Protocol Design for Output Consensus of Port-controlled Hamiltonian Multi-agent Systems

LI Chang-Sheng[1, 2] WANG Yu-Zhen[1]

Abstract This paper investigates the output consensus problem of port-controlled Hamiltonian (PCH) multi-agent systems with both fixed and switching topologies. Firstly, a distributed group output consensus protocol is designed via the energy shaping method to reach globally stability and group output consensus. Secondly, a new distributed control protocol is proposed by using the structural properties of the PCH systems. The advantage of this protocol is that it can transform the directed graph to the undirected graph by constructing a kind of virtual neighbors. Thirdly, a control protocol is designed with the extended LaSalle′s invariance principle developed for switched systems under the jointly connected topology condition to make all the agents reach output consensus when the topology is switching. Finally, some illustrative examples with simulations are provided to demonstrate the effectiveness of the protocols designed in this paper.

Key words Control design, Hamiltonian systems, multi-agent, consensus

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In recent years, the research of multi-agent systems has become an attractive topic and has drawn considerable attention from various scientific communities. This is partly due to broad practical applications of multi-agent systems in many areas such as formation control, flocking, distributed sensor networks, and attitude alignment of clusters of satellites. Consensus is a basic problem in the field of multi-agent systems. Such a problem has been studied extensively and abundant results have been obtained by efforts of many researchers[1−7]. The group consensus problem has been studied in [8]. As a kind of important consensus problems, the output consensus problem has found many practical applications in various fields such as unmanned aerial vehicles, sensor networks and mobile robots[9−10]. In [10], Chopra and Spong defined the output consensus problem, and using the passivity-based control method, designed several protocols for the coordination and synchronization problem of a class of multi-agent systems described by affine nonlinear input-output passive systems with balanced graphs. Later, Chopra and Spong discussed this problem on balanced graphs for weakly minimum phase nonlinear systems with relative degree one in [11]. The authors of [12−13] further considered this problem to relax the condition of balanced graph.

The port-controlled Hamiltonian (PCH) system, proposed by [14], has been well investigated in a series of papers, see, e.g., [15−17]. Such a system provides a framework with nice structure and clear physical meaning for the geometric description of many physical systems such as mechanical systems, electric systems, thermal, as well as mixtures of them. Based on PCH systems, several effective energy-based control methods were proposed in [16, 18], and successfully applied to the control of mechanical systems and power systems[19−21]. In [17], a new technique, called the energy-shaping plus damping injection, was proposed for PCH systems based on the passivity-based control (PBC) technique. However, to our best knowledge, there are few works in the literature on study of PCH multi-agent systems. Particularly, there are fewer results on the output consensus of such multi-agent systems.

The dynamic of robots has been studied generally by the Euler-Lagrange system[22]. In [23], the authors showed how to apply the new Hamiltonian formulation to rewrite uncertain mechanical systems as an augmented Hamiltonian system with dissipation. As the port-controlled Hamiltonian systems are a generalization of Euler-Lagrange systems, it is necessary to study the consensus problem of port-controlled Hamiltonian multi-agent systems. In this paper, we study the output consensus problem for PCH multi-agent systems under fixed and dynamically changed topologies. As an important kind of passive dynamical multi-agent systems, the PCH multi-agent systems may have numerous practical applications, such as mechanical systems such as robotic manipulators and rigid bodies, electric systems, and robot networks. The group consensus problem concerns a network, which is divided into multiple sub-networks. All the agents in it are consequently divided into multiple groups and keep some group characteristic by information exchange existing not only between two agents in a group but also between different groups. As the PCH systems may have numerous practical applications, it is necessary to study the group consensus of the PCH multi-agent systems. First, using the structural properties of the PCH systems, we propose a new distributed control protocol for the systems. The advantage of this protocol is that it can transform the directed graph to the undirected graph by constructing a kind of virtual neighborhood. It is shown that this new agreement protocol can solve the output consensus problem even for weakly connected unbalanced networks. Using the extended LaSalle′s invariance principle developed for switched systems, it is proved that under the jointly connected topology condition, the designed control protocol can make all the agents reach output consensus when the topology is switching. Second, we address the group output consensus problem of the PCH multi-agent systems via the energy shaping method, and design a distributed group output consensus protocol, under which the PCH multi-agent system is globally stable and all the agents can reach the group output consensus. Finally, we provide some illustrative examples with simulations to demonstrate the effectiveness of the protocols designed in this paper.

The main contributions of this paper are as follows. In

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1. School of Control Science and Engineering, Shandong University, Jinan 250061, China 2. Department of Information Science and Technology, Taishan College, Taian 271021, China

this paper, a class of multi-agent systems has been introduced, that is, PCH multi-agent systems, and several new output consensus protocols have been designed by exploiting the structural properties of this class of systems. The protocols obtained in this paper can not only apply to PCH multi-agent systems with balanced graphs, but also be applicable to those with weakly connected unbalanced graphs. The method used in this paper is different from the technique adopted by [10−13].

The remainder of this paper is organized as follows. In Section 1, we recall some necessary preliminaries on graph theory first, then present the problem formulation of this paper. In Section 2, we investigate the output consensus protocol design and present the main results of our paper. Section 3 is the illustrative examples with simulations, which is followed by the conclusion in Section 4.

1 Preliminaries and formulation

In this section, we first review some concepts and basic knowledge on graph theory[24] and then give the formulation of the output consensus problem of PCH multi-agent systems.

A graph $\mathcal{G}$ consists of a vertex (node) set $\mathcal{V} = \{v_1, v_2, \cdots, v_N\}$ and an edge set $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$, denoted by $\mathcal{G} = (\mathcal{V}, \mathcal{E})$. If each edge of $\mathcal{G}$, denoted by $e_{ij} = (v_i, v_j) \in \mathcal{E}$, is an ordered pair of distinct vertices of $\mathcal{V}$, we call $\mathcal{G}$ directed graph (or diagraph). If each edge $e_{ij} = (v_i, v_j) \in \mathcal{E}$ implies that $e_{ji} = (v_j, v_i) \in \mathcal{E}$, then we call $\mathcal{G}$ an undirected graph. In a digraph $\mathcal{G}$, a directed path is a sequence of ordered edges of the form $(v_{i_1}, v_{i_2}), (v_{i_2}, v_{i_3}), \cdots$. An undirected path in an undirected graph is defined accordingly. A weak path is a sequence of $v_{i_1}, \cdots, v_{i_r}$ of r distinct vertices such that for each $j \in \{i_1, \cdots, i_r\}$ either (v_j, v_{j+1}) or (v_{j+1}, v_j) is an edge. A digraph is called strongly connected if there is a path between any two distinct vertices (nodes), and called weakly connected if any two vertices can be joined by a weak path. An undirected graph is called connected if there is a path between any distinct pair of nodes. A directed tree is a digraph in which every node has only one parent except for the root. A spanning tree of a digraph is a directed tree formed by a subset of edges that can connect all the nodes of the digraph.

Denoting by M the total number of links, the $N \times M$ incidence matrix, D, of a graph with N nodes is defined as

$$d_{ik} = \begin{cases} 1, & \text{if the } k\text{-th link starts from the } i\text{-th node} \\ -1, & \text{if the } k\text{-th link ends at the } i\text{-th node} \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

The neighbor nodes set of the i-th node is defined as $N_i = \{j \in \mathcal{V} \,|(j, i) \in \mathcal{E}\}$. The Laplacian, $L = \{l_{ij}\}$, of a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ is a matrix with

$$l_{ij} = \begin{cases} -1, & \text{if } j \in N_i \\ |N_i|, & j = i \\ 0, & \text{otherwise} \end{cases}$$

where $|N_i|$ denotes the number of neighbors of the i-th node. In an undirected graph with an assigned direction to each edge, the graph Laplacian can be decomposed as $L = DD^{\mathrm{T}}$.

For an undirected graph, the Laplacian has a single zero eigenvalue and all the other eigenvalues are positive if and only if the graph is connected, and for a directed information exchange graph, its Laplacian has a single zero eigenvalue and all the other eigenvalues have positive real parts if and only if the digraph has a (directed) spanning tree. In both the cases, $\mathbf{1} = [1, \cdots, 1]^{\mathrm{T}}$ is the eigenvector of the graph (digraph) Laplacian with respect to the zero eigenvalue[24].

In the following, we give the formulation of our problem.

Consider the following multi-agent system with each agent described as a port-controlled Hamiltonian (PCH) system

$$\begin{cases} \dot{x}_i = [J_i(x_i) - R_i(x_i)]\nabla H_i(x_i) + g_i(x_i)u_i \\ y_i = g_i^{\mathrm{T}}(x_i)\nabla H_i(x_i) \qquad i = 1, 2, \cdots, N \end{cases} \tag{2}$$

where $x_i \in \mathbf{R}^n$ and $y_i \in \mathbf{R}^m$ are the state and the output of the i-th agent, respectively. $J_i(x_i) \in \mathbf{R}^{n\times n}$ is skew-symmetric and $R_i(x_i) \in \mathbf{R}^{n\times n}$ positive semi-definite, $\nabla H_i(x_i) = \frac{\partial H_i(x_i)}{\partial x_i}$, $g_i(x) \in \mathbf{R}^{n\times m}$, and $u_i \in \mathbf{R}^m$ is the control input of the i-th agent, $H_i(x_i)$ is the Hamiltonian function and throughout this paper we suppose each is a positive definite function.

For easy description, we call system (2) a PCH multi-agent system, and denote by $\mathcal{G}$ its communication topology/graph consisting of all the nodes and their links.

Definition 1. Consider system (2). Let $A_1 = \{i_1, i_2, \cdots, i_s\} \subset \{1, 2, \cdots, N\}$, $A_2 = \{1, 2, \cdots, N\}\backslash A_1$ and $d \in \mathbf{R}^m$ be a given constant vector denoting the required "distance" between the groups A_1 and A_2. The N agents are said to be output consensus if

$$\lim_{t\to\infty} \|y_j(t) - y_i(t)\| = 0, \quad \forall i, j = 1, \cdots, N \tag{3}$$

the agents are said to reach a group output consensus if

$$\lim_{t\to\infty} \|y_i(t) - y_j(t)\| = 0, \quad \forall i, j \in A_1 \tag{4}$$

$$\lim_{t\to\infty} \|y_i(t) - y_j(t)\| = 0, \quad \forall i, j \in A_2 \tag{5}$$

$$\lim_{t\to\infty} \|y_i(t) - y_j(t)\| = d, \quad \forall i \in A_1,\ j \in A_2 \tag{6}$$

where $\|\cdot\|$ denotes the Euclidean norm of the enclosed signal.

Remark 1. It is noted that the two concepts become the same if $d = 0$. If y_i is chosen as the position state of the agent i, $i = 1, 2, \cdots, N$, then the two group agents, A_1 and A_2, can keep the required distance $D = \|d\|$.

The objective of this paper is to investigate the (group) output consensus for system (2). We will consider these problems in two cases: 1) the graph of system (2) has a fixed topology, and 2) the topology is switching.

2 Main results

In this section, we consider the (group) output consensus problem of system (2) with both fixed and switching topologies, and present the main results on the protocol design for the (group) output consensus. In Subsection 2.1, we study the case of fixed topology, and In Subsection 2.2, we consider the case of switching topology.

2.1 Results for fixed topology

First, we investigate the group output consensus problem of system (2) with with fixed digraph $\mathcal{G}$. Toward this end, we turn to a well-known controller design method, i.e., the energy shaping and damping injection method, which can provide a desired Hamiltonian structure by shaping the system′s total energy and injecting some proper damping into the system′s structure. For details, please refer to [17].

In this section, we only apply the energy shaping method to group A_1 (not shaping the energies of the agents in group A_2). For agent $i \in A_1$, its desired Hamiltonian function, $H_{id}(x_i)$, is required to satisfy

$$g_i^{\mathrm{T}} \nabla H_{id} = y_i + d, \quad i \in A_1 \tag{7}$$

and the shaping feedback law $\bar{u}_i$ satisfies the following matching equation

$$(J_i - R_i)\nabla H_i + g_i \bar{u}_i = (J_i - R_i)\nabla H_{id}, \quad i \in A_1 \tag{8}$$

If $g_i(x)$ is full column rank, then the control can be directly calculated using the formula

$$u_i = (g_i^{\mathrm{T}} g_i)^{-1} g_i^{\mathrm{T}} (J_i - R_i)(\nabla H_{id} - \nabla H_i) \tag{9}$$

and in general, $\bar{u}_i$ can be designed such that

$$g_i \bar{u}_i = (J_i - R_i)\nabla H_{id} - (J_i - R_i)\nabla H_i, \quad i \in A_1 \tag{10}$$

which leads to the following match condition

$$g_i^{\perp}\Big((J_i - R_i)\nabla H_{id} - (J_i - R_i)\nabla H_i\Big) = 0 \tag{11}$$

for any choice of $\bar{u}_i$ such that (10) holds, where $g_i^{\perp}$ is a full rank left annihilator satisfying $g_i^{\perp} g_i = 0$.

Based on the above, with the energy shaping and damping injection method, we have the following result.

Theorem 1. Consider the PCH multi-agent system (2). For group A_1, suppose that the desired Hamiltonian function H_{id} and the shaping control law (pre-feedback) $\bar{u}_i$ satisfy (7) and (8), respectively. If the communication topology has a directed spanning tree, then under the following control law

$$u_i = \sum_{j \in \mathcal{N}_i} \Big((y_j + \delta_j d) - (y_i + \delta_i d)\Big) + \delta_i \bar{u}_i, \quad i = 1, \cdots, N \tag{12}$$

the PCH multi-agent system (2) is globally stable and all the agents can reach the group output consensus, where

$$\delta_i = \begin{cases} 1, & i \in A_1 \\ 0, & \text{otherwise} \end{cases}$$

Proof. With (8), substituting the control law (12) into (2) yields

$$\begin{aligned} \dot{x} = \; & (J - R)\nabla H - g(L \otimes I_m) y_d + g\left[\delta_1 \bar{u}_1^{\mathrm{T}}, \cdots, \delta_N \bar{u}_N^{\mathrm{T}}\right]^{\mathrm{T}} = \\ & (J - R)\nabla H_d - g(L \otimes I_m) y_d \end{aligned} \tag{13}$$

where $H = \sum_{i=1}^{N} H_i$, $H_d(x) = \sum_{i=1}^{N} H_{id}$, $y_d = \left[y_1^{\mathrm{T}} + \delta_1 d^{\mathrm{T}}, \cdots, y_N^{\mathrm{T}} + \delta_N d^{\mathrm{T}}\right]^{\mathrm{T}}$, and $g = \mathrm{diag}\{g_1, \cdots, g_N\}$. With (7), considering $H_d(x)$ as a Lyapunov function candidate of the entire system, the time derivative along the trajectory of (7) is

$$\begin{aligned} \dot{H}_d = \; & \nabla H_d^{\mathrm{T}} (J - R)\nabla H_d - \nabla H_d^{\mathrm{T}} g(L \otimes I_m) y_d = \\ & -\nabla H_d^{\mathrm{T}} R \nabla H_d - y_d^{\mathrm{T}} (L \otimes I_m) y_d \le 0 \end{aligned}$$

from which we know that the entire PCH multi-agent system is globally stable.

On the other hand, we know from Section 1 that if the communication topology has a directed spanning tree, then 0 is a simple eigenvalue of the Laplacian L with $\mathbf{1} = [1, \cdots, 1]^{\mathrm{T}}$ as the corresponding eigenvector. By setting $\dot{H}_d = 0$, it follows that $S = \{x \mid y_1 + \delta_1 d = y_2 + \delta_2 d = \cdots = y_N + \delta_N d\} \subset \{x \mid \dot{H}_d = 0\}$. By the LaSalle's invariance principle, the trajectory of the system converges to S. Thus, if $\forall i, j \in A_1$, $\delta_i = \delta_j = 1$, from which and set S,

$$\lim_{t \to \infty} \|y_i(t) - y_j(t)\| = 0, \quad \forall i, j \in A_1$$

if $\forall\, i, j \in A_2$, $\delta_i = \delta_j = 0$, from which

$$\lim_{t \to \infty} \|y_i(t) - y_j(t)\| = 0, \quad \forall i, j \in A_2$$

and if $\forall i \in A_1,\ j \in A_2$, then $\delta_i = 1$ and $\delta_j = 0$, from which

$$\lim_{t \to \infty} \|y_i(t) - y_j(t)\| = d, \quad \forall i \in A_1,\ j \in A_2$$

Therefore, the group output consensus is reached, and the proof is completed. □

To facilitate the analysis, we give several new definitions.

Definition 2. Consider a digraph $\mathcal{G} = \{\mathcal{V}, \varepsilon\}$. For node $i \in \mathcal{V}$, sets

$$\begin{aligned} \mathcal{N}_s^i &= \Big\{j \mid (v_j, v_i) \in \varepsilon, (v_i, v_j) \notin \varepsilon\Big\} \\ \mathcal{N}_d^i &= \Big\{j \mid (v_j, v_i) \in \varepsilon, (v_i, v_j) \in \varepsilon\Big\} \\ \mathcal{N}_p^i &= \Big\{j \mid (v_i, v_j) \in \varepsilon, (v_j, v_i) \notin \varepsilon\Big\} \end{aligned}$$

are called the single directional neighbor, double directional neighbor and potential neighbor of the i-th agent, respectively.

Remark 2. It is easy to see from Definition 2 that for any i, $j \in \mathcal{V}$, if $j \in \mathcal{N}_p^i$, then $i \in \mathcal{N}_s^j$.

Designing different control protocols for agents in the three different neighbors, we have the following result.

Theorem 2. Consider the PCH multi-agent system (2) with a fixed weakly connected graph. Then, under the control law

$$\begin{aligned} u_i = & \sum_{j \in \mathcal{N}_s^i} (2y_j - y_i) + \sum_{j \in \mathcal{N}_d^i} (y_j - y_i) - K_i y_i, \\ & i = 1, 2, \cdots, N \end{aligned} \tag{14}$$

the PCH multi-agent system (2) is globally stable and all the agents reach output consensus, where $K_i = |\mathcal{N}_p^i|$ stands for the number of the agents of $\mathcal{N}_p^i$.

Proof. Substituting (14) into agent i yields

$$\begin{aligned} \dot{x}_i = \; & (J_i - R_i)\nabla H_i + g_i \sum_{j \in \mathcal{N}_s^i} (2y_j - y_i) + \\ & g_i \sum_{j \in \mathcal{N}_d^i} (y_j - y_i) - g_i K_i y_i \end{aligned} \tag{15}$$

with which we obtain

$$\begin{aligned} \dot{x}_i = \; & (J_i - R_i)\nabla H_i - \sum_{j \in \mathcal{N}_p^i} g_i (y_j - y_i) + \sum_{j \in \mathcal{N}_p^i} g_i (y_j - y_i) + \\ & g_i \sum_{j \in \mathcal{N}_s^i} (2y_j - y_i) + g_i \sum_{j \in \mathcal{N}_d^i} (y_j - y_i) - g_i K_i y_i = \\ & (J_i - R_i)\nabla H_i + K_i g_i g_i^{\mathrm{T}} \nabla H_i - \sum_{j \in \mathcal{N}_p^i} g_i g_j^{\mathrm{T}} \nabla H_j + \end{aligned}$$

$$\sum_{j\in\mathcal{N}_p^i} g_i(y_j-y_i)+g_i\sum_{j\in\mathcal{N}_s^i} y_j+g_i\sum_{j\in\mathcal{N}_s^i}(y_j-y_i)+$$
$$g_i\sum_{j\in\mathcal{N}_d^i}(y_j-y_i)-g_iK_ig_i^{\mathrm{T}}\nabla H_i=$$
$$(J_i-R_i)\nabla H_i-\sum_{j\in\mathcal{N}_p^i} g_ig_j^{\mathrm{T}}\nabla H_j+\sum_{j\in\mathcal{N}_s^i} g_ig_j^{\mathrm{T}}\nabla H_j+$$
$$g_i\sum_{j\in\mathcal{N}_d^i\cup\mathcal{N}_p^i\cup\mathcal{N}_s^i}(y_j-y_i) \tag{16}$$

Let $H(x)=\sum_{i=1}^N H_i(x_i)$, $x=[x_1^{\mathrm{T}},\cdots,x_N^{\mathrm{T}}]^{\mathrm{T}}$, $y=\left[y_1^{\mathrm{T}},\cdots,y_N^{\mathrm{T}}\right]^{\mathrm{T}}$, and $R(x)=\mathrm{diag}\{R_1,R_2,\cdots,R_N\}$. Then, by Remark 2, the entire closed-loop system can be expressed as

$$\dot{x}=[\tilde{J}(x)-R(x)]\nabla H-g(\tilde{L}\otimes I_m)y \tag{17}$$

where

$$\tilde{J}(x)=\begin{bmatrix} J_1 & \delta_{12}g_1g_2^{\mathrm{T}} & \cdots & \delta_{1N}g_1g_N^{\mathrm{T}}\\ \delta_{21}g_2g_1^{\mathrm{T}} & J_2 & \cdots & \delta_{2N}g_2g_N^{\mathrm{T}}\\ \vdots & \vdots & \ddots & \vdots\\ \delta_{N1}g_Ng_1^{\mathrm{T}} & \delta_{N2}g_Ng_2^{\mathrm{T}} & \cdots & J_N\end{bmatrix}$$
$$g=\mathrm{diag}\{g_1,g_2,\cdots,g_N\}$$

$$\delta_{ij}=\begin{cases}-1, & j\in\mathcal{N}_p^i\\ 1, & j\in\mathcal{N}_s^i\\ 0, & \text{otherwise}\end{cases} \tag{18}$$

and $\tilde{L}$ is the Laplacian of the connected undirected graph obtained by adding potential neighbors to each agent.

It is easy to see from (18) that $\delta_{ij}=-\delta_{ji}$, and thus $\tilde{J}^{\mathrm{T}}=-\tilde{J}$. Moreover, matrix $\tilde{L}$ can be decomposed as $\tilde{L}=\tilde{D}\tilde{D}^{\mathrm{T}}$ with $\tilde{D}$ being the corresponding incidence matrix.

Choosing H as a Lyapunov function candidate, we have

$$\dot{H}=-\nabla H^{\mathrm{T}}R(x)\nabla H-y^{\mathrm{T}}(\tilde{L}\otimes I_m)y=$$
$$-\nabla H^{\mathrm{T}}R(x)\nabla H-\|(\tilde{D}\otimes I_m)^{\mathrm{T}}y\|^2\le 0$$

from which we know that the entire PCH multi-agent system is globally stable.

Considering the set $S=\{x\mid \dot{H}(x)=0\}=\{x\mid \nabla H^{\mathrm{T}}R(x)\nabla H=0,\ (\tilde{D}\otimes I_m)^{\mathrm{T}}y=0\}$, and using the LaSalle′s invariant principle and the connectivity of the digraph, we obtain

$$\lim_{t\to\infty}\|y_i-y_j\|=0,\quad \forall i,j=1,\cdots,N$$

and thus the proof is completed. □

Remark 3. In particular, the proposed control u_i forces the multi-agent system to reach the output consensus even if the network is weakly connected, while it requires the knowledge of the cardinality of the potential neighbor set of the i-th agent. When the multi-agent systems use visions as the communication devices, this control algorithm may not be used.

Remark 4. If the topology of the PCH multi-agent system (14) is an undirected graph, then the agreement control law (14) becomes as follows

$$u_i=\sum_{j\in\mathcal{N}_i}(y_j-y_i),\qquad i=1,\cdots,N \tag{19}$$

Remark 5. If the above graph is balanced, then $|N_s^i|=|N_p^i|$, with which the control law (2) becomes

$$u_i=2\sum_{j\in\mathcal{N}_i\backslash\mathcal{N}_d^i}(y_j-y_i)+\sum_{j\in\mathcal{N}_d^i}(y_j-y_i) \tag{20}$$

This protocol can be regarded as a weighting protocol, where 1 is for the undirected information and 2 for the unidirectional one.

2.2 Results for switching topology

In this subsection, we investigate the output consensus problem of the PCH multi-agent system (2) with its topology switching. By the extended LaSalle′s invariance principle for switched systems[25−28] and based on Subsection 2.1, we present several control protocols for this case.

Consider the PCH multi-agent system (2) with a switching topology $\mathcal{G}$. Assume that the switching is caused by a switching signal: $\sigma(t):[0,+\infty)\to\Lambda=\{1,2,\cdots,l\}$, which is a piecewise constant right continuous function. For each $i\in\Lambda$, the resulted corresponding topology is assumed to be a simple graph, denoted by $\mathcal{G}_i=\{\mathcal{V},\varepsilon_i\}$. Moreover, the union of a collection of graphs $\{\mathcal{G}_1,\mathcal{G}_2,\cdots,\mathcal{G}_l\}$ is defined as $\cup\mathcal{G}_i=\{\mathcal{V},\ \cup\varepsilon_i\}$. If the union is a connected graph, then the switching topology is called jointly connected[29]. It is noted that a switching topology can be jointly connected even if none of its members are connected[29]. Accordingly, neighbors $\mathcal{N}_i$, N_s^i, N_d^i and N_p^i of the i-th agent defined in Subsection 2.1 will be modified as $\mathcal{N}_i(t)$, $N_s^i(t)$, $N_d^i(t)$ and $N_p^i(t)$, respectively.

To this end, we give a realistic assumption and a lemma first.

Assumption 1. The minimal dwell time τ_0 of the switching topology $\mathcal{G}$ satisfies $\tau_0>0$. Moreover, for any $T>0$ and any $\lambda\in\Lambda$, there exists $t>T$ such that $\sigma(t)=\lambda$.

Lemma 1[28]. Assume that the switched system

$$\dot{x}=f_{\sigma(t)}(x),\quad x\in\mathbf{R}^n \tag{21}$$

has a common weak Lyapunov function $V(x)$. Then, every solution of the switched system is attracted to the largest weakly invariant set contained in $\bigcup_{\sigma\in\Lambda}Z_\sigma$, where $Z_\sigma=\{x\ :\ \dot{V}(x)|_{f_\sigma}=0\}$, $\forall\sigma\in\Lambda$.

For the PCH multi-agent system (2) with the switching topology $\mathcal{G}$, we have the following result on the output consensus.

Theorem 3. Consider the PCH multi-agent system (2) with the dynamically changed topology $\mathcal{G}$. Suppose Assumption 1 holds and $\mathcal{G}$ is jointly connected. Then, under the control law

$$u_i=\sum_{j\in\mathcal{N}_s^i(t)}(2y_j-y_i)+\sum_{j\in\mathcal{N}_d^i(t)}(y_j-y_i)-|\mathcal{N}_p^i(t)|y_i,$$
$$i=1,\cdots,N \tag{22}$$

the entire system is globally stable and all the agents can reach output consensus.

Proof. Substituting (22) into the PCH multi-agent system (2), and using the similar argument to the proof of Theorem 2, we have

$$\dot{x}=(J_\sigma(x)-R(x))\nabla H(x)-g(L_\sigma\otimes I_m)y \tag{23}$$

where

$x=[x_1^{\mathrm{T}},\cdots,x_N^{\mathrm{T}}]^{\mathrm{T}}$, $y=[y_1^{\mathrm{T}},\cdots,y_N^{\mathrm{T}}]^{\mathrm{T}}$, $H(x)=\sum_{i=1}^N H_i$

(x_i), $g = \mathrm{diag}\{g_1, g_2, \cdots, g_N\}$, $R = \mathrm{diag}\{R_1, R_2, \cdots, R_N\}$,

$$J_\sigma = \begin{bmatrix} J_1 & \delta_{12}^\sigma g_1 g_2^{\mathrm{T}} & \cdots & \delta_{1N}^\sigma g_1 g_N^{\mathrm{T}} \\ \delta_{21}^\sigma g_2 g_1^{\mathrm{T}} & J_2 & \cdots & \delta_{2N}^\sigma g_2 g_N^{\mathrm{T}} \\ \vdots & \vdots & \ddots & \vdots \\ \delta_{N1}^\sigma g_N g_1^{\mathrm{T}} & \delta_{N2}^\sigma g_N g_2^{\mathrm{T}} & \cdots & J_N \end{bmatrix} \tag{24}$$

L_σ is the Lapalacian matrix of the resulted undirected graph by adding the potential neighbor to $\mathcal{G}_\sigma$, and L_σ can be decomposed as $L_\sigma = D_\sigma D_\sigma^{\mathrm{T}}$.

By choosing $H(x)$ as a common Lyapunov function candidate, its time derivative along the trajectory of (23) is

$$\begin{aligned} \dot{H}(x) = & -\nabla H^{\mathrm{T}} R(x) \nabla H - \nabla H^{\mathrm{T}} g (L_\sigma \otimes I_m) y = \\ & -\nabla H^{\mathrm{T}} R(x) \nabla H - y^{\mathrm{T}} (L_\sigma \otimes I_m) y = \\ & -\nabla H^{\mathrm{T}} R(x) \nabla H - \|(D_\sigma \otimes I_m)^{\mathrm{T}} y\|^2 \le 0 \end{aligned}$$

from which we know that $H(x)$ is a common Lyapunov function of the entire system (23) and thus the closed-loop system is globally stable. Let

$$\begin{aligned} Z_\sigma = & \ \{x \mid \dot{H}(x) = 0\} = \\ & \ \{x \mid \nabla H^{\mathrm{T}} R(x) \nabla H = 0,\ (D_\sigma \otimes I_m)^{\mathrm{T}} y = 0\} \end{aligned}$$

According to Lemma 1, the trajectory converges to

$$\bigcup_{\sigma \in \Lambda} Z_\sigma = \left\{x \mid \nabla H^{\mathrm{T}} R(x) \nabla H = 0,\ \cup_{\sigma \in \Lambda} (D_\sigma \otimes I_m)^{\mathrm{T}} y = 0\right\}$$

Therefore, from the joint connectivity of the switching topology $\mathcal{G}$, we have

$$\lim_{t \to \infty} \|y_i(t) - y_j(t)\| = 0, \quad \forall i, j = 1, \cdots, N$$

and thus the proof is completed. □

When $\mathcal{G}$ is a switching undirected graph, we have the following corollary.

Corollary 1. Consider the PCH multi-agent system (2) with the switching undirected topology $\mathcal{G}$. Suppose Assumption 1 holds and the topology is jointly connected. Then, under the control law

$$u_i = \sum_{j \in \mathcal{N}_i(t)} (y_j - y_i), \quad i = 1, \cdots, N \tag{25}$$

the entire system is globally stable and all the agents can reach output consensus.

3 Illustrative examples

In this section, we give several illustrative examples with simulations to demonstrate the effectiveness of the protocols designed in this paper.

Example 1. Consider the following PCH multi-agent system

$$\begin{cases} \dot{x}_i = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \nabla H_i(x_i) + g_i(x_i) u_i \\ y_i = g_i^{\mathrm{T}}(x_i) \nabla H_i(x_i), \qquad i = 1, 2, \cdots, n \end{cases} \tag{26}$$

where $x_i \in \mathbf{R}^3$, $g_i \in \mathbf{R}^3$,

$$H_i(x_i) = \frac{x_{i1}^2}{2L_i} + \frac{x_{i2}^2}{2M_i} + \frac{x_{i3}^2}{2N_i}$$

and L_i, M_i, N_i are positive real numbers.

In the following, we use Theorem 1 to design a group output consensus protocol for system (26) with $n = 7$.

The communication graph of the seven agents is shown in Fig. 1. We divide the seven agents into two groups: $A_1 = \{1, 2, 3, 4, 5\}$ and $A_2 = \{6, 7\}$, and the desired distance of the two groups is $d = 6$.

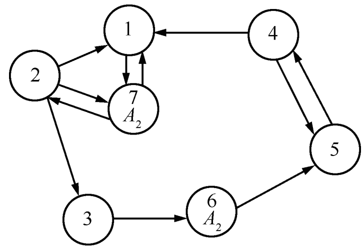

Fig. 1 The topology of Example 1

Let $L_1 = 1$, $M_1 = 1$, $N_1 = 1$, $g_1 = [0\ 1\ 0]^{\mathrm{T}}$; $L_2 = 1$, $M_2 = 2$, $N_2 = 3$, $g_2 = [1\ 0\ 3]^{\mathrm{T}}$; $L_3 = 1$, $M_3 = 3$, $N_3 = 4$, $g_3 = [1\ 3\ 4]^{\mathrm{T}}$; $L_4 = 4$, $M_4 = 4$, $N_4 = 5$, $g_4 = [1\ 0\ 5]^{\mathrm{T}}$; $L_5 = 2$, $M_5 = 2$, $N_5 = 1$, $g_5 = [2\ 0\ 1]^{\mathrm{T}}$; $L_6 = 1$, $M_6 = 3$, $N_6 = 4$, $g_6 = [1\ 3\ 0]^{\mathrm{T}}$; $L_7 = 1$, $M_7 = 4$, $N_7 = 5$, $g_7 = [1\ 0\ 5]^{\mathrm{T}}$.

By choosing

$$H_{6d} = \frac{x_{61}^2}{2} + \frac{(x_{62} + 6)^2}{6} + \frac{(x_{63} + 8)^2}{8}$$

we have $\nabla H_{6d} - \nabla H_6 = [0\ 2\ 2]^{\mathrm{T}}$ and $g_6(\nabla H_{6d} - \nabla H_6)^{\mathrm{T}} = d$. From (9), we obtain $\bar{u}_6 = 0$. Similarly, by choosing

$$H_{7d} = \frac{(x_{71} + 1)^2}{2} + \frac{(x_{72} - 16)^2}{8} + \frac{(x_{73} + 5)^2}{10}$$

it is easy to obtain $\bar{u}_7 = 0$. Therefore, according to Theorem 1, the desired group output consensus protocol can be given as

$$\begin{cases} u_1 = y_4 - y_1 + y_2 - y_1 \\ u_2 = y_1 - y_2 + y_7 + 6 - y_2 \\ u_3 = y_2 - y_3 \\ u_4 = y_5 - y_4 \\ u_5 = y_6 + 6 - y_5 \\ u_6 = y_3 - (y_6 + 6) \\ u_7 = y_1 - (y_7 + 6) \end{cases} \tag{27}$$

To demonstrate the effectiveness of the protocol in (27), we carry out some simulation with the following initial conditions: $x_1^{(0)} = [1, 1, 1]^{\mathrm{T}}$, $x_2^{(0)} = [1, 1, 2]^{\mathrm{T}}$, $x_3^{(0)} = [1, 1, 2]^{\mathrm{T}}$, $x_4^{(0)} = [1, 1, 2]^{\mathrm{T}}$, $x_5^{(0)} = [1, 1, 2]^{\mathrm{T}}$, $x_6^{(0)} = [0, 1, 3]^{\mathrm{T}}$, and $x_7^{(0)} = [1, 1, 2]^{\mathrm{T}}$. The simulation results are shown in Fig. 2, which are the responses of the differences between the outputs of the seven agents.

It can be observed from Fig. 2 that the output signals reach the group output consensus and two groups keep the desired distance eventually. Simulation shows that the protocol in (27) is very effective in the group output consensus control of system (26).

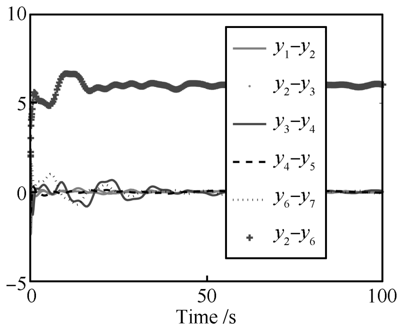

Fig. 2 The simulation results of Example 1

Example 2. Consider system (26) with $n = 7$, whose communication graph is assumed to be given in Fig. 3.

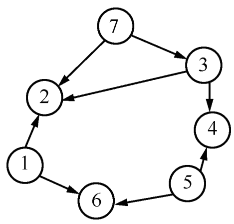

Fig. 3 The topology of Example 2

In the following, we use Theorem 2 to design the protocol.

According to Definition 2, it is easy to see that $N_s^1 = \emptyset$, $N_d^1 = \emptyset$, $N_p^1 = \{2, 6\}$; $N_s^2 = \{3, 7, 1\}$, $N_d^2 = \emptyset$, $N_p^2 = \emptyset$; $N_s^3 = \{7\}$, $N_d^3 = \emptyset$, $N_p^3 = \{2, 4\}$; $N_s^4 = \{3, 5\}$, $N_d^4 = \emptyset$, $N_p^4 = \emptyset$; $N_s^5 = \emptyset$, $N_d^5 = \emptyset$, $N_p^5 = \{4, 6\}$; $N_s^6 = \{1, 5\}$, $N_d^6 = \emptyset$, $N_p^6 = \emptyset$; $N_s^7 = \emptyset$, $N_d^7 = \emptyset$, $N_p^7 = \{2, 3\}$. Then, using Theorem 3, the desired output consensus protocol can be given as

$$\left\{\begin{array}{l} u_1 = -2y_1 \\ u_2 = 2y_1 + 2y_3 - 3y_2 + 2y_7 \\ u_3 = -3y_3 + 2y_7 \\ u_4 = 2y_3 + 2y_5 - 2y_4 \\ u_5 = -2y_5 \\ u_6 = 2y_5 + 2y_1 - 2y_6 \\ u_7 = -2y_7 \end{array}\right. \tag{28}$$

To demonstrate the effectiveness of the protocol in (28), we carry out some simulation with the following choices: Initial conditions: $x_1^{(0)} = [1, 2, 3]^{\rm T}$, $x_2^{(0)} = [2, 3, 4]^{\rm T}$, $x_3^{(0)} = [1, 1, 1]^{\rm T}$, $x_4^{(0)} = [2, 3, 1]^{\rm T}$, $x_5^{(0)} = [2, 2, 4]^{\rm T}$, $x_6^{(0)} = [3, 4, 5]^{\rm T}$, $x_7^{(0)} = [4, 5, 6]^{\rm T}$; parameters: $L_1 = 1$, $M_1 = 1$, $N_1 = 1$, $g_1 = [0\ 1\ 0]^{\rm T}$, $L_2 = 1$, $M_2 = 2$, $N_2 = 3$, $g_2 = [1\ 0\ 3]^{\rm T}$, $L_3 = 1$, $M_3 = 3$, $N_3 = 4$, $g_3 = [1\ 3\ 4]^{\rm T}$, $L_4 = 4$, $M_4 = 4$, $N_4 = 5$, $g_4 = [1\ 0\ 5]^{\rm T}$; $L_5 = 2$, $M_5 = 2$, $N_5 = 1$, $g_5 = [2\ 0\ 1]^{\rm T}$, $L_6 = 1$, $M_6 = 3$, $N_6 = 4$, $g_6 = [1\ 3\ 0]^{\rm T}$, $L_7 = 1$, $M_7 = 4$, $N_7 = 5$, $g_7 = [1\ 0\ 5]^{\rm T}$.

The simulation results are shown in Fig. 4, which are the responses of the differences between the outputs of the seven agents. It can be observed from Fig. 4 that the output signals reach the output consensus eventually and the protocol in (27) is very effective.

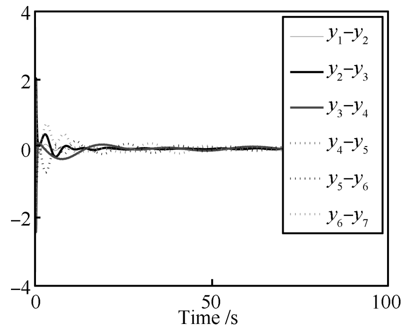

Fig. 4 The simulation results of Example 2

Example 3. Consider system (26) with $n = 5$, whose communication topology is assumed to switch between graphs (a) and (b) randomly, which is shown in Fig. 5.

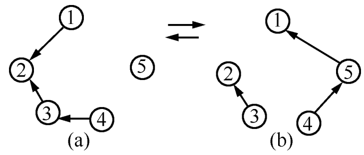

Fig. 5 The switching communication topology of Example 3

In the following, we use Theorem 2 to design the protocol.

According to Definition 2, it is easy to see that for the topology graph (a): $N_s^1 = \emptyset$, $N_d^1 = \emptyset$, $N_p^1 = \{2\}$; $N_s^2 = \{3, 1\}$, $N_d^2 = \emptyset$, $N_p^2 = \emptyset$; $N_s^3 = \{4\}$, $N_d^3 = \emptyset$, $N_p^3 = \{2\}$; $N_s^4 = \emptyset$, $N_d^4 = \emptyset$, $N_p^4 = \{3\}$; $N_s^5 = \emptyset$, $N_d^5 = \emptyset$, $N_p^5 = \emptyset$; and for the topology (b), $N_s^1 = \{5\}$, $N_d^1 = \emptyset$, $N_p^1 = \emptyset$; $N_s^2 = \{3\}$, $N_d^2 = \emptyset$, $N_p^2 = \emptyset$; $N_s^3 = \emptyset$, $N_d^3 = \emptyset$, $N_p^3 = \{2\}$; $N_s^4 = \emptyset$, $N_d^4 = \emptyset$, $N_p^4 = \{5\}$; $N_s^5 = \{4\}$, $N_d^5 = \emptyset$, $N_p^5 = \{1\}$. Based on the above and using Theorem 3, the desired output consensus protocol can be designed as follows.

Case 1. When the communication topology is (a),

$$\begin{array}{l} u_1 = -y_1, u_2 = 2y_1 - 3y_2 + 2y_3 \\ u_3 = 2y_4 - 2y_3, u_4 = -y_4, u_5 = 0 \end{array} \tag{29}$$

Case 2. When the communication topology is (b),

$$\begin{array}{l} u_1 = 2y_5 - y_1, u_2 = 2y_3 - y_2 \\ u_3 = y_3, u_4 = -y_4, u_5 = 2y_4 - 2y_5 \end{array} \tag{30}$$

To demonstrate the effectiveness of the protocol in (29) and (30), we carry out some simulation with the following choices: initial conditions: $x_1^{(0)} = [2, 1, -3]^{\rm T}$, $x_2^{(0)} = [2, 1, 3]^{\rm T}$, $x_3^{(0)} = [4, 1, 1]^{\rm T}$, $x_4^{(0)} = [1, 0, 1]^{\rm T}$, $x_5^{(0)} = [5, 6, 7]^{\rm T}$; parameters: $L_1 = 1$, $M_1 = 1$, $N_1 = 1$, $g_1 = [0\ 1\ 0]^{\rm T}$; $L_2 = 1$, $M_2 = 2$, $N_2 = 3$, $g_2 = [1\ 0\ 3]^{\rm T}$; $L_3 = 1$, $M_3 = 3$, $N_3 = 4$, $g_3 = [1\ 3\ 4]^{\rm T}$; $L_4 = 4$, $M_4 = 4$, $N_4 = 5$, $g_4 = [1\ 0\ 5]^{\rm T}$; $L_5 = 2$, $M_5 = 2$, $N_5 = 1$, $g_5 = [2\ 0\ 1]^{\rm T}$; topology switching: random path generated by the random function of Matlab. The dwell time is at random. The simulation results are shown in Fig. 6, which are the responses of the differences between the outputs of the five agents. It can be observed from Fig. 6 that the output signals reach the output consensus eventually. Simulation shows that the protocol in (30) is very effective.

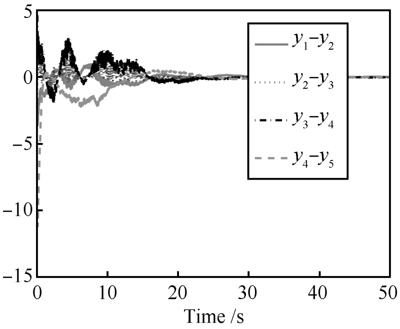

Fig. 6 The simulation result of Example 3

4 Conclusion

In this paper, we have investigated the output consensus problem of port-controlled Hamiltonian (PCH) multi-agent systems with both fixed and switching topologies. Using the structural properties of the PCH systems as well as the extended LaSalle′s invariance principle, several new distributed control protocols have been presented. It is shown that our new agreement protocols can solve the output consensus problem even for weakly connected unbalanced graph in the case of fixed topology. Under the jointly connected topology condition, our designed control protocol can make all the agents reach output consensus when the topology is switching. Furthermore, the group output consensus problem of the PCH multi-agent system has also been studied via the energy shaping method, and a distributed group output consensus protocol has been proposed. The study of several illustrative examples with simulations has shown the protocols designed in this paper work very well.

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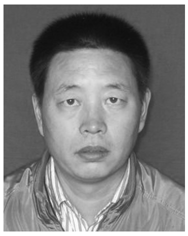

LI Chang-Sheng Received his master degree in applied mathematics from Shandong University of Science and Technology in 2004. His research interest covers nonlinear control, Hamiltonian systems, and multi-agent systems. Corresponding author of this paper. E-mail: lcsheng2004@163.com

WANG Yu-Zhen Graduated from Tai′an Teachers College in 1986, received his master degree from Shandong University of Science Technology in 1995 and his Ph. D. degree from the Institute of Systems Science, Chinese Academy of Sciences in 2001. From 2001 to 2003, he worked as a postdoctoral fellow in Tsinghua University, China. Since 2003, he is a professor with the School of Control Science and Engineering, Shandong University, China, and the director of the Institute of Automatic Control, Shandong University. From March 2004 to June 2004, from February 2006 to May 2006 and from November 2008 to January 2009, he visited City University of Hong Kong as a research fellow. From September 2004 to May 2005, he worked as a visit research fellow at the National University of Singapore.

His research interest covers nonlinear control systems, Hamiltonian systems, Boolean networks, and multi-agent systems.
E-mail: yzwang@sdu.edu.cn