Stability Analysis for Uncertain Nonlinear Time-delay Systems with Quasi-one-sided Lipschitz Condition

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Abstract This paper deals with the robust stability of a class of uncertain nonlinear time-delay systems. A quasi-one-sided Lipschitz condition is introduced to estimate the influence of nonlinear vector function on the stability analysis. Delay-independent/delay-dependent stability criteria formulated in the form of linear matrix inequalities are presented. Furthermore, these stability criteria are available even if the system parameter is unstable, because the unnecessary positive quasi-one-sided Lipschitz constant matrix includes much useful information of the nonlinear part. Numerical examples show the advantage of the results obtained in this paper.

Key words Robust stability, uncertain nonlinear time-delay systems, quasi-one-sided Lipschitz condition, delay-independent/delay-dependent, linear matrix inequality (LMI)

The stability problems of nonlinear time-delay systems are of great importance due to the fact that nonlinearities and time delays are inherent characteristics of many real physical systems such as mechanical and chemical processes, power and water distribution networks, air pollution systems, and econometric systems. There are a lot of reports about stability conditions for nonlinear time-delay systems in the published works[1−9] and the references therein. For example, the neutral-delay-dependent and discrete-delay-dependent stability criteria for neutral systems with mixed and nonlinear perturbation were derived in [8]. Robust stability of uncertain linear systems and uncertain neutral systems with time-varying delay and nonlinear perturbations were investigated in [1−2], respectively. Especially, a so-called one-sided Lipschitz condition and a quasi-one-sided Lipschitz condition were introduced in [10−11], respectively, instead of the classical Lipschitz condition for observer design of nonlinear ordinary differential systems. It is interesting that the quasi-one-sided Lipschitz condition introduced in [10] includes much useful information of the nonlinear part and contributes to the observer design. However, for nonlinear time-delay systems, the nonlinear part is generally treated as a perturbation causing instability and performance degradation. This motivates the authors to exploit a quasi-one-sided Lipschitz condition for nonlinear time-delay systems, which can contribute to getting less conservative stability criteria.

In practice, the nonlinear part may contribute to the stability of nonlinear systems. For example, the system $\dot{x} = x + \phi(x)$ is unstable when $\phi(x) = 0$. But it is asymptotically stable when $\phi(x) = -2\sigma x/(1+\sin^2 x)$ with $\sigma > 1$, because the formula $\dot{V} = 2x^2(1-2\sigma/(1+\sin^2 x)) \leq -2x^2(\sigma - 1) < 0$ holds if the Lyapunov function is cho-

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sen as $V = x^2$. Thus, in order to make more use of the useful information of the nonlinear part, a quasi-one-sided Lipschitz condition is introduced in this paper for a class of uncertain nonlinear time-delay systems. It is the extension of the quasi-one-sided Lipschitz condition employed in [10]. Based on the quasi-one-sided Lipschitz condition and Lyapunov-Krasovskii stability theory, delay-independent/delay-dependent stability criteria that can be formulated in the form of linear matrix inequalities (LMIs) are presented. It is interesting that these stability criteria are available even if the system parameter is unstable, because the unnecessary positive quasi-one-sided Lipschitz constant matrix includes much useful information of the nonlinear part. Furthermore, we also give some examples to show the effectiveness of the criteria.

Throughout this paper, $\mathbf{R}^n$ denotes n-dimensional Euclidean space, $\mathbf{R}^{n\times m}$ is the set of all $n \times m$ real matrices, I denotes identity matrix of appropriate order, and "$*$" represents the elements below the main diagonal of a symmetric matrix.

1 Delay-independent stability analysis

Consider the following uncertain nonlinear time-delay system:

$$\dot{\boldsymbol{x}} = (A + \Delta A)\boldsymbol{x} + (B + \Delta B)\boldsymbol{x}_\tau + \boldsymbol{\Phi}(\boldsymbol{x}, \boldsymbol{x}_\tau) \qquad (1)$$

where $\boldsymbol{x} \in \mathbf{R}^n$, $A, B \in \mathbf{R}^{n\times n}$ are constant matrices, $\boldsymbol{x}_\tau = \boldsymbol{x}(t-\tau)$, and τ is the positive constant time delay. $\boldsymbol{\Phi}(\boldsymbol{x}, \boldsymbol{x}_\tau)$ is a nonlinear function with respect to $\boldsymbol{x}$, $\boldsymbol{x}_\tau$ and $\boldsymbol{\Phi}(\boldsymbol{0}, \boldsymbol{0}) = \boldsymbol{0}$. The parameter uncertainties ΔA and ΔB are assumed to be of the form

$$[\Delta A \;\; \Delta B] = DF[E \;\; E_1] \qquad (2)$$

where D, E, and $E_1$are known real constant matrices of appropriate dimensions, and F is an unknown matrix which satisfies

$$F^{\mathrm{T}}F \leq I \qquad (3)$$

Then, we can represent system (1) as

$$\begin{cases} \dot{\boldsymbol{x}} &= A\boldsymbol{x} + B\boldsymbol{x}_\tau + \boldsymbol{\Phi}(\boldsymbol{x}, \boldsymbol{x}_\tau) + D\boldsymbol{p} \\ \boldsymbol{q} &= E\boldsymbol{x} + E_1\boldsymbol{x}_\tau \end{cases} \qquad (4)$$

with constraint

$$\boldsymbol{p} = F\boldsymbol{q} \qquad (5)$$

We further rewrite (4) as

$$\begin{cases} \dot{\boldsymbol{x}} &= A\boldsymbol{x} + B\boldsymbol{x}_\tau + \boldsymbol{\Phi}(\boldsymbol{x}, \boldsymbol{x}_\tau) + D\boldsymbol{p} \\ \boldsymbol{p}^{\mathrm{T}}\boldsymbol{p} &\leq (E\boldsymbol{x} + E_1\boldsymbol{x}_\tau)^{\mathrm{T}}(E\boldsymbol{x} + E_1\boldsymbol{x}_\tau) \end{cases} \qquad (6)$$

Next, we present the quasi-one-sided Lipschitz condition which plays an important role in the following stability analysis.

Definition 1[10]**.** For any $\boldsymbol{x}, \boldsymbol{y} \in \mathbf{R}^n$,

$$\langle \boldsymbol{f}, \boldsymbol{x} \rangle \leq \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{y} \end{bmatrix}^{\mathrm{T}} \Omega \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{y} \end{bmatrix} \qquad (7)$$

where $\boldsymbol{f} = P\boldsymbol{\Phi}(\boldsymbol{x}, \boldsymbol{y})$, P is some symmetric positive-definite matrix to be determined later. (7) is called the quasi-one-sided Lipschitz condition and the symmetric matrix $\Omega = \begin{bmatrix} M & 0 \\ 0 & N \end{bmatrix}$, which can be indefinite, is called the quasi-one-side Lipschitz constant matrix for $\boldsymbol{f}$ with respect to $\boldsymbol{x}$ and $\boldsymbol{y}$, where M and N are any real symmetric matrices.

The quasi-one-sided Lipschitz condition extends a general version of the one-sided Lipschitz condition, which is investigated frequently and plays an useful role in the numerical stability analysis for nonlinear ordinary differential equations[12]. Of course, it is intimately connected with the Lyapunov function $V(\boldsymbol{x}) = \boldsymbol{x}^{\mathrm{T}}\boldsymbol{x}$.

Theorem 1. The null solution of the nonlinear time-delay system (1) with the quasi-one-sided Lipschitz condition (7) is robustly asymptotically stable for all values of the delay if there exist a scalar solution $\varepsilon > 0$ and symmetric positive-definite solutions P and Q such that the following LMI holds:

$$\begin{bmatrix} \Sigma_{11} & \Sigma_{12} & PD \\ * & \Sigma_{22} & 0 \\ * & * & -\varepsilon I \end{bmatrix} < 0 \qquad (8)$$

where

$$\begin{aligned} \Sigma_{11} &= A^{\mathrm{T}}P + PA + \varepsilon E^{\mathrm{T}}E + Q + 2M \\ \Sigma_{12} &= PB + \varepsilon E^{\mathrm{T}}E_1 \\ \Sigma_{22} &= -Q + \varepsilon E_1^{\mathrm{T}}E_1 + 2N \end{aligned}$$

Proof. Define a Lyapunov-Krasovskii functional as

$$V = \boldsymbol{x}^{\mathrm{T}}P\boldsymbol{x} + \int_{t-\tau}^{t} \boldsymbol{x}^{\mathrm{T}}(s)Q\boldsymbol{x}(s)\mathrm{d}s \qquad (9)$$

The weight matrices P and Q are positive definite. From (6) and (7), the derivative of V with respect to t along the solution of the nonlinear time-delay system (1) gives

$$\begin{aligned} \dot{V} = \; & 2\boldsymbol{x}^{\mathrm{T}}P\dot{\boldsymbol{x}} + \boldsymbol{x}^{\mathrm{T}}Q\boldsymbol{x} - \boldsymbol{x}_\tau^{\mathrm{T}}Q\boldsymbol{x}_\tau \leq \\ & 2\boldsymbol{x}^{\mathrm{T}}P(A\boldsymbol{x} + B\boldsymbol{x}_\tau^{\mathrm{T}} + \boldsymbol{\Phi}(\boldsymbol{x}, \boldsymbol{x}_\tau) + D\boldsymbol{p}) + \\ & \boldsymbol{x}^{\mathrm{T}}Q\boldsymbol{x} - \boldsymbol{x}_\tau^{\mathrm{T}}Q\boldsymbol{x}_\tau + \\ & \varepsilon((E\boldsymbol{x} + E_1\boldsymbol{x}_\tau)^{\mathrm{T}}(E\boldsymbol{x} + E_1\boldsymbol{x}_\tau) - \boldsymbol{p}^{\mathrm{T}}\boldsymbol{p}) \leq \\ & \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{x}_\tau \\ \boldsymbol{p} \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} \Sigma_{11} & \Sigma_{12} & PD \\ * & \Sigma_{22} & 0 \\ * & * & -\varepsilon I \end{bmatrix} \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{x}_\tau \\ \boldsymbol{p} \end{bmatrix} \end{aligned}$$

Clearly, (8) implies $\dot{V} < 0$, i.e., it guarantees the robust stability of system (1) by the Lyapunov-Krasovskii theory. □

Furthermore, based on the proof of Theorem 1, the following corollary can be obtained easily.

Corollary 1. The null solution of the nonlinear time-delay system (1) with $\Delta A = \Delta B = 0$ and the quasi-one-sided Lipschitz condition (7) is asymptotically stable for all values of the delay if there exist symmetric positive-definite solutions P and Q such that the following LMI holds:

$$\begin{bmatrix} A^{\mathrm{T}}P + PA + Q + 2M & PB \\ B^{\mathrm{T}}P & -Q + 2N \end{bmatrix} < 0 \qquad (10)$$

Remark 1. The inequalities (8) and (10) imply $A^{\mathrm{T}}P + PA < 0$ is necessary, i.e., the parameter A must be stable if $M > 0$. Considering M is unnecessarily positive, the inequalities (8) and (10) are still available even if the parameter A of nonlinear time-delay system (1) is unstable.

2 Delay-dependent stability analysis

In general, the delay-dependent criteria are less conservative than the delay-independent criteria when the delay is small. Delay-dependent stability criteria are obtained in this section, which can be formulated in the form of LMI.

In this section, we assume that $\boldsymbol{\Phi}(\boldsymbol{x},\boldsymbol{x}_\tau)$ of nonlinear time-delay system (1) satisfies the quasi-one-sided Lipschitz condition (7) and the following bounded condition:

$$\langle\boldsymbol{\Phi}(\boldsymbol{x},\boldsymbol{y}),\boldsymbol{\Phi}(\boldsymbol{x},\boldsymbol{y})\rangle \le a^2\boldsymbol{x}^{\mathrm{T}}\boldsymbol{x}+b^2\boldsymbol{y}^{\mathrm{T}}\boldsymbol{y},\quad \forall \boldsymbol{x},\boldsymbol{y}\in\mathbf{R}^n \tag{11}$$

where $a,b\in\mathbf{R}$.

Lemma 1[1]**.** For any constant symmetric matrix $P>0$, scalar $\tau>0$, and vector function $\dot{\boldsymbol{x}}(\cdot):[-\tau,0]\to\mathbf{R}^n$ such that the following integral is well defined, then

$$-\tau\int_{t-\tau}^{t}\dot{\boldsymbol{x}}(s)^{\mathrm{T}}P\dot{\boldsymbol{x}}(s)\mathrm{d}s \le \begin{bmatrix}\boldsymbol{x}\\ \boldsymbol{x}_\tau\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}-P & P\\ P & -P\end{bmatrix}\begin{bmatrix}\boldsymbol{x}\\ \boldsymbol{x}_\tau\end{bmatrix} \tag{12}$$

Theorem 2. The null solution of the nonlinear time-delay system (1) with the quasi-one-sided Lipschitz condition (7) and the bounded condition (11) is robustly asymptotically stable if there exist scalar solutions $\varepsilon,\delta>0$ and symmetric positive-definite solutions P,Q, and S such that the following LMI holds:

$$\begin{bmatrix}\Gamma_{11} & \Gamma_{12} & \tau^2A^{\mathrm{T}}S & PD+\tau^2A^{\mathrm{T}}SD\\ * & \Gamma_{22} & \tau^2B^{\mathrm{T}}S & \tau^2B^{\mathrm{T}}SD\\ * & * & \tau^2S-\delta I & \tau^2SD\\ * & * & * & \tau^2D^{\mathrm{T}}SD-\varepsilon I\end{bmatrix}<0 \tag{13}$$

where

$$\begin{aligned}\Gamma_{11}= &\ A^{\mathrm{T}}P+PA+\tau^2A^{\mathrm{T}}SA+\varepsilon E^{\mathrm{T}}E+Q-S+\\ &\ 2M+\delta a^2I\\ \Gamma_{12}= &\ PB+\tau^2A^{\mathrm{T}}SB+\varepsilon E^{\mathrm{T}}E_1+S\\ \Gamma_{22}= &\ -Q+\tau^2B^{\mathrm{T}}SB+\varepsilon E_1^{\mathrm{T}}E_1-S+2N+\delta b^2I\end{aligned}$$

Proof. Define a Lyapunov-Krasovskii functional as

$$V=V_1+V_2+V_3$$

where

$$\begin{aligned}V_1 &= \boldsymbol{x}^{\mathrm{T}}P\boldsymbol{x},\quad V_2=\textstyle\int_{t-\tau}^{t}\boldsymbol{x}^{\mathrm{T}}(\alpha)Q\boldsymbol{x}(\alpha)\mathrm{d}\alpha\\ V_3 &= \tau\textstyle\int_{-\tau}^{0}\int_{t+\beta}^{t}\dot{\boldsymbol{x}}^{\mathrm{T}}(\alpha)S\dot{\boldsymbol{x}}(\alpha)\mathrm{d}\alpha\mathrm{d}\beta\end{aligned}$$

The derivative of V with respect to t along the nonlinear time-delay system (1) is

$$\dot{V}=\dot{V}_1+\dot{V}_2+\dot{V}_3$$

where

$$\begin{aligned}\dot{V}_1 &= 2\boldsymbol{x}^{\mathrm{T}}P\dot{\boldsymbol{x}}=2\boldsymbol{x}^{\mathrm{T}}P(A\boldsymbol{x}+B\boldsymbol{x}_\tau+\boldsymbol{\Phi}(\boldsymbol{x},\boldsymbol{x}_\tau)+D\boldsymbol{p})\\ \dot{V}_2 &= \boldsymbol{x}^{\mathrm{T}}Q\boldsymbol{x}-\boldsymbol{x}_\tau^{\mathrm{T}}Q\boldsymbol{x}_\tau\\ \dot{V}_3 &= \tau^2\dot{\boldsymbol{x}}^{\mathrm{T}}S\dot{\boldsymbol{x}}-\tau\textstyle\int_{t-\tau}^{t}\dot{\boldsymbol{x}}^{\mathrm{T}}(\alpha)S\dot{\boldsymbol{x}}(\alpha)\mathrm{d}\alpha\end{aligned}$$

It follows from Lemma 1, (6), (7), and (11) that

$$\begin{aligned}\dot{V}\le &\ \textstyle\sum_{i=1}^{3}\dot{V}_i+\varepsilon(E\boldsymbol{x}+E_1\boldsymbol{x}_\tau)^{\mathrm{T}}(E\boldsymbol{x}+E_1\boldsymbol{x}_\tau)-\varepsilon\boldsymbol{p}^{\mathrm{T}}\boldsymbol{p}+\\ &\ \delta(a^2\boldsymbol{x}^{\mathrm{T}}\boldsymbol{x}+b^2\boldsymbol{x}_\tau^{\mathrm{T}}\boldsymbol{x}_\tau-\boldsymbol{\Phi}(\boldsymbol{x},\boldsymbol{x}_\tau)^{\mathrm{T}}\boldsymbol{\Phi}(\boldsymbol{x},\boldsymbol{x}_\tau))\le\\ &\ \boldsymbol{\xi}^{\mathrm{T}}\begin{bmatrix}\Gamma_{11} & \Gamma_{12} & \tau^2A^{\mathrm{T}}S & PD+\tau^2A^{\mathrm{T}}SD\\ * & \Gamma_{22} & \tau^2B^{\mathrm{T}}S & \tau^2B^{\mathrm{T}}SD\\ * & * & \tau^2S-\delta I & \tau^2SD\\ * & * & * & \tau^2D^{\mathrm{T}}SD-\varepsilon I\end{bmatrix}\boldsymbol{\xi}\end{aligned} \tag{14}$$

where

$$\boldsymbol{\xi}^{\mathrm{T}}=[\ \boldsymbol{x}^{\mathrm{T}}\quad \boldsymbol{x}_\tau^{\mathrm{T}}\quad \boldsymbol{\Phi}(\boldsymbol{x},\boldsymbol{x}_\tau)^{\mathrm{T}}\quad \boldsymbol{p}^{\mathrm{T}}\ ]$$

Thus, (13) guarantees the robust stability of system (1) by the Lyapunov-Krasovskii stability theory. □

Corollary 2. The null solution of the nonlinear time-delay system (1) with $\Delta A=\Delta B=0$, the quasi-one-sided Lipschitz condition (7) and the bounded condition (11) is asymptotically stable if there exist a scalar solution $\delta>0$ and symmetric positive-definite solutions P,Q, and S such that the following LMI holds:

$$\begin{bmatrix}\Pi_{11} & \Pi_{12} & \tau^2A^{\mathrm{T}}S\\ * & \Pi_{22} & \tau^2B^{\mathrm{T}}S\\ * & * & \tau^2S-\delta I\end{bmatrix}<0 \tag{15}$$

where

$$\begin{aligned}\Pi_{11} &= A^{\mathrm{T}}P+PA+\tau^2A^{\mathrm{T}}SA+Q-S+2M+\delta a^2I\\ \Pi_{12} &= PB+\tau^2A^{\mathrm{T}}SB+S\\ \Pi_{22} &= -Q+\tau^2B^{\mathrm{T}}SB-S+2N+\delta b^2I\end{aligned}$$

For system (1) with (7) and (11), more delay-dependent stability criteria can be obtained by applying the methods employed in the published works[2, 6, 9].

Remark 2. The inequalities (13) and (15) imply the inequality $A^{\mathrm{T}}P+PA-S<0$ is not necessary, i.e., the parameter A of nonlinear system (1) can be unstable, because there exists an unnecessarily positive matrix M in the first order principal minor. That is, the inequalities (13) and (15) are still available even if the parameter A of nonlinear time-delay system (1) is unstable.

Remark 3. Applying the same methods employed in Theorems 1 and 2 and based on some bounded conditions (e.g., (11)), we can obtain the corresponding stability criteria. But the parameter A must be stable and $A^{\mathrm{T}}P+PA-S$ must be negative in the delay-independent criteria and delay-dependent criteria, respectively. Thus, we can say that the quasi-one-sided Lipschitz condition (7) is less conservative than the bounded conditions from Remarks 1 and 2.

3 Numerical examples

In this section, we give some examples to show the effectiveness of the stability criteria above and to verify the less conservatism of the quasi-one-sided Lipschitz condition (7).

Example 1. Consider system (1) with

$$A=\begin{bmatrix}0.3 & 0.1\\ 0.1 & 0.3\end{bmatrix},\quad B=0.4I,\quad D=I,\quad E=E_1=0.1I$$

$$\boldsymbol{\Phi}(\boldsymbol{x},\boldsymbol{x}_\tau)=\begin{bmatrix}\left(-\dfrac{2}{1+(0.5\sqrt{2}-1)\sin^2x_1}\right)x_1+x_{\tau1}\sin x_2\\ \left(-\dfrac{2}{1+(0.5\sqrt{2}-1)\sin^2x_2}\right)x_2+x_{\tau2}\sin x_1\end{bmatrix}$$

where $\boldsymbol{x}=[x_1\ x_2]^{\mathrm{T}}$ and $\boldsymbol{x}_\tau=[x_{\tau1}\ x_{\tau2}]^{\mathrm{T}}$.

Observe that $-2\sqrt{2}\le-2/(1+(0.5\sqrt{2}-1)\sin^2x_i)\le-2$ for all $x_i\in\mathbf{R}$ $(i=1,2)$ and let $P=\begin{bmatrix}P_1 & 0\\ 0 & P_2\end{bmatrix}>0$. Then, for all $\boldsymbol{x},\boldsymbol{x}_\tau\in\mathbf{R}^2$, we have

$$\langle P\boldsymbol{\Phi}(\boldsymbol{x},\boldsymbol{x}_\tau),\boldsymbol{x}\rangle\le-1.5\boldsymbol{x}^{\mathrm{T}}P\boldsymbol{x}+0.5\boldsymbol{x}_\tau^{\mathrm{T}}P\boldsymbol{x}_\tau$$

and

$$\langle\boldsymbol{\Phi}(\boldsymbol{x},\boldsymbol{x}_\tau),\boldsymbol{\Phi}(\boldsymbol{x},\boldsymbol{x}_\tau)\rangle\le16\boldsymbol{x}^{\mathrm{T}}\boldsymbol{x}+2\boldsymbol{x}_\tau^{\mathrm{T}}\boldsymbol{x}_\tau$$

i.e., $M = -1.5P$, $N = 0.5P$, and $a = 4, b = \sqrt{2}$.

Theorem 1 is unavailable for Example 1. Using Theorem 2, we get the maximum allowed time-delay $\tau_{\max} = 0.14$.

The next example shows that N can also be negative.

Example 2. Consider system (1) with

$$A = 0.6,\quad B = 0.9,\quad D = 1,\quad E = E_1 = 0.1$$

$$\Phi(x, x_\tau) = \left(-\frac{2}{1+\cos^2 x}\right)x + \left(-\frac{3}{2x+\sin^2 x}\right)x_\tau^2$$

Using the mean value theory, we have $2x + \sin^2 x = (2 + \sin 2\zeta)x$, where $\zeta \in [0, x]$. Note that $-2 \le -2/(1+\cos^2 x) \le -1$ for all $x \in \mathbf{R}$ and $-3 \le -3/(2+\sin 2\zeta) \le -1$ for all $\zeta \in \mathbf{R}$. Let $P > 0$. Then, we have $\langle P\Phi(x, x_\tau), x\rangle \le -x^{\mathrm{T}}Px - x_\tau^{\mathrm{T}}Px_\tau$ for all $x, x_\tau \in \mathbf{R}$, i.e., $M = -P$, $N = -P$. Applying Theorem 1, Example 2 is asymptotically stable for all values of the delay.

Remark 4. The parameter A must be stable[7] or $A+B$ must be stable[6] in some existing results which are based on bounded conditions. However, we verify that the criteria obtained in this research are available even if A or $A+B$ are unstable from the numerical examples. That is, the quasi-one-sided Lipschitz condition (7) is less conservative than the bounded conditions.

4 Conclusion

In general, for nonlinear time-delay systems, the nonlinear part is treated as a perturbation causing instability and performance degradation in the published works. However, the nonlinear part may contribute to the stability of the nonlinear time-delay systems. This motivated the authors to exploit a mathematical technique, which can make more use of the useful information of the nonlinear part. Consequently, the quasi-one-sided Lipschitz condition (7) is introduced and plays an important role in the stability analysis. The stability criteria obtained in the study are presented in the form of LMI and are still available even if the parameter A of nonlinear system (1) is unstable. Thus, the quasi-one-sided Lipschitz condition (7) is less conservative.

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