf{R}^n$ 是神经元激活函数向量, 其中, $f_i(x_i(t)) = f_i(x_i(t) + \boldsymbol{y}^*) - f_i(\boldsymbol{y}^*)$, 且 $f_i(0) = 0, \boldsymbol{\varphi}(t) = \boldsymbol{\phi}(t) - \boldsymbol{y}^*$. 由文中假设 1, 可得激活函数 $f_i(\cdot)$ $(i = 1, 2, \cdots, n)$ 满足:

$$l_i^- \le \frac{f_i(\xi)}{\xi} \le l_i^+,\ \forall\xi\in\mathbf{R},\ i = 1, 2, \cdots n, f_i(0) = 0 \tag{7}$$

其中, l_i^-, l_i^+ 可取正数、零和负数.

下面, 给出在本文推导稳定性判定准则的过程中, 将使用的几个引理.

引理 1 (Schur 补充条件). 对给定的常对称阵 $\sum_1, \sum_2$ 和 $\sum_3$, 若 $\sum_1 = \sum_1^{\mathrm{T}}$ 且 $0 < \sum_2 = \sum_2^{\mathrm{T}}$,

那么 $\sum_1 + \sum_3^{\rm T} \sum_2^{-1} \sum_3 < 0$, 当且仅当:

$$\begin{bmatrix} \sum_1 & \sum_3^{\rm T} \\ \sum_3 & -\sum_2 \end{bmatrix} < 0 \begin{bmatrix} -\sum_2 & \sum_3 \\ \sum_3^{\rm T} & \sum_1 \end{bmatrix} < 0 \tag{8}$$

引理 2. 对任意常对称矩阵 M, E 和 $F(t)$, 当 $F^{\rm T}(t)F(t) \le I$ 且 υ 为正常数, 则以下不等式成立:

$$MF(t)E + E^{\rm T}F^{\rm T}(t)M^{\rm T} \le \upsilon^{-1}MM^{\rm T} + \upsilon E^{\rm T}E \tag{9}$$

引理 3. 对任意常对称矩阵 $M = M^{\rm T} > 0$, 任意常数 a 和 b 满足 $a < b$, 向量函数 $x(t) : [a, b] \to \mathbf{R}^n$, 则:

$$\left[\int_a^b \boldsymbol{x}(s){\rm d}s\right]^{\rm T} M \left[\int_a^b \boldsymbol{x}(s){\rm d}s\right] \le (b-a)\int_a^b \boldsymbol{x}^{\rm T}(s)M\boldsymbol{x}(s){\rm d}s \tag{10}$$

2 稳定性分析

定理 1. 对于给定的常量 τ_1, τ_2, τ_d 和 h_d 满足式 (3), 系统 (6) 在均方意义下是全局渐近鲁棒稳定的, 如果存在正定矩阵 $P, Q_i, i = 1, 2, R_i, i = 1, 2, 3, W_i, i = 1, 2, 3$, 任意矩阵 $M_i, N_i, U_i, V_i, i = 1, 2, \cdots, 9$, 正定对角矩阵 K, Z_1, Z_2, 以及 4 个正常量 $\varepsilon_i, i = 1, 2, 3, 4$, 使得如下的 LMI 成立:

$$\Psi = \begin{bmatrix} \Psi_{11} & \Psi_{12} & \Psi_{13} & \Psi_{14} & \Psi_{15} & \Psi_{16} & \Psi_{17} & \Psi_{18} \\ * & \Psi_{22} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & \Psi_{33} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \Psi_{44} & 0 & 0 & 0 & 0 \\ * & * & * & * & -\varepsilon_1 I & 0 & 0 & 0 \\ * & * & * & * & * & -\varepsilon_2 I & 0 & 0 \\ * & * & * & * & * & * & -\varepsilon_3 I & 0 \\ * & * & * & * & * & * & * & -\varepsilon_4 I \end{bmatrix} < 0 \tag{11}$$

式中

$$\Psi_{11} = \begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} & \Xi_{15} & \Xi_{16} & \Xi_{17} & \Xi_{18} & \Xi_{19} \\ * & \Xi_{22} & \Xi_{23} & \Xi_{24} & \Xi_{25} & \Xi_{26} & \Xi_{27} & \Xi_{28} & \Xi_{29} \\ * & * & \Xi_{33} & \Xi_{34} & \Xi_{35} & \Xi_{36} & \Xi_{37} & \Xi_{38} & \Xi_{39} \\ * & * & * & \Xi_{44} & \Xi_{45} & \Xi_{46} & \Xi_{47} & \Xi_{48} & \Xi_{49} \\ * & * & * & * & \Xi_{55} & \Xi_{56} & \Xi_{57} & \Xi_{58} & \Xi_{59} \\ * & * & * & * & * & \Xi_{66} & \Xi_{67} & \Xi_{68} & \Xi_{69} \\ * & * & * & * & * & * & \Xi_{77} & \Xi_{78} & \Xi_{79} \\ * & * & * & * & * & * & * & \Xi_{88} & \Xi_{89} \\ * & * & * & * & * & * & * & * & \Xi_{99} \end{bmatrix}$$

$$\Psi_{12} = [-M_i]_{9\times1}, \Psi_{13} = [-N_i]_{9\times1}, \Psi_{14} = [-U_i]_{9\times1}$$
$$\Psi_{15} = H_1[V_i]_{9\times1}, \Psi_{16} = H_2[V_i]_{9\times1}$$
$$\Psi_{17} = H_3[V_i]_{9\times1}, \Psi_{18} = H_4[V_i]_{9\times1}$$
$$\Psi_{22} = -W_1, \Psi_{33} = -W_2$$
$$\Psi_{44} = -W_1 - \tau_2(\tau_2-\tau_1)^{-1}W_2$$

且

$$\Xi_{11} = Q_1 + R_1 + R_2 + (\tau_2-\tau_1)^2R_3 - V_1A - A^{\rm T}V_1^{\rm T} + M_1 + M_1^{\rm T} - L_1Z_1 - Z_1^{\rm T}L_1^{\rm T} + \varepsilon_1T_1^{\rm T}T_1$$
$$\Xi_{12} = -A^{\rm T}V_2^{\rm T} + M_2^{\rm T} - M_1 - N_1 + U_1$$
$$\Xi_{13} = -A^{\rm T}V_3^{\rm T} + V_1B + M_3^{\rm T} + L_2Z_2$$
$$\Xi_{14} = -A^{\rm T}V_4^{\rm T} + M_4^{\rm T} + N_1$$
$$\Xi_{15} = -A^{\rm T}V_5^{\rm T} + M_5^{\rm T} - U_1$$
$$\Xi_{16} = P - A^{\rm T}V_6^{\rm T} - V_1 + M_6^{\rm T}$$
$$\Xi_{17} = -A^{\rm T}V_7^{\rm T} + V_1D + M_7^{\rm T}$$
$$\Xi_{18} = -A^{\rm T}V_8^{\rm T} + V_1C + M_8^{\rm T}$$
$$\Xi_{19} = -A^{\rm T}V_9^{\rm T} + M_9^{\rm T}$$
$$\Xi_{22} = -(1-\tau_d)Q_1 - M_2 - M_2^{\rm T} - N_2 - N_2^{\rm T} + U_2 + U_2^{\rm T} - L_1Z_2 - Z_2^{\rm T}L_1^{\rm T}$$
$$\Xi_{23} = V_2B - M_3^{\rm T} - N_3^{\rm T} + U_3^{\rm T}$$
$$\Xi_{24} = -M_4^{\rm T} + N_2 + U_4^{\rm T} - N_4^{\rm T}$$
$$\Xi_{25} = -M_5^{\rm T} + U_5^{\rm T} - N_5^{\rm T} - U_2$$
$$\Xi_{26} = -V_2 - M_6^{\rm T} - N_6^{\rm T} + U_6^{\rm T}$$
$$\Xi_{27} = V_2D - M_7^{\rm T} - N_7^{\rm T} + U_7^{\rm T}$$
$$\Xi_{28} = V_2C - M_8^{\rm T} - N_8^{\rm T} + U_8^{\rm T} + L_1Z_2$$
$$\Xi_{29} = -M_9^{\rm T} - N_9^{\rm T} + U_9^{\rm T}$$
$$\Xi_{33} = Q_2 + V_3B + B^{\rm T}V_3^{\rm T} - Z_1 - Z_1^{\rm T} + \varepsilon_2T_2^{\rm T}T_2$$
$$\Xi_{34} = -M_4^{\rm T} + B^{\rm T}V_4^{\rm T} + N_3, \quad \Xi_{35} = B^{\rm T}V_5^{\rm T} - U_3$$
$$\Xi_{36} = B^{\rm T}V_6^{\rm T} - V_3 + K$$
$$\Xi_{37} = B^{\rm T}V_7^{\rm T} + V_3D, \Xi_{38} = B^{\rm T}V_8^{\rm T} + V_3C$$
$$\Xi_{39} = B^{\rm T}V_9^{\rm T}\Xi_{44} = -R_1 + N_4 + N_4^{\rm T}$$
$$\Xi_{45} = N_5^{\rm T} - U_4, \quad \Xi_{46} = -V_4 + N_6^{\rm T}$$
$$\Xi_{47} = V_4D + N_7^{\rm T}, \quad \Xi_{48} = V_4C + N_8^{\rm T}, \quad \Xi_{49} = N_9^{\rm T}$$
$$\Xi_{55} = -R_2 - U_5 - U_5^{\rm T}, \quad \Xi_{56} = -V_5 - U_6^{\rm T}$$
$$\Xi_{57} = V_5D - U_7^{\rm T}$$
$$\Xi_{58} = V_5C - U_8^{\rm T}, \quad \Xi_{59} = -U_9^{\rm T}$$
$$\Xi_{66} = \tau_2^2W_1 + (\tau_2-\tau_1)^2W_2 + h^2W_3 - V_6 - V_6^{\rm T}$$
$$\Xi_{67} = V_6D - V_7^{\rm T}, \quad \Xi_{68} = V_6C - V_8^{\rm T}, \quad \Xi_{69} = -V_9^{\rm T}$$
$$\Xi_{77} = -h(1-h_d)W_3 + V_7D + D^{\rm T}V_7^{\rm T} + \varepsilon_3T_3^{\rm T}T_3$$
$$\Xi_{78} = V_7C + D^{\rm T}V_8^{\rm T}, \quad \Xi_{79} = D^{\rm T}V_9^{\rm T}$$
$$\Xi_{88} = -(1-\tau_d)Q_2 + V_8C + C^{\rm T}V_8^{\rm T} - Z_2 - Z_2^{\rm T} + \varepsilon_4T_4^{\rm T}T_4$$

$$\Xi_{89}=C^{\rm T}V_9^{\rm T},\quad \Xi_{99}=-R_3$$

证明. 由引理 1, 如果 $\Psi<0$, 知下式成立.

$$\Psi_{11}=\begin{bmatrix}\Theta_{11}&\Xi_{12}&\Xi_{13}&\Xi_{14}&\Xi_{15}&\Xi_{16}&\Xi_{17}&\Xi_{18}&\Xi_{19}\\ *&\Xi_{22}&\Xi_{23}&\Xi_{24}&\Xi_{25}&\Xi_{26}&\Xi_{27}&\Xi_{28}&\Xi_{29}\\ *&*&\Theta_{33}&\Xi_{34}&\Xi_{35}&\Xi_{36}&\Xi_{37}&\Xi_{38}&\Xi_{39}\\ *&*&*&\Xi_{44}&\Xi_{45}&\Xi_{46}&\Xi_{47}&\Xi_{48}&\Xi_{49}\\ *&*&*&*&\Xi_{55}&\Xi_{56}&\Xi_{57}&\Xi_{58}&\Xi_{59}\\ *&*&*&*&*&\Xi_{66}&\Xi_{67}&\Xi_{68}&\Xi_{69}\\ *&*&*&*&*&*&\Theta_{77}&\Xi_{78}&\Xi_{79}\\ *&*&*&*&*&*&*&\Theta_{88}&\Xi_{89}\\ *&*&*&*&*&*&*&*&\Xi_{99}\end{bmatrix}+$$
$$\varepsilon_1^{-1}\vartheta_1\vartheta_1^{\rm T}+\varepsilon_1\eta_1^{\rm T}\eta_1+\varepsilon_2^{-1}\vartheta_2\vartheta_2^{\rm T}+\varepsilon_2\eta_2^{\rm T}\eta_2+$$
$$\varepsilon_3^{-1}\vartheta_3\vartheta_3^{\rm T}+\varepsilon_3\eta_3^{\rm T}\eta_3+\varepsilon_4^{-1}\vartheta_4\vartheta_4^{\rm T}+\varepsilon_4\eta_4^{\rm T}\eta_4<0 \tag{12}$$

式中

$$\vartheta_1=\Psi_{15},\ \vartheta_2=\Psi_{16},\ \vartheta_3=\Psi_{17},\vartheta_4=\Psi_{18}$$
$$\eta_1=\begin{bmatrix}-T_1&0&0&0&0&0&0&0&0\end{bmatrix}^{\rm T}$$
$$\eta_2=\begin{bmatrix}0&0&T_2&0&0&0&0&0&0\end{bmatrix}^{\rm T}$$
$$\eta_3=\begin{bmatrix}0&0&0&0&0&0&T_3&0&0\end{bmatrix}^{\rm T}$$
$$\eta_4=\begin{bmatrix}0&0&0&0&0&0&0&T_4&0\end{bmatrix}^{\rm T}$$

且

$$\Theta_{11}=Q_1+R_1+R_2+(\tau_2-\tau_1)^2R_3-V_1A-$$
$$A^{\rm T}V_1^{\rm T}+M_1+M_1^{\rm T}-L_1Z_1-Z_1^{\rm T}L_1^{\rm T}$$
$$\Theta_{33}=Q_2+V_3B+B^{\rm T}V_3^{\rm T}-Z_1-Z_1^{\rm T}$$
$$\Theta_{77}=-h\left(1-h_d\right)Q_5+V_7D+D^{\rm T}V_7^{\rm T}$$
$$\Theta_{88}=-(1-\tau_d)Q_2+V_8C+C^{\rm T}V_8^{\rm T}-Z_2-Z_2^{\rm T}$$

注意到式 (4) 和式 (5), 由引理 2 知, 下面的不等式成立.

$$\begin{bmatrix}\Delta\Xi_{11}&\Delta\Xi_{12}&\Delta\Xi_{13}&\Delta\Xi_{14}&\Delta\Xi_{15}&\Delta\Xi_{16}&\Delta\Xi_{17}&\Delta\Xi_{18}&\Delta\Xi_{19}\\ *&0&\Delta\Xi_{23}&0&0&0&\Delta\Xi_{27}&\Delta\Xi_{28}&0\\ *&*&\Delta\Xi_{33}&\Delta\Xi_{34}&\Delta\Xi_{35}&\Delta\Xi_{36}&\Delta\Xi_{37}&\Delta\Xi_{38}&\Delta\Xi_{39}\\ *&*&*&0&0&0&\Delta\Xi_{47}&\Delta\Xi_{48}&0\\ *&*&*&*&0&0&\Delta\Xi_{57}&\Delta\Xi_{58}&0\\ *&*&*&*&*&0&\Delta\Xi_{67}&\Delta\Xi_{68}&0\\ *&*&*&*&*&*&\Delta\Xi_{77}&\Delta\Xi_{78}&\Delta\Xi_{79}\\ *&*&*&*&*&*&*&\Delta\Xi_{88}&\Delta\Xi_{89}\\ *&*&*&*&*&*&*&*&0\end{bmatrix}=$$
$$\vartheta_1F_1^{\rm T}(t)\eta_1^{\rm T}+\eta_1F_1(t)\vartheta_1^{\rm T}+\vartheta_2F_2^{\rm T}(t)\eta_2^{\rm T}+$$
$$\eta_2F_2(t)\vartheta_2^{\rm T}+\vartheta_3F_3^{\rm T}(t)\eta_3^{\rm T}+\eta_3F_3(t)\vartheta_3^{\rm T}+$$
$$\vartheta_4F_4^{\rm T}(t)\eta_4^{\rm T}+\eta_4F_4(t)\vartheta_4^{\rm T}\le\varepsilon_1^{-1}\vartheta_1\vartheta_1^{\rm T}+$$
$$\varepsilon_1\eta_1^{\rm T}\eta_1+\varepsilon_2^{-1}\vartheta_2\vartheta_2^{\rm T}+\varepsilon_2\eta_2^{\rm T}\eta_2+\varepsilon_3^{-1}\vartheta_3\vartheta_3^{\rm T}+$$
$$\varepsilon_3\eta_3^{\rm T}\eta_3+\varepsilon_4^{-1}\vartheta_4\vartheta_4^{\rm T}+\varepsilon_4\eta_4^{\rm T}\eta_4<0$$

式中

$$\Delta\Xi_{11}=-V_1(\Delta A)-(\Delta A)^{\rm T}V_1^{\rm T}$$
$$\Delta\Xi_{12}=-(\Delta A)^{\rm T}V_2^{\rm T},\Delta\Xi_{13}=-(\Delta A)^{\rm T}V_3^{\rm T}$$
$$\Delta\Xi_{14}=-(\Delta A)^{\rm T}V_4^{\rm T},\Delta\Xi_{15}=-(\Delta A)^{\rm T}V_5^{\rm T}$$
$$\Delta\Xi_{16}=-(\Delta A)^{\rm T}V_6^{\rm T},\Delta\Xi_{17}=-(\Delta A)^{\rm T}V_7^{\rm T}$$
$$\Delta\Xi_{18}=-(\Delta A)^{\rm T}V_8^{\rm T},\Delta\Xi_{19}=-(\Delta A)^{\rm T}V_9^{\rm T}$$
$$\Delta\Xi_{23}=V_2(\Delta B),\Delta\Xi_{27}=V_2(\Delta D)$$
$$\Delta\Xi_{28}=V_2(\Delta C)$$
$$\Delta\Xi_{33}=V_3(\Delta B)+(\Delta B)^{\rm T}V_3^{\rm T}$$
$$\Delta\Xi_{34}=(\Delta B)^{\rm T}V_4^{\rm T},\Delta\Xi_{35}=(\Delta B)^{\rm T}V_5^{\rm T}$$
$$\Delta\Xi_{36}=(\Delta B)^{\rm T}V_6^{\rm T}$$
$$\Delta\Xi_{37}=(\Delta B)^{\rm T}V_7^{\rm T}+V_3(\Delta D)$$
$$\Delta\Xi_{38}=(\Delta B)^{\rm T}V_8^{\rm T}+V_3(\Delta C)$$
$$\Delta\Xi_{39}=(\Delta B)^{\rm T}V_9^{\rm T},\Delta\Xi_{47}=V_4(\Delta D)$$
$$\Delta\Xi_{48}=V_4(\Delta C),\Delta\Xi_{57}=V_5(\Delta D)$$
$$\Delta\Xi_{58}=V_5(\Delta C),\Delta\Xi_{67}=V_6(\Delta D)$$
$$\Delta\Xi_{68}=V_6(\Delta C)$$
$$\Delta\Xi_{77}=V_7(\Delta D)+(\Delta D)^{\rm T}V_7^{\rm T}$$
$$\Delta\Xi_{78}=V_7(\Delta C)+(\Delta D)^{\rm T}V_8^{\rm T}$$
$$\Delta\Xi_{79}=(\Delta D)^{\rm T}V_9^{\rm T}$$
$$\Delta\Xi_{88}=V_8(\Delta C)+(\Delta C)^{\rm T}V_8^{\rm T}$$
$$\Delta\Xi_{89}=(\Delta C)^{\rm T}V_9^{\rm T}$$

因此, 存在如下不等式成立.

$$\Pi=\begin{bmatrix}\tilde\Xi_{11}&\tilde\Xi_{12}&\tilde\Xi_{13}&\tilde\Xi_{14}&\tilde\Xi_{15}&\tilde\Xi_{16}&\tilde\Xi_{17}&\tilde\Xi_{18}&\tilde\Xi_{19}\\ *&\Xi_{22}&\Xi_{23}&\Xi_{24}&\Xi_{25}&\Xi_{26}&\tilde\Xi_{27}&\tilde\Xi_{28}&\Xi_{29}\\ *&*&\tilde\Xi_{33}&\tilde\Xi_{34}&\tilde\Xi_{35}&\tilde\Xi_{36}&\tilde\Xi_{37}&\tilde\Xi_{38}&\tilde\Xi_{39}\\ *&*&*&\Xi_{44}&\Xi_{45}&\Xi_{46}&\tilde\Xi_{47}&\tilde\Xi_{48}&\Xi_{49}\\ *&*&*&*&\Xi_{55}&\Xi_{56}&\tilde\Xi_{57}&\tilde\Xi_{58}&\Xi_{59}\\ *&*&*&*&*&\Xi_{66}&\tilde\Xi_{67}&\tilde\Xi_{68}&\Xi_{69}\\ *&*&*&*&*&*&\tilde\Xi_{77}&\tilde\Xi_{78}&\tilde\Xi_{79}\\ *&*&*&*&*&*&*&\tilde\Xi_{88}&\tilde\Xi_{89}\\ *&*&*&*&*&*&*&*&\Xi_{99}\end{bmatrix}<0 \tag{13}$$

其中

$$\tilde\Xi_{11}=Q_1+R_1+R_2+(\tau_2-\tau_1)^2R_3-V_1A(t)-$$
$$A(t)^{\rm T}V_1^{\rm T}+M_1+M_1^{\rm T}-L_1Z_1-Z_1^{\rm T}L_1^{\rm T}$$

$$\tilde{\Xi}_{12} = -A^{\mathrm{T}}(t)V_2^{\mathrm{T}} + M_2^{\mathrm{T}} - M_1 - N_1 + U_1$$
$$\tilde{\Xi}_{13} = -A^{\mathrm{T}}(t)V_3^{\mathrm{T}} + V_1B + M_3^{\mathrm{T}} + L_2Z_2$$
$$\tilde{\Xi}_{14} = -A^{\mathrm{T}}(t)V_4^{\mathrm{T}} + M_4^{\mathrm{T}} + N_1$$
$$\tilde{\Xi}_{15} = -A^{\mathrm{T}}(t)V_5^{\mathrm{T}} + M_5^{\mathrm{T}} - U_1$$
$$\tilde{\Xi}_{16} = P - A^{\mathrm{T}}(t)V_6^{\mathrm{T}} - V_1 + M_6^{\mathrm{T}}$$
$$\tilde{\Xi}_{17} = -A^{\mathrm{T}}(t)V_7^{\mathrm{T}} + V_1D(t) + M_7^{\mathrm{T}}$$
$$\tilde{\Xi}_{18} = -A^{\mathrm{T}}(t)V_8^{\mathrm{T}} + V_1C(t) + M_8^{\mathrm{T}}$$
$$\tilde{\Xi}_{19} = -A^{\mathrm{T}}(t)V_9^{\mathrm{T}} + M_9^{\mathrm{T}}$$
$$\tilde{\Xi}_{23} = V_2B(t) - M_3^{\mathrm{T}} - N_3^{\mathrm{T}} + U_3^{\mathrm{T}}$$
$$\tilde{\Xi}_{27} = V_2D(t) - M_7^{\mathrm{T}} - N_7^{\mathrm{T}} + U_7^{\mathrm{T}}$$
$$\tilde{\Xi}_{28} = V_2C(t) - M_8^{\mathrm{T}} - N_8^{\mathrm{T}} + U_8^{\mathrm{T}} + L_1Z_2$$
$$\tilde{\Xi}_{33} = Q_2 + V_3B(t) + B^{\mathrm{T}}(t)V_3^{\mathrm{T}} - Z_1 - Z_1^{\mathrm{T}}$$
$$\tilde{\Xi}_{34} = -M_4^{\mathrm{T}} + B^{\mathrm{T}}(t)V_4^{\mathrm{T}} + N_3$$
$$\tilde{\Xi}_{35} = B^{\mathrm{T}}(t)V_5^{\mathrm{T}} - U_3,\ \tilde{\Xi}_{36} = B^{\mathrm{T}}(t)V_6^{\mathrm{T}} - V_3 + K$$
$$\tilde{\Xi}_{37} = B^{\mathrm{T}}(t)V_7^{\mathrm{T}} + V_3D(t)$$
$$\tilde{\Xi}_{38} = B^{\mathrm{T}}(t)V_8^{\mathrm{T}} + V_3C(t),\ \tilde{\Xi}_{39} = B^{\mathrm{T}}(t)V_9^{\mathrm{T}}$$
$$\tilde{\Xi}_{47} = V_4D(t) + N_7^{\mathrm{T}}$$
$$\tilde{\Xi}_{48} = V_4C(t) + N_8^{\mathrm{T}},\ \tilde{\Xi}_{57} = V_5D(t) - U_7^{\mathrm{T}}$$
$$\tilde{\Xi}_{58} = V_5C(t) - U_8^{\mathrm{T}},\ \tilde{\Xi}_{67} = V_6D(t) - V_7^{\mathrm{T}}(t)$$
$$\tilde{\Xi}_{68} = V_6C(t) - V_8^{\mathrm{T}}$$
$$\Xi_{77} = -h\left(1 - h_d\right)W_3 + V_7D(t) + D^{\mathrm{T}}(t)V_7^{\mathrm{T}}$$
$$\tilde{\Xi}_{78} = V_7C(t) + D^{\mathrm{T}}(t)V_8^{\mathrm{T}},\ \tilde{\Xi}_{79} = D^{\mathrm{T}}(t)V_9^{\mathrm{T}}$$
$$\tilde{\Xi}_{88} = -(1-\tau_d)Q_2 + V_8C(t) + C^{\mathrm{T}}(t)V_8^{\mathrm{T}} - Z_2 - Z_2^{\mathrm{T}}$$
$$\tilde{\Xi}_{89} = C^{\mathrm{T}}(t)V_9^{\mathrm{T}}$$

构造如下的 Lyapunov-Krasovskii 泛函:

$$V(x_t) = V_1(x_t) + V_2(x_t) + V_3(x_t) + V_4(x_t) \quad (14)$$

式中

$$V_1(x_t) = \boldsymbol{x}^{\mathrm{T}}(t)P\boldsymbol{x}(t) + 2\sum_{i=1}^{n} k_i \int_0^{x_i} f_i(s)\mathrm{d}s$$
$$V_2(x_t) = \int_{t-\tau(t)}^{T} \boldsymbol{x}^{\mathrm{T}}(s)\,Q_1\boldsymbol{x}(s)\mathrm{d}s + \int_{t-\tau(t)}^{T} \boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}(s))\,Q_2\boldsymbol{f}(\boldsymbol{x}(s))\mathrm{d}s$$
$$V_3(x_t) = \int_{t-\tau_1}^{T} \boldsymbol{x}^{\mathrm{T}}(s)R_1\boldsymbol{x}(s)\mathrm{d}s + \int_{t-\tau_2}^{T} \boldsymbol{x}^{\mathrm{T}}(s)R_2\boldsymbol{x}(s)\mathrm{d}s + (\tau_2-\tau_1)\int_{t-\tau_2}^{t-\tau_1}\int_s^T \boldsymbol{x}^{\mathrm{T}}(s)R_3\boldsymbol{x}(s)\mathrm{d}\theta\mathrm{d}s$$
$$V_4(x_t) = \tau_2\int_{t-\tau_2}^{T}\int_s^T \dot{\boldsymbol{x}}^{\mathrm{T}}(\theta)W_1\dot{\boldsymbol{x}}(\theta)\mathrm{d}\theta\mathrm{d}s + (\tau_2-\tau_1)\int_{t-\tau_2}^{t-\tau_1}\int_s^T \dot{\boldsymbol{x}}^{\mathrm{T}}(\theta)W_2\dot{\boldsymbol{x}}(\theta)\mathrm{d}\theta\mathrm{d}s + h\int_{t-h(t)}^{T} \dot{\boldsymbol{x}}^{\mathrm{T}}(s)W_3\dot{\boldsymbol{x}}(s)\mathrm{d}s$$

其中

$$P = P^{\mathrm{T}} > 0, Q_i = Q_i^{\mathrm{T}} > 0,\ i = 1,2,\ R_i = R_i^{\mathrm{T}} > 0,\ i = 1,2,3,\ W_i = W_i^{\mathrm{T}} > 0_i, i = 1,2,3$$
$$K = \mathrm{diag}\{k_1, k_2, \cdots, k_n\} > 0$$

对 $V(x_t)$ 的各项沿系统 (6) 的轨迹求导有

$$\dot{V}(x_t) = \dot{V}_1(x_t) + \dot{V}_2(x_t) + \dot{V}_3(x_t) + \dot{V}_4(x_t) \quad (15)$$

式中

$$\dot{V}_1(\boldsymbol{x}_t) = 2\boldsymbol{x}^{\mathrm{T}}(t)P\dot{\boldsymbol{x}}(t) + 2\boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}(t))K\dot{\boldsymbol{x}}(t) \quad (16)$$

$$\dot{V}_2(\boldsymbol{x}_t) = \boldsymbol{x}^{\mathrm{T}}(t)Q_1\boldsymbol{x}(t) - (1-\dot{\tau}(t))\boldsymbol{x}^{\mathrm{T}}(t-\tau(t)) \times Q_1\boldsymbol{x}(t-\tau(t)) + \boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}(t))Q_2\boldsymbol{f}(x(t)) - (1-\dot{\tau}(t)) \times \boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}(t-\tau(t)))Q_2\boldsymbol{f}(\boldsymbol{x}(t-\tau(t))) \le \boldsymbol{x}^{\mathrm{T}}(t)Q_1\boldsymbol{x}(t) - (1-\tau_d)\boldsymbol{x}^{\mathrm{T}}(t-\tau(t))Q_1\boldsymbol{x}(t-\tau(t)) + \boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}(t))Q_2\boldsymbol{f}(\boldsymbol{x}(t)) - (1-\tau_d) \times \boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}(t-\tau(t)))Q_2\boldsymbol{f}(\boldsymbol{x}(t-\tau(t))) \quad (17)$$

$$\dot{V}_3(x_t) = \boldsymbol{x}^{\mathrm{T}}(t)R_1\boldsymbol{x}(t) - \boldsymbol{x}^{\mathrm{T}}(t-\tau_1)R_1\boldsymbol{x}(t-\tau_1) + \boldsymbol{x}^{\mathrm{T}}(t)R_2\boldsymbol{x}(t) - \boldsymbol{x}^{\mathrm{T}}(t-\tau_2)R_2\boldsymbol{x}(t-\tau_2) + (\tau_2-\tau_1)^2\boldsymbol{x}^{\mathrm{T}}(t)R_3\boldsymbol{x}(t) - (\tau_2-\tau_1) \times \int_{t-\tau_2}^{t-\tau_1} \boldsymbol{x}^{\mathrm{T}}(s)R_3\boldsymbol{x}(s)\mathrm{d}s \quad (18)$$

$$\dot{V}_4(x_t) = \tau_2^2\dot{\boldsymbol{x}}^{\mathrm{T}}(t)W_1\dot{\boldsymbol{x}}(t) - \tau_2 \times \int_{t-\tau_2}^{T} \dot{\boldsymbol{x}}^{\mathrm{T}}(s)W_1\dot{\boldsymbol{x}}(s)\mathrm{d}s + \left(\tau_2-\tau_1\right)^2\dot{\boldsymbol{x}}^{\mathrm{T}}(t)W_2\dot{\boldsymbol{x}}(t) - \left(\tau_2-\tau_1\right) \times \int_{t-\tau_2}^{t-\tau_1} \dot{\boldsymbol{x}}^{\mathrm{T}}(s)W_2\dot{\boldsymbol{x}}(s)\mathrm{d}s + h(t)h\dot{\boldsymbol{x}}^{\mathrm{T}}(t)W_3\dot{\boldsymbol{x}}(t) - h(1-\dot{h}(t))\dot{\boldsymbol{x}}^{\mathrm{T}}(t-h(t)) \times W_3\dot{\boldsymbol{x}}(t-h(t)) \le$$
$$\tau_2^2\dot{\boldsymbol{x}}^{\mathrm{T}}(t)W_1\dot{\boldsymbol{x}}(t) - \tau_2\int_{t-\tau_2}^{T} \dot{\boldsymbol{x}}^{\mathrm{T}}(s)W_1\dot{\boldsymbol{x}}(s)\mathrm{d}s + \left(\tau_2-\tau_1\right)^2\dot{\boldsymbol{x}}^{\mathrm{T}}(t)W_2\dot{\boldsymbol{x}}(t) - \left(\tau_2-\tau_1\right) \times \int_{t-\tau_2}^{t-\tau_1} \dot{\boldsymbol{x}}^{\mathrm{T}}(s)W_2\dot{\boldsymbol{x}}(s)\mathrm{d}s + h^2\dot{\boldsymbol{x}}^{\mathrm{T}}(t)W_3\dot{\boldsymbol{x}}(t) - h(1-h_d)\dot{\boldsymbol{x}}^{\mathrm{T}}(t-h(t)) \times$$

$$W_3\dot{\boldsymbol{x}}(t-h(t)) \tag{19}$$

由引理 3 知, 以下不等式成立:

$$-(\tau_2-\tau_1)\int_{t-\tau_2}^{t-\tau_1}\boldsymbol{x}^{\rm T}(s)R_3\boldsymbol{x}(s){\rm d}s \leq -\left[\int_{t-\tau_2}^{t-\tau_1}\boldsymbol{x}(s){\rm d}s\right]^{\rm T}R_3\left[\int_{t-\tau_2}^{t-\tau_1}\boldsymbol{x}(s){\rm d}s\right] \tag{20}$$

$$\begin{aligned}&-\tau_2\int_{t-\tau_2}^{T}\dot{\boldsymbol{x}}^{\rm T}(s)W_1\dot{\boldsymbol{x}}(s){\rm d}s= -\tau_2\left[\int_{t-\tau(t)}^{T}\dot{\boldsymbol{x}}^{\rm T}(s)W_1\dot{\boldsymbol{x}}(s){\rm d}s+\right.\\ &\left.\int_{t-\tau_2}^{t-\tau(t)}\dot{\boldsymbol{x}}^{\rm T}(s)W_1\dot{\boldsymbol{x}}(s){\rm d}s\right]\leq\\ &-\left[\int_{t-\tau(t)}^{T}\dot{\boldsymbol{x}}(s){\rm d}s\right]^{\rm T}W_1\left[\int_{t-\tau(t)}^{T}\dot{\boldsymbol{x}}(s){\rm d}s\right]-\\ &\tau_2\left(\tau_2-\tau_1\right)^{-1}\left[\int_{t-\tau_2}^{t-\tau(t)}\dot{\boldsymbol{x}}(s){\rm d}s\right]^{\rm T}\times\\ &W_1\left[\int_{t-\tau_2}^{t-\tau(t)}\dot{\boldsymbol{x}}(s){\rm d}s\right]\end{aligned} \tag{21}$$

$$\begin{aligned}&-(\tau_2-\tau_1)\int_{t-\tau_2}^{t-\tau_1}\dot{\boldsymbol{x}}^{\rm T}(s)W_2\dot{\boldsymbol{x}}(s){\rm d}s=\\ &-(\tau_2-\tau_1)\left[\int_{t-\tau(t)}^{t-\tau_1}\dot{\boldsymbol{x}}^{\rm T}(s)W_2\dot{\boldsymbol{x}}(s){\rm d}s+\right.\\ &\left.\int_{t-\tau_2}^{t-\tau(t)}\dot{\boldsymbol{x}}^{\rm T}(s)W_2\dot{\boldsymbol{x}}(s){\rm d}s\right]\leq\\ &-\left[\int_{t-\tau(t)}^{t-\tau_1}\dot{\boldsymbol{x}}(s){\rm d}s\right]^{\rm T}W_2\left[\int_{t-\tau(t)}^{t-\tau_1}\dot{\boldsymbol{x}}(s){\rm d}s\right]-\\ &\left[\int_{t-\tau_2}^{t-\tau(t)}\dot{\boldsymbol{x}}(s){\rm d}s\right]^{\rm T}W_2\left[\int_{t-\tau_2}^{t-\tau(t)}\dot{\boldsymbol{x}}(s){\rm d}s\right]\end{aligned} \tag{22}$$

由牛顿–莱布尼茨公式和式 (6), 对任意适当维数矩阵 M, N, U, V, 则有以下等式成立.

$$2\zeta^{\rm T}(t)M\left[\boldsymbol{x}(t)-\boldsymbol{x}(t-\tau(t))-\int_{t-\tau(t)}^{T}\dot{\boldsymbol{x}}(s){\rm d}s\right]=0 \tag{23}$$

$$2\zeta^{\rm T}(t)N\left[\boldsymbol{x}(t-\tau_1)-\boldsymbol{x}(t-\tau(t))-\int_{t-\tau(t)}^{t-\tau_1}\dot{\boldsymbol{x}}(s){\rm d}s\right]=0 \tag{24}$$

$$2\zeta^{\rm T}(t)U\left[\boldsymbol{x}(t-\tau(t))-\boldsymbol{x}(t-\tau_2)-\int_{t-\tau_2}^{t-\tau(t)}\dot{\boldsymbol{x}}(s){\rm d}s\right]=0 \tag{25}$$

$$\begin{aligned}&2\zeta^{\rm T}V\left[-A(t)\boldsymbol{x}(t)+B(t)\boldsymbol{f}(\boldsymbol{x}(t))+\right.\\ &C(t)\boldsymbol{f}(\boldsymbol{x}(t-\tau(t)))+\\ &\left.D(t)\dot{\boldsymbol{x}}(t-h(t))-\dot{\boldsymbol{x}}(t)\right]=0\end{aligned} \tag{26}$$

式中

$$\begin{aligned}\boldsymbol{\zeta}^{\rm T}(t)=\Big[&\boldsymbol{x}^{\rm T}(t)\ \ \boldsymbol{x}^{\rm T}(t-\tau(t))\ \ \boldsymbol{f}^{\rm T}(\boldsymbol{x}(t))\ \ \boldsymbol{x}^{\rm T}(t-\tau_1)\\ &\boldsymbol{x}^{\rm T}(t-\tau_2)\ \ \dot{\boldsymbol{x}}^{\rm T}(t)\ \ \dot{\boldsymbol{x}}^{\rm T}(t-h(t))\ \ \boldsymbol{f}^{\rm T}(\boldsymbol{x}(t-\tau(t)))\\ &\left(\int_{t-\tau_2}^{t-\tau_1}\boldsymbol{x}(s){\rm d}s\right)^{\rm T}\Big]\end{aligned}$$

$M^{\rm T}=[M_i]_{1\times 9}\,, N^{\rm T}=[N_i]_{1\times 9}\,,\ U^{\rm T}=[U_i]_{1\times 9}$
$V^{\rm T}=[V_i]_{1\times 9}\,, i=1,2,3,\cdots,9, A(t)=A+\Delta A(t)$
$B(t)=B+\Delta B(t), C(t)=C+\Delta C(t)$
$D(t)=D+\Delta D(t)$

由式 (7) 知, 对于正定对角矩阵 Z_1, Z_2, 下面的不等式成立.

$$\begin{aligned}&0\leq-2\sum_{i=1}^{n}z_{i1}\left[f_i(x_i(t))-l_i^-x_i(t)\right]\times\\ &\left[f_i(x_i(t))-l_i^+x_i(t)\right]=\\ &-2\boldsymbol{f}^{\rm T}(\boldsymbol{x}(t))Z_1\boldsymbol{f}(\boldsymbol{x}(t))+2\boldsymbol{x}^{\rm T}(t)L_2Z_1\boldsymbol{f}(\boldsymbol{x}(t))-\\ &2\boldsymbol{x}^{\rm T}(t)L_1Z_1\boldsymbol{x}(t)\end{aligned} \tag{27}$$

$$\begin{aligned}&0\leq-2\sum_{i=1}^{n}z_{i2}\left[f_i(x_i(t-\tau(t))-l_i^-x_i(t-\tau(t))\right]\times\\ &\left[f_i(x_i(t-\tau(t)))-l_i^+x_i(t-\tau(t))\right]=\\ &-2\boldsymbol{f}^{\rm T}(\boldsymbol{x}(t-\tau(t)))Z_2\boldsymbol{f}(\boldsymbol{x}(t-\tau(t)))+\\ &2\boldsymbol{x}^{\rm T}(t-\tau(t))L_2Z_2\times\\ &\boldsymbol{f}(\boldsymbol{x}(t-\tau(t)))-2\boldsymbol{x}^{\rm T}(t-\tau(t))L_1Z_2\boldsymbol{x}(t-\tau(t))\end{aligned} \tag{28}$$

式中, $L_1={\rm diag}\{l_1^-l_1^+, l_2^-l_2^+,\cdots,l_n^-l_n^+\}, L_2={\rm diag}\{l_1^-+l_1^+, l_2^-+l_2^+,\cdots,l_n^-+l_n^+\}$.

将式 (16) ∼ (28) 带入式 (15), 经过一系列计算得:

$$\dot{V}(x_t)\leq\boldsymbol{\xi}^{\rm T}(t)\Pi\boldsymbol{\xi}(t) \tag{29}$$

式中, $\boldsymbol{\xi}^{\rm T}(t)=\left[\boldsymbol{\zeta}^{\rm T}(t),\ \left(\int_{t-\tau(t)}^{T}\dot{\boldsymbol{x}}(s){\rm d}s\right)^{\rm T},\ \left(\int_{t-\tau(t)}^{t-\tau_1}\dot{\boldsymbol{x}}(s){\rm d}s\right)^{\rm T},\ \left(\int_{t-\tau_2}^{t-\tau(t)}\dot{\boldsymbol{x}}(s){\rm d}s\right)^{\rm T}\right]$.

若 $\Pi < 0$, 则 $\dot{V}(x_t) \leq \boldsymbol{\xi}^{\mathrm{T}}(t)\Pi\boldsymbol{\xi}(t) < 0$ 对于任意的 $\boldsymbol{\xi}(t) \neq 0$, 也就意味着系统 (6) 是全局鲁棒稳定. □

注 3. 当 $\tau_1 = 0$, 这种情况经常出现在很多学者的结论中, 依据定理 1, 得到下面的推论.

推论 1. 对于给定的常量 τ_2, τ_d, h 和 h_d, 系统 (6) 在均方意义下是全局渐近鲁棒稳定的, 如果存在正定矩阵 $P, Q_i, i=1,2, R_i, i=2,3, W_i, i=1,3$, 任意矩阵 M_i, $N_i, U_i, V_i, i=1,2,\cdots,8$, 正定对角矩阵 K, Z_1, Z_2, 以及 4 个正常量 $\varepsilon_i, i=1,2,3,4$, 使得如下的 LMI 成立:

$$\bar{\Psi} = \begin{bmatrix} \bar{\Psi}_{11} & \bar{\Psi}_{12} & \bar{\Psi}_{14} & \bar{\Psi}_{15} & \bar{\Psi}_{16} & \bar{\Psi}_{17} & \bar{\Psi}_{18} \\ * & -W_2 & 0 & 0 & 0 & 0 & 0 \\ * & * & -W_1 & 0 & 0 & 0 & 0 \\ * & * & * & -\varepsilon_1 I & 0 & 0 & 0 \\ * & * & * & * & -\varepsilon_2 I & 0 & 0 \\ * & * & * & * & * & -\varepsilon_3 I & 0 \\ * & * & * & * & * & * & -\varepsilon_4 I \end{bmatrix} < 0 \tag{30}$$

式中

$$\bar{\Psi}_{11} = \begin{bmatrix} \bar{\Xi}_{11} & \bar{\Xi}_{12} & \bar{\Xi}_{13} & \bar{\Xi}_{14} & \bar{\Xi}_{15} & \bar{\Xi}_{16} & \bar{\Xi}_{17} & \bar{\Xi}_{18} \\ * & \bar{\Xi}_{22} & \bar{\Xi}_{23} & \bar{\Xi}_{24} & \bar{\Xi}_{25} & \bar{\Xi}_{26} & \bar{\Xi}_{27} & \bar{\Xi}_{28} \\ * & * & \bar{\Xi}_{33} & \bar{\Xi}_{34} & \bar{\Xi}_{35} & \bar{\Xi}_{36} & \bar{\Xi}_{37} & \bar{\Xi}_{38} \\ * & * & * & \bar{\Xi}_{44} & \bar{\Xi}_{45} & \bar{\Xi}_{46} & \bar{\Xi}_{47} & \bar{\Xi}_{48} \\ * & * & * & * & \bar{\Xi}_{55} & \bar{\Xi}_{56} & \bar{\Xi}_{57} & \bar{\Xi}_{58} \\ * & * & * & * & * & \bar{\Xi}_{66} & \bar{\Xi}_{67} & \bar{\Xi}_{68} \\ * & * & * & * & * & * & \bar{\Xi}_{77} & \bar{\Xi}_{78} \\ * & * & * & * & * & v & * & \bar{\Xi}_{88} \end{bmatrix}$$

$$\begin{aligned} &\bar{\Psi}_{12} = [-M_i]_{8\times1}, \bar{\Psi}_{14} = [-N_i]_{8\times1} \\ &\bar{\Psi}_{15} = H_1[V_i]_{8\times1}, \bar{\Psi}_{16} = H_2[V_i]_{8\times1} \\ &\bar{\Psi}_{17} = H_3[V_i]_{8\times1}, \bar{\Psi}_{18} = H_4[V_i]_{8\times1} \\ &\bar{\Psi}_{22} = -W_1, \bar{\Psi}_{33} = -W_1 \end{aligned}$$

且

$$\begin{aligned}
&\bar{\Xi}_{11} = Q_1 + R_2 + \tau_2^2 R_3 - V_1 A - A^{\mathrm{T}} V_1^{\mathrm{T}} + M_1 + \\
&\qquad M_1^{\mathrm{T}} - L_1 Z_1 - Z_1^{\mathrm{T}} L_1^{\mathrm{T}} + \varepsilon_1 T_1^{\mathrm{T}} T_1 \\
&\bar{\Xi}_{12} = -A^{\mathrm{T}} V_2^{\mathrm{T}} + M_2^{\mathrm{T}} - M_1 \\
&\bar{\Xi}_{13} = -A^{\mathrm{T}} V_3^{\mathrm{T}} + V_1 B + M_3^{\mathrm{T}} + L_2 Z_2 \\
&\bar{\Xi}_{14} = -A^{\mathrm{T}} V_4^{\mathrm{T}} + M_4^{\mathrm{T}} - U_1 \\
&\Xi_{15} = P - A^{\mathrm{T}} V_5^{\mathrm{T}} - V_1 + M_5^{\mathrm{T}} \\
&\bar{\Xi}_{16} = -A^{\mathrm{T}} V_6^{\mathrm{T}} + V_1 D + M_6^{\mathrm{T}} \\
&\bar{\Xi}_{17} = -A^{\mathrm{T}} V_7^{\mathrm{T}} + V_1 C + M_7^{\mathrm{T}} \\
&\bar{\Xi}_{18} = -A^{\mathrm{T}} V_8^{\mathrm{T}} + M_8^{\mathrm{T}} \\
&\bar{\Xi}_{22} = -(1-\tau_d) Q_1 - M_2 - M_2^{\mathrm{T}} + U_2 + U_2^{\mathrm{T}} - \\
&\qquad L_1 Z_2 - Z_2^{\mathrm{T}} L_1^{\mathrm{T}} \\
&\bar{\Xi}_{23} = V_2 B - M_3^{\mathrm{T}} + U_3^{\mathrm{T}},\ \bar{\Xi}_{24} = -M_4^{\mathrm{T}} + U_4^{\mathrm{T}} - U_2 \\
&\bar{\Xi}_{25} = -V_2 - M_5^{\mathrm{T}} + U_5^{\mathrm{T}},\ \bar{\Xi}_{26} = V_2 D - M_6^{\mathrm{T}} + U_6^{\mathrm{T}} \\
&\bar{\Xi}_{27} = V_2 C - M_7^{\mathrm{T}} + U_7^{\mathrm{T}} + L_1 Z_2 \\
&\bar{\Xi}_{28} = -M_8^{\mathrm{T}} + U_8^{\mathrm{T}} \\
&\bar{\Xi}_{33} = Q_2 + V_3 B + B^{\mathrm{T}} V_3^{\mathrm{T}} - Z_1 - Z_1^{\mathrm{T}} + \varepsilon_2 T_2^{\mathrm{T}} T_2 \\
&\bar{\Xi}_{34} = B^{\mathrm{T}} V_4^{\mathrm{T}} - U_3 \\
&\Xi_{35} = B^{\mathrm{T}} V_5^{\mathrm{T}} - V_3 + K,\ \bar{\Xi}_{36} = B^{\mathrm{T}} V_6^{\mathrm{T}} + V_3 D \\
&\bar{\Xi}_{37} = B^{\mathrm{T}} V_7^{\mathrm{T}} + V_3 C,\ \bar{\Xi}_{38} = B^{\mathrm{T}} V_8^{\mathrm{T}} \\
&\bar{\Xi}_{44} = -R_2 - U_4 - U_4^{\mathrm{T}},\ \bar{\Xi}_{45} = -V_4 - U_5^{\mathrm{T}} \\
&\bar{\Xi}_{46} = V_4 D - U_6^{\mathrm{T}},\ \bar{\Xi}_{47} = V_4 C - U_7^{\mathrm{T}},\ \bar{\Xi}_{48} = -U_8^{\mathrm{T}} \\
&\bar{\Xi}_{55} = \tau_2^2 W_1 + h^2 W_3 - V_5 - V_5^{\mathrm{T}} + \varepsilon_3 T_3^{\mathrm{T}} T_3 \\
&\bar{\Xi}_{56} = V_5 D - V_6^{\mathrm{T}},\ \bar{\Xi}_{57} = V_5 C - V_7^{\mathrm{T}},\ \bar{\Xi}_{58} = -V_8^{\mathrm{T}} \\
&\bar{\Xi}_{66} = -h(1-h_d) W_3 + V_6 D + D^{\mathrm{T}} V_6^{\mathrm{T}} \\
&\bar{\Xi}_{67} = V_6 C + D^{\mathrm{T}} V_7^{\mathrm{T}},\ \bar{\Xi}_{68} = D^{\mathrm{T}} V_8^{\mathrm{T}} \\
&\Xi_{77} = -(1-\tau_d) Q_2 + V_7 C + C^{\mathrm{T}} V_7^{\mathrm{T}} - Z_2 - Z_2^{\mathrm{T}} + \\
&\qquad \varepsilon_4 T_4^{\mathrm{T}} T_4 \\
&\bar{\Xi}_{78} = C^{\mathrm{T}} V_8^{\mathrm{T}},\ \bar{\Xi}_{88} = -R_3
\end{aligned}$$

证明. 此证明过程类似定理 1, 构造一个 Lyapunov-Krasovskii 泛函如下:

$$\begin{aligned}
V(x_t) = &\boldsymbol{x}^{\mathrm{T}}(t) P \boldsymbol{x}(t) + 2\sum_{i=1}^{n} k_i \int_0^{x_i} f_i(s)\mathrm{d}s + \\
&\int_{t-\tau(t)}^{T} \boldsymbol{x}^{\mathrm{T}}(s) Q_1 \boldsymbol{x}(s)\mathrm{d}s + \\
&\int_{t-\tau(t)}^{T} \boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}(s)) Q_2 \boldsymbol{f}(\boldsymbol{x}(s))\mathrm{d}s + \\
&\tau_2 \int_{t-\tau_2}^{T} \boldsymbol{x}^{\mathrm{T}}(s) R_2 \boldsymbol{x}(s)\mathrm{d}s + \\
&\tau_2 \int_{t-\tau_2}^{T} \int_s^{T} \boldsymbol{x}^{\mathrm{T}}(s) R_3 \boldsymbol{x}(s)\mathrm{d}\theta\mathrm{d}s + \\
&\tau_2 \int_{t-\tau_2}^{T} \int_s^{T} \dot{\boldsymbol{x}}^{\mathrm{T}}(\theta) W_1 \dot{\boldsymbol{x}}(\theta)\mathrm{d}\theta\mathrm{d}s + \\
&h \int_{t-h(t)}^{T} \dot{\boldsymbol{x}}^{\mathrm{T}}(s) W_3 \dot{\boldsymbol{x}}(s)\mathrm{d}s
\end{aligned}$$

余下证明过程类似证明定理 1, 由于篇幅所限, 在此省略. □

3 数值算例及仿真

例 1. 考虑如下一个含 4 个神经元的不确定中立型 Hopfield 神经网络:

$$\begin{aligned}
\dot{\boldsymbol{x}}(t) = &-(A + \Delta A(t))\boldsymbol{x}(t) + \\
&(B + \Delta B(t))\boldsymbol{f}(\boldsymbol{x}(t)) + \\
&(C + \Delta C(t))\boldsymbol{f}(\boldsymbol{x}(t-\tau(t))) + \\
&(D + \Delta D(t))\dot{\boldsymbol{x}}(t-h(t))
\end{aligned} \tag{31}$$

式中

$$A=\begin{bmatrix}1.6 & 0 & 0 & 0\\ 0 & 1.9 & 0 & 0\\ 0 & 0 & 2.5 & 0\\ 0 & 0 & 0 & 1.4\end{bmatrix}$$

$$B=\begin{bmatrix}-2.8 & -1.4 & 2 & -1.1\\ -1 & -0.9 & 0.5 & -0.2\\ 1 & 1.6 & 0.7 & 1.2\\ -0.4 & 0.7 & -0.3 & -2.3\end{bmatrix}$$

$$C=\begin{bmatrix}0.3 & -0.1 & 0.5 & 0.5\\ -0.6 & 1.3 & -1 & -0.6\\ -0.1 & -0.6 & -0.2 & -0.2\\ -0.4 & 0.7 & -0.6 & -2.3\end{bmatrix}$$

$$D=\begin{bmatrix}0.03 & 0.1 & 0.06 & -0.1\\ 0.3 & -0.1 & -0.1 & 0.1\\ 0.2 & -0.2 & 0.01 & 0.1\\ -0.3 & -0.2 & -0.07 & 0.4\end{bmatrix}$$

$$T_1=T_2=0.2I$$
$$T_3=T_4=0.3I$$
$$H_1=0.1, H_2=0.2$$
$$H_3=H_4=0.5$$

本例中, 假设 $l_1^-=l_2^-=l_3^-=l_4^-=0.3$, $l_1^+=l_2^+=l_3^+=l_4^+=0.7$, $\tau_1=0, h=0.2, h_d=0.1$. 然后, 使用 Matlab 中 LMI 控制工具箱, 计算最大允许的时滞 $\tau(t)$ 满足 LMI(11) 是 $\tau_m=\tau_2=4\,645.99$. 在此种情况下, 当 $\tau_2=5\,000, \tau_d=1.2$, $L_1=0.21I, L_2=I$, 得到一个可行解, 由于篇幅所限, 仅给出一部分可行解如下:

$$P=10^7\begin{bmatrix}6.9991 & 2.8090 & -2.8219 & -0.4920\\ 2.8090 & 8.1048 & -3.9601 & -0.8374\\ -2.8219 & -3.9601 & 7.8853 & -0.9545\\ -0.4920 & -0.8374 & -0.9545 & 6.7326\end{bmatrix}$$

$$Q_1=10^5\begin{bmatrix}1.9726 & 0.8214 & -0.8125 & 0.2270\\ 0.8214 & 2.3593 & -1.3477 & -0.4695\\ -0.8125 & \text{-}1.3477 & 1.4939 & -0.0336\\ 0.2270 & -0.4695 & -0.0336 & 0.6876\end{bmatrix}$$

$$Q_2=10^5\begin{bmatrix}5.6644 & -0.2710 & 0.5354 & -0.2014\\ -0.2710 & 1.0111 & 0.2208 & -0.8936\\ 0.5354 & 0.2208 & 0.3402 & -0.3845\\ -0.2014 & -0.8936 & -0.3845 & 1.3263\end{bmatrix}$$

$$R_1=10^5\begin{bmatrix}4.9520 & 2.4471 & -2.3044 & 1.1421\\ 2.4471 & 5.3096 & -3.2186 & -0.3711\\ -2.3044 & -3.2186 & 3.4413 & -0.5578\\ 1.1421 & -0.3711 & -0.5578 & 1.7716\end{bmatrix}$$

$$R_2=10^5\begin{bmatrix}4.9520 & 2.4471 & -2.3044 & 1.1421\\ 2.4471 & 5.3096 & -3.2186 & -0.3711\\ -2.3044 & -3.2186 & 3.4413 & -0.5578\\ 1.1421 & -0.3711 & -0.5578 & 1.7716\end{bmatrix}$$

$$R_3=\begin{bmatrix}1.0131 & 0.0072 & -0.0066 & 0.0029\\ 0.0072 & 1.0143 & -0.0089 & -0.0006\\ -0.0066 & -0.0089 & 1.0092 & -0.0015\\ 0.0029 & -0.0006 & -0.0015 & 1.0043\end{bmatrix}$$

$$W_1=\begin{bmatrix}1.0008 & -0.0000 & 0.0001 & 0.0004\\ -0.0000 & 1.0005 & -0.0001 & 0.0002\\ 0.0001 & -0.0001 & 1.0002 & -0.0000\\ 0.0004 & 0.0002 & -0.0000 & 1.0003\end{bmatrix}$$

$$W_2=\begin{bmatrix}1.0008 & -0.0000 & 0.0001 & 0.0004\\ -0.0000 & 1.0005 & -0.0001 & 0.0002\\ 0.0001 & -0.0001 & 1.0002 & -0.0000\\ 0.0004 & 0.0002 & -0.0000 & 1.0003\end{bmatrix}$$

$$W_3=10^8\begin{bmatrix}2.9537 & 0.0124 & 0.1126 & -0.9144\\ 0.0124 & 1.6548 & 0.1882 & -1.0750\\ 0.1126 & 0.1882 & 0.8623 & -0.3634\\ -0.9144 & -1.0750 & -0.3634 & 2.3919\end{bmatrix}$$

$$K=10^8\text{diag}\left\{1.1670\ 1.1670\ 1.1670\ 1.1670\right\}$$

$$Z_1=10^8\text{diag}\left\{5.2456\ 5.2456\ 5.2456\ 5.2456\right\}$$

$$Z_2=10^8\text{diag}\left\{5.9538\ 5.9538\ 5.9538\ 5.9538\right\}$$

$$\varepsilon_1=8.8968\text{E}+007,\quad \varepsilon_2=1.4106\text{E}+008$$
$$\varepsilon_3=1.3725\text{E}+008,\quad \varepsilon_4=2.5338\text{E}+008$$

通过例 1, 可以得出文中所提稳定性判据是可行的, 文献 [14−15, 17] 中, 作者所考虑的判据是在 $\tau_d<1$ 的情况下得到的, 但是本文中的判据已经取消了这一限制条件. 因此, 本文中定理 1 的结果与文献 [14−15, 17] 的结果相比具有较少的保守性.

例 2. 考虑如下一个含 2 个神经元的不确定中立型 Hopfield 神经网络:

$$\begin{aligned}\dot{x}(t)=&-(A+\Delta A(t))\boldsymbol{x}(t)+(B+\Delta B(t))\times\\ &\boldsymbol{f}(\boldsymbol{x}(t))+(C+\Delta C(t))\boldsymbol{f}(\boldsymbol{x}(t-\tau(t)))+\end{aligned}$$

$$(D+\Delta D(t))\,\dot{\boldsymbol{x}}(t-h(t)) \tag{32}$$

式中

$$A=\begin{bmatrix}4.2 & 0\\ 0 & 4.8\end{bmatrix},\ B=\begin{bmatrix}1.2 & -0.2\\ -0.06 & 0.7\end{bmatrix}$$

$$C=\begin{bmatrix}0.1 & -0.02\\ -0.02 & 0.1\end{bmatrix},\ D=\begin{bmatrix}0.02 & 0\\ 0 & 0.01\end{bmatrix}$$

$$H_1=\begin{bmatrix}-0.5 & 0\\ 0 & -0.5\end{bmatrix},\ H_1=\begin{bmatrix}-0.5 & 0\\ 0 & -0.5\end{bmatrix}$$

$$H_3=\begin{bmatrix}-0.5 & 0\\ 0 & -0.5\end{bmatrix},H_4=\begin{bmatrix}-0.5 & 0\\ 0 & -0.5\end{bmatrix}$$

$$T_1=\begin{bmatrix}0.21 & 0\\ 0 & 0.21\end{bmatrix},\ T_2=\begin{bmatrix}0.21 & 0\\ 0 & 0.21\end{bmatrix}$$

$$T_3=\begin{bmatrix}0.21 & 0\\ 0 & 0.21\end{bmatrix},\ T_4=\begin{bmatrix}0.21 & 0\\ 0 & 0.21\end{bmatrix}$$

$$f(x)=\tanh(x),\ \tau(t)=h(t)=0.6+0.6\sin^2(t)$$

本例中, 假设 $l_1^-=l_2^-=l_3^-=l_4^-=0.2$, $l_1^+=l_2^+=l_3^+=l_4^+=0.8$, $\tau_1=0.6, h=0.2, h_d=0.1, \tau_2=1.2, \tau_d=0.6$, 则有 $L_1=0.16I$, $L_2=I$, 由式 (11), 得到一个可行解, 由于篇幅所限, 仅给出一部分可行解如下:

$$P=\begin{bmatrix}373.4367 & 16.6427\\ 16.6427 & 482.9860\end{bmatrix}$$

$$Q_1=\begin{bmatrix}197.6283 & -1.6258\\ -1.6258 & 197.4515\end{bmatrix}$$

$$Q_2=\begin{bmatrix}132.9914 & 13.2327\\ 13.2327 & 186.0378\end{bmatrix}$$

$$R_1=\begin{bmatrix}195.0800 & -1.0200\\ -1.0200 & 195.1619\end{bmatrix}$$

$$R_2=\begin{bmatrix}196.6967 & -1.0690\\ -1.0690 & 196.9739\end{bmatrix}$$

$$R_3=\begin{bmatrix}200.7375 & -0.2987\\ -0.2987 & 200.8084\end{bmatrix}$$

$$W_1=\begin{bmatrix}29.4275 & 2.9830\\ 2.9830 & 41.1155\end{bmatrix}$$

$$W_2=\begin{bmatrix}91.4655 & 2.2681\\ 2.2681 & 99.9895\end{bmatrix}$$

$$W_3=\begin{bmatrix}296.1800 & 3.5136\\ 3.5136 & 308.9565\end{bmatrix}$$

$$K=\mathrm{diag}\left\{40.9429\ 40.9429\right\}$$

$$Z_1=\mathrm{diag}\left\{303.4016\ 303.4016\right\}$$

$$Z_2=\mathrm{diag}\left\{142.7503\ 142.7503\right\}$$

$$\varepsilon_1=\ 214.5411,\ \ \varepsilon_2=213.9355$$
$$\varepsilon_3=205.4030,\ \ \varepsilon_4=216.1562$$

可以得出文中所提稳定性判据是可行的, 然后, 通过 Matlab/Simulink 仿真, 得到 x_1, x_2 的曲线, 如图 1 所示.

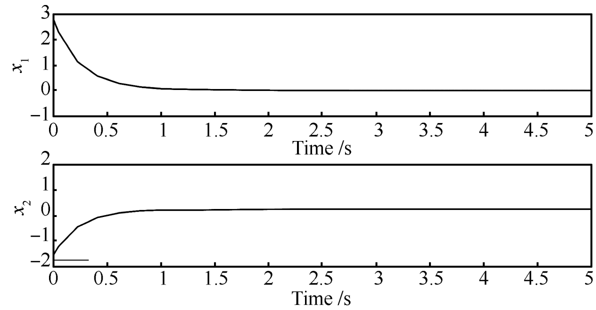

图 1 $\boldsymbol{x}(t)$ 的状态响应曲线

Fig. 1 The state response of $\boldsymbol{x}(t)$

由图 1, 可知系统 (32) 是全局鲁棒渐近稳定的.

4 结论

本文中, 通过构造适当 Lyapunov-Krasovskii 泛函和使用牛顿–莱布尼茨公式, 利用几个自由矩阵项, 给出了一类带有时变时滞和不确定参数的中立型 Hopfield 神经网络的鲁棒稳定性充分条件. 同时, 通过两个数值例子证明所得判据的有效性和可行性.

目前, 关于含有变时滞的不确定中立型神经网络稳定性的研究相应成果较少, 然而由于含有时变时滞的不确定中立型神经网络具有一般性, 更加接近于实际存在的系统模型, 其研究有助于理解神经网络数学理论的依据与背景、提供应用的基本思想.

另外, 关于中立型时滞神经网络的研究方法还有待进一步发掘. 半自由权矩阵方法、Razumikhin 方法均有望用于中立型神经网络的研究. 在将来注重实际背景问题探讨的同时, 还需进一步加强中立型神经网络模型与实际工程的结合.

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