Distributed Adaptive Tracking Control for Unknown Nonlinear Networked Systems

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Abstract In this paper, we investigate the cooperative tracking problem for a class of nonlinear uncertain networked systems subject to an active leader, whose state can merely be partially measured and input channel is disturbed as well. By virtue of neural network (NN) technique, the dynamics of followers are properly modeled on certain basis functions and their input channels are assumed to be disturbed as well. In this work, an observer-based adaptive control is proposed for the nonlinear networked systems which may have non-identical dynamics. It is shown via Lyapunov theory that the overall system is cooperatively uniformly ultimately bounded (UUB) by appropriately choosing the parameters under some graph condition. In the end, several numerical simulations are elaborated for validation of the proposed adaptive controller.

Key words Cooperative control, tracking control, nonlinear networked system, neural network (NN), adaptive control

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The study of multi-agent system is inspired by some natural phenomena, such as herd of land animals, swarm of bees, flocking of birds and school of fish. Great attention has been attracted to worldwide researchers in engineering, economic, and biology, etc[1]. Particularly among them, cooperative control algorithm plays a key role in the decision making process.

Compared with centralized cooperative control, distributed cooperative control takes more advantages of robustness, flexibility, and scalability[2]. In recent years, consensus-based distributed cooperative control has been sufficiently explored. In these works, the communication graph is considered as either directed or undirected[3−4] in presence of fixed, varying topologies[5], or communication delays[6]. Moreover, most of approaches are derived from a stability perspective, whereas [7] provided a unified optimal approach for the consensus problem as well as the obstacle avoidance.

Traditional models of interest in multi-agent system are autonomous mobile vehicles, unmanned air vehicles or robot manipulator, to name a few. However, the dynamic of the agent in the literature is mostly assumed linear and known for the ease of analysis, for example in [2] and [5], the agent is described as an integrator, either first-order or second-order. It is sometimes impractical due to the nonlinear nature of the real environment[8−9]. In addition, cooperative control of multi-agent systems with non-identical and highly nonlinear dynamics is currently a more challenging problem. It becomes more difficult to handle when they are all required to distributedly track an active leader with partial measurement and disturbance. Das et al.[10] investigated the problem but they did not consider the case that leader and followers have different orders. Whereas, Hong et al.[11] considered the tracking problem but the followers in their paper were considered as single-integrator dynamics. Moreover, the controllers in [11] required each follower to know the leader′s acceleration information even they were not connected. In [12], a unified inverse optimal control approach was firstly proposed for the cooperative tracking problem as well as obstacle avoidance. The issue was the same as [11, 13−14], which assumed homogeneous integrator dynamics.

In this paper, we consider the leader-follower cooperative tracking problem in both strongly connected and spanning tree cases, in which the active leader′s position information is sent to the followers but without velocity information. The follower can get the leader′s position if there is a connection between them. The followers have unknown nonlinear dynamics, which may be non-identical and be disturbed in the input channel. The equivalent dynamics will be attained on certain basis by virtue of neural network (NN) modeling technique in [15].

The contributions of this work can be summarized as follows: Firstly, a distributed velocity estimator is designed for each follower to estimate the active leader′s velocity, which extends [11]′s result to diagraph and removes the constraint that each follower must know the leader′s information. Then, a distributed adaptive controller is designed to guarantee the overall networked system being cooperatively uniformly ultimately bounded (UUB), when the leader and followers are in different order, which extends [10]′s result. Also, a distributed adaptive disturbance compensator is considered as well.

The rest of paper is organized as follows. Section 1 provides some preliminaries and Section 2 formulates the problem. In what follows, the main result of this paper is presented in Section 3. Section 4 provides several numerical examples to illustrate the effectiveness of proposed adaptive controller and Section 5 concludes this paper and points out the future direction.

1 Preliminary

1.1 Matrix basis

Throughout this paper, $\mathbf{R}^{m\times n}$ denotes the family of $m\times n$ real matrices and I_n is $n\times n$ identity matrix, $M>(<)\ 0$ means that M is a positive (negative) definite matrix. Its largest eigenvalue is denoted by $\lambda_{\max}(M)$ and smallest eigenvalue is denoted by $\lambda_{\min}(M)$. $\bar{\sigma}(M)$ and $\underline{\sigma}(M)$ denote the maximum and minimum singular value of matrix M. The Frobenius norm of matrix M is $\|M\|_F=\mathrm{tr}\sqrt{M^{\mathrm{T}}M}$ with $\mathrm{tr}\{\cdot\}$ being the trace of a matrix.

Definition 1[9]**.** Let Ξ denote the set of square matrices whose off-diagonal elements are non-positive, then matrix $\mathcal{M}$ is called a non-singular M matrix if $\mathcal{M}\in\Xi$ and all its principal minors are positive.

1.2 Graph theory

A weighted graph is denoted by $\mathcal{G}=(\mathcal{V},\mathcal{E})$, where $\mathcal{V}=\{v_1,v_2,\cdots,v_N\}$ is a nonempty finite set of N nodes, an edge set $\mathcal{E}\subseteq\mathcal{V}\times\mathcal{V}$ is used to model the communica-

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tions among agents. A weighted adjacent matrix $A = [a_{ij}] \in \mathbf{R}^{N\times N}$, $a_{ii} = 0, \forall i$ and $i \neq j$, $a_{ij} > 0$ if $(v_i, v_j) \in \mathcal{E}$ and 0 otherwise. The neighbor set of node i is denoted by $\mathcal{N}_i = \{j|j \in \mathcal{V}, (i,j) \in \mathcal{E}\}$. $j \notin \mathcal{N}_i$ means that there is no information flow from node j to node i. In undirected graph, $a_{ij} = a_{ji}$. Define the in-degree of node i as $d_i = \sum_j a_{ij}$ and $D = \mathrm{diag}\{d_i\} \in \mathbf{R}^{N\times N}$ is the in-degree matrix. Then, the Laplacian matrix of graph $L = D - A$. Let $\mathbf{1}_N = [1, 1, \cdots, 1]^{\mathrm{T}} \in \mathbf{R}^N$, it is well-known that 0 is one of eigenvalues of the Laplacian matrix L associated with the eigenvector $\mathbf{1}_N$. $B = \mathrm{diag}\{b_i\} \in \mathbf{R}^{N\times N}$ with $b_i > 0$ if there is a connection between agent i and leader, otherwise, $b_i = 0$.

In a digraph, a sequence of successive edges in the form $\{(v_i, v_k), (v_k, v_l), \cdots, (v_m, v_j)\}$ is a directed path from node i to node j. An undirected path is defined similarly. A digraph is said to have a spanning tree if there is a node (called the root) and there is a path from the root to any other node in the graph; it is strongly connected if there is a path from node i to node j, for all the distinct nodes $v_i, v_j \in \mathcal{V}$. For the undirected graph, it is said to be connected if there is a path from node i to node j, for all the distinct nodes $v_i, v_j \in \mathcal{V}$.

1.3 Neural network modeling

It is stated in [15] that multilayers feedforward neural networks is a class of universal approximation functions, which can be used to approximate a smooth nonlinear function in a compact set with any degree of accuracy by a set of certain basis functions in the neural network provided that sufficient hidden units are available.

2 Problem formulation

Most existing works like [3−4, 10] assumed that the dynamics of leader and followers were all first-order integrators. In this case, due to lack of velocity information, dynamical consensus cannot be reached, i.e., $\lim_{t\to\infty} x_i(t) = \lim_{t\to\infty} x_j(t) = f(t) \neq c$, where c is a constant and $f(t)$ is a function of time t. In order to solve the dynamical consensus problem, we consider the dynamics of leader and followers with different orders.

Consider a network of N agents with the dynamic of agent i being

$$\dot{x}_i = f_i(x_i) + u_i + n_i(t) \tag{1}$$

in which $u_i \in \mathbf{R}$ is control input of agent i, $f_i(x_i) \in \mathbf{R}$ is an unknown nonlinear function, $n_i(t) \in \mathbf{R}$ is the bounded disturbance in the input channel.

The active leader is

$$\begin{cases} \dot{x}_0 = v_0 \\ \dot{v}_0 = a_0(t) + \delta(t) \end{cases} \tag{2}$$

where $x_0 \in \mathbf{R}$, $v_0 \in \mathbf{R}$, $a_0(t) \in \mathbf{R}$ are the position, velocity, and acceleration of the leader, respectively. $\delta(t)$ is the bounded disturbance in the acceleration channel. x_0 can be sent to agent i once there is a connection between them whereas v_0 is assumed unmeasurable in this paper.

By virtue of the NN techniques[15], assume the unknown nonlinearity $f_i(x_i)$ in (1) is locally smooth and thus it can be approximated in a compact set by

$$f_i(x_i) = \boldsymbol{W}_i^{*\mathrm{T}} \boldsymbol{\varphi}_i(x_i) + \xi_i \tag{3}$$

where $\boldsymbol{\varphi}_i(x_i) \in \mathbf{R}^{l_i}$ is a set of basis and $\boldsymbol{W}_i^* \in \mathbf{R}^{l_i}$ is a set of coefficients, $\xi_i \in \mathbf{R}$ is the approximation error. According to the Weierstrass approximation theory, given any approximation error bound ξ_M, there always exist a suitable set of $\boldsymbol{\varphi}_i(x_i)$ and $\boldsymbol{W}_i^*$ such that $|\xi_i| \leq \xi_M$. Just for simplicity of notation, we assume $l_i = l_j = l$ in this paper and rewrite (1) as

$$\dot{x}_i = \boldsymbol{W}_i^{*\mathrm{T}} \boldsymbol{\varphi}_i(x_i) + u_i + n_i(t) + \xi_i \tag{4}$$

The following standard assumptions are required in this paper. The following bounds requirements do not show up in the adaptive controllers, they just appear in the proof of the main results.

Assumption 1.

1) $a_0(t)$, $\delta(t)$ are bounded signals with $\|a_0(t)\| \leq a_M$, $\|\delta(t)\| \leq \delta_M$ where a_M, δ_M are positive constants.

2) $n_i(t)$ is a bounded disturbance with bounded derivative $\forall i$, $|n_i(t)| \leq n_M, |\dot{n}_i(t)| \leq n_h$, where n_M, n_h are positive constants.

3) $\boldsymbol{W}_i^*$, $\boldsymbol{\varphi}_i$, and ξ_i are bounded by W_M, φ_M, and ξ_M such that $\forall i$, $\|\boldsymbol{W}_i^*\| \leq W_M$, $\|\boldsymbol{\varphi}_i\| \leq \varphi_M$, and $|\xi_i| \leq \xi_M$, respectively, where W_M, φ_M, ξ_M are positive constants.

With the above assumptions, the control object of this paper is ready to be shown in the next definition.

Definition 2[10]**.** The nonlinear networked system is cooperatively uniformly ultimately bounded (UUB) with respect to the solutions of node dynamics (4) and (2) if there exists a compact set $\Theta \subset \mathbf{R}$ such that $\forall\, (x_i(t_0) - x_0(t_0)) \in \Theta$, there exists a bound $\mathcal{B}$ and a time $t_f(\mathcal{B}, (x_i(t_0) - x_0(t_0)))$, both independent of $t_0 \geq 0$, such that $\forall\, i$ and $\forall\, t \geq t_0 + t_f$, $\|x_i(t) - x_0(t)\| \leq \mathcal{B}$.

3 Main result

In this section, the main result of this paper will be presented. The communication graph investigated in this paper could be strongly connected or having a spanning tree. We will firstly focus on the strongly connected graph and thus the following lemmas are straightforward.

Lemma 1[11]**.** $H = L + B \in \mathbf{R}^{N\times N}$, all the eigenvalues of H have positive real parts if the digraph is strongly connected and there is at least one $b_i \neq 0$ in diagonal matrix B.

Lemma 2[9]**.** Define $P = \mathrm{diag}\{p_i\} \in \mathbf{R}^{N\times N}$ with $p_i = 1/q_i$ and $\boldsymbol{q} = [q_1, q_2, \cdots, q_N]^{\mathrm{T}} = (L + B)^{-1}\mathbf{1}_N$, then P and $Q = (L + B)^{\mathrm{T}} P + P(L + B)$ are positive definite matrices if $(L + B)$ is non-singular M matrix.

Definition 3. The neighborhood synchronization error of agent i is

$$e_{ix} = \sum_{j\in\mathcal{N}_i} a_{ij}(x_i - x_j) + b_i(x_i - x_0) \tag{5}$$

The main result of the paper details how to design distributed control u_i to guarantee that all the nodes in the network are synchronized to the active leader with a bounded tracking error, i.e., the networked system is cooperatively UUB.

Consider the following distributed adaptive controller for agent i

$$u_i = v_{i0} - k_v e_{ix} - \hat{\boldsymbol{W}}_i^{\mathrm{T}} \boldsymbol{\varphi}_i(x_i) - \hat{n}_i \tag{6}$$

with v_{i0} is an adaptive estimator for agent i to estimate the leader′s velocity v_0, $\hat{\boldsymbol{W}}_i$ is the adaptive updating parameter for $\boldsymbol{W}_i^*$, $\hat{n}_i$ is a compensation for disturbance n_i, $k_v > 0$ is a control gain to be designed.

Remark 1. It can be seen that the controller (6) contains the following 4 parts:

1) v_{i0} is a velocity estimator;

2) $-k_v e_{ix}$ is feedback control for tracking;

3) $\hat{\boldsymbol{W}}_i$ is adaptive updater for unknown parameter $\boldsymbol{W}_i^*$;

4) $\hat{n}_i$ is adaptive compensator for disturbance.

Compared to the existing estimator in [11], for instance, $\dot{v}_{i0} = a_0 - r_v k_v e_{ix}$, it indicates that a_0 is needed no matter whether there is a connection between agent i and the active leader. Whereas, the estimator (7) in this paper is totally distributed (see b_i and a_0).

Let the distributed adaptive updating law for agent i be

$$\begin{aligned}
&\dot{v}_{i0} = b_i a_0 - r_v k_v e_{ix} \\
&\dot{\hat{\boldsymbol{W}}}_i = \frac{p_i}{b_i + d_i} k_{wi} e_{ix} \boldsymbol{\varphi}_i - r_w k_{wi} \hat{\boldsymbol{W}}_i \\
&\dot{\hat{n}}_i = p_i (b_i + d_i) k_{ni} e_{ix} - r_n k_{ni} \hat{n}_i
\end{aligned} \tag{7}$$

with $r_v, r_w, r_n, k_v, k_{wi}, k_{ni}$ are positive constants to be designed, $b_i \geq 0$ is the diagonal element in B, d_i is the diagonal element in in-degree matrix D, p_i is obtained from Lemma 2, $a_0(t)$ is the desired acceleration of the active leader, e_{ix} is defined in (5) and $\boldsymbol{\varphi}_i$ is a suitable set of basis.

Remark 2. It can be seen that $b_i + d_i \neq 0$ when the graph is strongly connected and the leader is connected to any agent in the network.

Then, the first result of this paper is presented in the following theorem.

Theorem 1. Consider a network system of N agents given by (4) and an active leader in (2) under Assumption 1 by selecting the control gains as follows:

$$\begin{aligned}
&0 < r_v < 1 \\
&k_v > \frac{2 r_v \underline{\sigma}(P) + 3\bar{\sigma}^2(P)}{r_v(1 - r_v^2)\underline{\sigma}(P)\underline{\sigma}(Q)} \\
&r_n > \frac{1}{2}\left\{\bar{\sigma}^2(P - H^{\mathrm{T}} P(B + D)) + 3 r_v \frac{\bar{\sigma}^2(P)}{\underline{\sigma}(P)}\right\} \\
&r_w > \frac{1}{2}\left\{\varphi_M^2 \bar{\sigma}^2(P(B + D)^{-1} A)) + 3 r_v \varphi_M^2 \frac{\bar{\sigma}^2(P)}{\underline{\sigma}(P)}\right\}
\end{aligned} \tag{8}$$

the distributed adaptive controllers (6) and (7) for each agent in the network is able to render the overall system cooperatively UUB, i.e., $\forall\, i$, x_i is synchronized to x_0 with a bounded tracking error if the graph is strongly connected and there is at least one follower connected to leader.

Proof. When the graph is strongly connected and there is at least one follower connected to leader, $H = L + B$ is non-singular M matrix according to Lemma 1 and Definition 1. Then, $P > 0$ and $Q > 0$ can be obtained by Lemma 2.

Define the error state $\boldsymbol{\varepsilon} = [(\boldsymbol{x} - \boldsymbol{x_0})^{\mathrm{T}}, (\boldsymbol{v} - \boldsymbol{v_0})^{\mathrm{T}}, \tilde{\boldsymbol{n}}^{\mathrm{T}}]^{\mathrm{T}}$, where $\boldsymbol{x} = [x_1, \cdots, x_N]^{\mathrm{T}}$, $\boldsymbol{v} = [v_{10}, \cdots, v_{N0}]^{\mathrm{T}}$, $\boldsymbol{x_0} = x_0 \mathbf{1}_N$, $\boldsymbol{v_0} = v_0 \mathbf{1}_N$ and $\tilde{\boldsymbol{n}} = \boldsymbol{n} - \hat{\boldsymbol{n}}$, $\boldsymbol{n} = [n_1, \cdots, n_N]^{\mathrm{T}}$, $\hat{\boldsymbol{n}} = [\hat{n}_1, \cdots, \hat{n}_N]^{\mathrm{T}}$. Thus, the error can be obtained as follows:

$$\begin{aligned}
\dot{\boldsymbol{\varepsilon}} = &\begin{bmatrix} -k_v H & I & I \\ -r_v k_v H & 0 & 0 \\ -K_n P(B + D)H & 0 & -r_n K_n \end{bmatrix} \boldsymbol{\varepsilon} + \\
&\begin{bmatrix} \boldsymbol{\xi} \\ (B - I) a_0 \mathbf{1}_N - \delta \mathbf{1}_N \\ r_n K_n \boldsymbol{n} + \dot{\boldsymbol{n}} \end{bmatrix} + \begin{bmatrix} \tilde{\boldsymbol{W}}^{\mathrm{T}} \boldsymbol{\varphi} \\ 0 \\ 0 \end{bmatrix}
\end{aligned} \tag{9}$$

with $\tilde{\boldsymbol{W}} = \mathrm{diag}\{\tilde{\boldsymbol{W}}_i\} \in \mathbf{R}^{Nl \times N}$, $\tilde{\boldsymbol{W}}_i = \boldsymbol{W}_i^* - \hat{\boldsymbol{W}}_i \in \mathbf{R}^l$, $K_n = \mathrm{diag}\{k_{ni}\} \in \mathbf{R}^{N \times N}$, $\boldsymbol{\varphi} = [\varphi_{11}, \cdots, \varphi_{1l}, \cdots, \varphi_{N1}, \cdots, \varphi_{Nl}]^{\mathrm{T}} \in \mathbf{R}^{Nl}$, $\boldsymbol{\xi} = [\xi_1, \cdots, \xi_N]^{\mathrm{T}}$.

Choosing the Lyapunov function as

$$V = \boldsymbol{\varepsilon}^{\mathrm{T}} P_1 \boldsymbol{\varepsilon} + \mathrm{tr}\{\tilde{\boldsymbol{W}} K_w^{-1} \tilde{\boldsymbol{W}}^{\mathrm{T}}\} \tag{10}$$

where

$$P_1 = \begin{bmatrix} P & -r_v P & 0 \\ -r_v P & P & 0 \\ 0 & 0 & K_n^{-1} \end{bmatrix}$$

is a positive definite matrix when $0 < r_v < 1$, P can be seen in Lemma 2, $K_w = \mathrm{diag}\{k_{wi}\}$ and $K_n = \mathrm{diag}\{k_{ni}\}$, where the diagonal elements k_{wi} and k_{ni} are positive constants.

The derivative of V with respect to time t along (9) is shown in (11) with $Q > 0$ defined in Lemma 2. From (11), (12) is obtained and then (12) can be recast as (13), where $\boldsymbol{z} = [\|\boldsymbol{x} - \boldsymbol{x}_0\|, \|\boldsymbol{v} - \boldsymbol{v}_0\|, \|\tilde{\boldsymbol{n}}\|, \|\tilde{\boldsymbol{W}}\|_F]^{\mathrm{T}}$.

From Lyapunov function V in (10), (14) can be rewritten in the form of $\boldsymbol{z}^{\mathrm{T}} S_1 \boldsymbol{z} \leq V \leq \boldsymbol{z}^{\mathrm{T}} S_2 \boldsymbol{z}$.

$$\begin{aligned}
\dot{V} = &\boldsymbol{\varepsilon}^{\mathrm{T}} \begin{bmatrix} k_v(r_v^2 - 1)Q & P & P - H^{\mathrm{T}} P(B + D) \\ P & -2 r_v P & -r_v P \\ P - (B + D)PH & -r_v P & -2 r_n I \end{bmatrix} \boldsymbol{\varepsilon} + 2(\boldsymbol{\varphi}^{\mathrm{T}} \tilde{\boldsymbol{W}} P(\boldsymbol{x} - \boldsymbol{x}_0) - r_v \boldsymbol{\varphi}^{\mathrm{T}} \tilde{\boldsymbol{W}} P(\boldsymbol{v} - \boldsymbol{v}_0)) + \\
&2\Big[\boldsymbol{\xi}^{\mathrm{T}} P - r_v((B - I) a_0 \mathbf{1}_N - \delta \mathbf{1}_N)^{\mathrm{T}} P \quad -r_v \boldsymbol{\xi}^{\mathrm{T}} P + ((B - I) a_0 \mathbf{1}_N - \delta \mathbf{1}_N)^{\mathrm{T}} P \quad (r_n \boldsymbol{n} + K_n^{-1} \dot{\boldsymbol{n}})^{\mathrm{T}}\Big] \boldsymbol{\varepsilon} + 2\mathrm{tr}\{\tilde{\boldsymbol{W}} K_w^{-1} \dot{\tilde{\boldsymbol{W}}}^{\mathrm{T}}\}
\end{aligned} \tag{11}$$

$$\begin{aligned}
\dot{V} \leq &\begin{bmatrix} \|\boldsymbol{x} - \boldsymbol{x_0}\| \\ \|\boldsymbol{v} - \boldsymbol{v_0}\| \\ \|\tilde{\boldsymbol{n}}\| \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} k_v(r_v^2 - 1)\underline{\sigma}(Q) & \bar{\sigma}(P) & \bar{\sigma}(P - H^{\mathrm{T}} P(B + D)) \\ \bar{\sigma}(P) & -2 r_v \underline{\sigma}(P) & r_v \bar{\sigma}(P) \\ \bar{\sigma}(P - (B + D)PH) & r_v \bar{\sigma}(P) & -2 r_n \end{bmatrix} \begin{bmatrix} \|\boldsymbol{x} - \boldsymbol{x_0}\| \\ \|\boldsymbol{v} - \boldsymbol{v_0}\| \\ \|\tilde{\boldsymbol{n}}\| \end{bmatrix} - \\
&2 r_v \boldsymbol{\varphi}^{\mathrm{T}} \tilde{\boldsymbol{W}} P(\boldsymbol{v} - \boldsymbol{v}_0) + 2\mathrm{tr}\{\tilde{\boldsymbol{W}}(-P(B + D)^{-1}(B + D - A)(\boldsymbol{x} - \boldsymbol{x}_0)\boldsymbol{\varphi}^{\mathrm{T}} + r_w \hat{\boldsymbol{W}} + P(\boldsymbol{x} - \boldsymbol{x}_0)\boldsymbol{\varphi}^{\mathrm{T}})\} + \\
&2\left[\xi_M \bar{\sigma}(P) + r_v(2a_M + \delta_M)\bar{\sigma}(P) \quad r_v \xi_M \bar{\sigma}(P) + (2a_M + \delta_M)\bar{\sigma}(P) \quad r_n n_M + \frac{n_h}{\min\{k_{ni}\}}\right] \begin{bmatrix} \|\boldsymbol{x} - \boldsymbol{x_0}\| \\ \|\boldsymbol{v} - \boldsymbol{v_0}\| \\ \|\tilde{\boldsymbol{n}}\| \end{bmatrix}
\end{aligned} \tag{12}$$

$$\dot{V} \leq \boldsymbol{z}^{\mathrm{T}} \begin{bmatrix} k_v(r_v^2-1)\underline{\sigma}(Q) & \bar{\sigma}(P) & \bar{\sigma}(P-H^{\mathrm{T}}P(B+D)) & \varphi_M\bar{\sigma}(P(B+D)^{-1}A) \\ \bar{\sigma}(P) & -2r_v\underline{\sigma}(P) & r_v\bar{\sigma}(P) & r_v\varphi_M\bar{\sigma}(P) \\ \bar{\sigma}(P-(B+D)PH) & r_v\bar{\sigma}(P) & -2r_n & 0 \\ \varphi_M\bar{\sigma}(P(B+D)^{-1}A) & r_v\varphi_M\bar{\sigma}(P) & 0 & -2r_w \end{bmatrix} \boldsymbol{z} +$$
$$2\begin{bmatrix} \xi_M\bar{\sigma}(P)+r_v(2a_M+\delta_M)\bar{\sigma}(P) & r_v\xi_M\bar{\sigma}(P)+(2a_M+\delta_M)\bar{\sigma}(P) & r_nn_M+\dfrac{n_h}{\min\{k_{ni}\}} & r_wW_M \end{bmatrix}\boldsymbol{z} =$$
$$-\boldsymbol{z}^{\mathrm{T}}\Omega\boldsymbol{z}+\boldsymbol{m}^{\mathrm{T}}\boldsymbol{z} \tag{13}$$

$$\begin{bmatrix} \|\boldsymbol{x}-\boldsymbol{x_0}\| \\ \|\boldsymbol{v}-\boldsymbol{v_0}\| \\ \|\tilde{\boldsymbol{n}}\| \\ \left\|\tilde{\boldsymbol{W}}\right\|_F \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} (1-r_v)\lambda_{\min}(P) & 0 & 0 & 0 \\ 0 & (1-r_v)\lambda_{\min}(P) & 0 & 0 \\ 0 & 0 & \dfrac{1}{\max(k_{ni})} & 0 \\ 0 & 0 & 0 & \dfrac{1}{\max(k_{wi})} \end{bmatrix} \begin{bmatrix} \|\boldsymbol{x}-\boldsymbol{x_0}\| \\ \|\boldsymbol{v}-\boldsymbol{v_0}\| \\ \|\tilde{\boldsymbol{n}}\| \\ \left\|\tilde{\boldsymbol{W}}\right\|_F \end{bmatrix} \leq V \leq$$
$$\begin{bmatrix} \|\boldsymbol{x}-\boldsymbol{x_0}\| \\ \|\boldsymbol{v}-\boldsymbol{v_0}\| \\ \|\tilde{\boldsymbol{n}}\| \\ \left\|\tilde{\boldsymbol{W}}\right\|_F \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} (1+r_v)\lambda_{\max}(P) & 0 & 0 & 0 \\ 0 & (1+r_v)\lambda_{\max}(P) & 0 & 0 \\ 0 & 0 & \dfrac{1}{\min(k_{ni})} & 0 \\ 0 & 0 & 0 & \dfrac{1}{\min(k_{wi})} \end{bmatrix} \begin{bmatrix} \|\boldsymbol{x}-\boldsymbol{x_0}\| \\ \|\boldsymbol{v}-\boldsymbol{v_0}\| \\ \|\tilde{\boldsymbol{n}}\| \\ \left\|\tilde{\boldsymbol{W}}\right\|_F \end{bmatrix} \tag{14}$$

Then

$$\underline{\sigma}(S_1)\|\boldsymbol{z}\|^2 \leq V \leq \bar{\sigma}(S_2)\|\boldsymbol{z}\|^2 \tag{15}$$

From (13) and (15),

$$\dot{V} \leq -\frac{\lambda_{\min}(\Omega)}{\bar{\sigma}(S_2)}V + \frac{\|\boldsymbol{m}^{\mathrm{T}}\|}{\sqrt{\underline{\sigma}(S_1)}}\sqrt{V} \tag{16}$$

It can be seen that $\Omega > 0$ if (8) holds. Thus, (16) can be rewritten as $\dot{V} \leq -\alpha V + \beta\sqrt{V}$ with $\alpha > 0$, $\beta > 0$, then

$$\sqrt{V(t)} \leq \sqrt{V(0)}\mathrm{e}^{-\frac{\alpha}{2}t} + \frac{\beta}{\alpha}(1-\mathrm{e}^{-\frac{\alpha}{2}t}) \leq \sqrt{V(0)} + \frac{\beta}{\alpha} \tag{17}$$

Therefore,

$$\|\boldsymbol{x}(t)-\boldsymbol{x}_0(t)\| \leq \|\boldsymbol{z}(t)\| \leq \frac{\bar{\sigma}(S_2)}{\underline{\sigma}(S_1)}\frac{\|\boldsymbol{m}^{\mathrm{T}}\|}{\lambda_{\min}(\Omega)} + \sqrt{\frac{\bar{\sigma}(S_2)}{\underline{\sigma}(S_1)}} \times$$
$$\sqrt{\|\boldsymbol{x}(0)-\boldsymbol{x}_0(0)\|^2+\|\boldsymbol{v}(0)-\boldsymbol{v}_0(0)\|^2+\|\tilde{W}(0)\|_F^2+\|\tilde{\boldsymbol{n}}(0)\|^2} \tag{18}$$

with

$$\|\boldsymbol{m}^{\mathrm{T}}\| = 2\{(1+r_v^2)\bar{\sigma}^2(P)[(2a_M+\delta_M)^2+\xi_M^2] + r_w^2W_M^2 + r_n^2n_M^2 + \left(\frac{n_h}{\min\{k_{ni}\}}\right)^2 + \frac{2r_nn_Mn_h}{\min\{k_{ni}\}} + 4r_v\xi_M(2a_M+\delta_M)\bar{\sigma}^2(P)\}^{\frac{1}{2}} \tag{19}$$

It can be seen that $\|\boldsymbol{x}-\boldsymbol{x}_0\|$ is bounded by a constant determined by the bounds in Assumption 1, the structural properties of the digraph, and the control gains in the controller. It can be obtained from (17) that for a bound $\mathcal{B} = \frac{\sqrt{2}}{2}\sqrt{V(0)} + \frac{\beta}{\alpha}$, there exists a $t_f = In(2)/\alpha$ such that $\|x_i(t)-x_0(t)\| \leq \mathcal{B}, \forall\, t \geq t_0+t_f$, $t_0 = 0$, according to Definition 2, the overall system is cooperatively UUB, which completes the proof. □

Remark 3. It can be seen that this result extends the result in [11] from undirected graph to strongly connected digraph, which is non-trivial.

Remark 4. Considering the constraint of communication and sensor, velocity information of the leader is sometimes unavailable. Therefore, the controller proposed in (6) loosens the requirement of transferring velocity information by adding a distributed velocity estimator to each agent.

Next, we will further consider the problem under a digraph with a spanning tree. The following lemma is needed:

Lemma 3. $H = L + B \in \mathbf{R}^{N\times N}$, all the eigenvalues of H have positive real parts if there is a spanning tree in the graph and the leader is connected to the root node.

Proof. the proof is a straight consequence of Lemma 2 and Definition 1, and thus omitted. □

With Theorem 1 and Lemma 3, the following corollary is straightforward:

Corollary 2. Consider a network system of N agents given by (4) and an active leader in (2) under Assumptions 1, by selecting the control gains as (8), the distributed adaptive controllers (6) and (7) for each agent in the network is also able to render the overall system cooperatively UUB, i.e., $\forall\, i$, x_i is synchronized to x_0 with a bounded tracking error if there is a spanning tree in the graph and the active leader is connected to the root node.

Proof. The proof is omitted. Please refer the proof of Theorem 1. □

4 Simulation results

This section will provide several scenarios to show the effectiveness of the proposed controllers. In the scenario, we consider 1 active leader and 3 followers. The active leader, denoted by "0" in Fig. 1, is

$$\begin{cases} \dot{x}_0 = v_0 \\ \dot{v}_0 = a_0(t) + \delta(t) = 0.5\sin(0.2t) + 0.1\sin(t) \end{cases}$$

The dynamics of 3 agents are

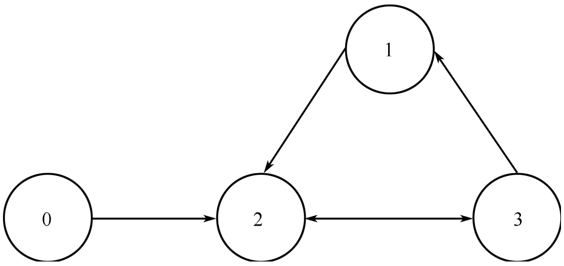

Fig. 1 Fixed strongly connected graph with an active leader

$$\dot{x}_1 = f_1(x_1) + u_1 + n_1 = \sin(x_1) + u_1 + n_1$$
$$\dot{x}_2 = f_i(x_2) + u_2 + n_2 = 0.8\sin(x_2) + u_2 + n_2$$
$$\dot{x}_3 = f_i(x_3) + u_3 + n_3 = \sin(x_3^2) + u_3 + n_3$$

and denoted by "1" ~ "3", respectively.

The following parameters are used for all the simulations: Control gains $k_{wi} = 1$, $k_{ni} = 1$, the set of basis function is $\varphi_i(x) = \begin{bmatrix} 1/(1+\exp(-x_i)) & 2/(2+\exp(-x_i)) \end{bmatrix}^{\mathrm{T}}$, $\forall\, i =$ 1, 2, 3.

4.1 Fixed topology (strongly connected and the leader connected to any follower)

In this subsection, we consider the followers under the strongly connected topology shown in Fig. 1. The weights of edges are all set to 1. According to the definition in Section 2, the following matrices are deduced:

$$L = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix},\quad B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

such that Lemma 1 can be verified. From Lemma 2, P and Q can be found as follows:

$$P = \begin{bmatrix} 0.1667 & 0 & 0 \\ 0 & 0.25 & 0 \\ 0 & 0 & 0.2 \end{bmatrix}$$

$$Q = \begin{bmatrix} 0.3333 & -0.25 & -0.1667 \\ -0.25 & 1.5 & -0.45 \\ -0.1667 & -0.45 & 0.4 \end{bmatrix}$$

The disturbances are $n_1 = 0.13, n_2 = 0.21, n_3 = 0.14$, respectively. The initial states of leader and followers are $[x_0(0), x_1(0), x_2(0), x_3(0)] = [7.5, -1.5, 4.5, -5]$ and $v_0(0) =$ 0.23. Based on (8), the control gains are chosen as $r_v =$ 0.75, $k_v = 250 > 245.3038$, $r_n = 3.5 > 3.001$, and $r_w = 2$ > 0.4419.

The error state in the top figure of Fig. 2 and zoom-in view of bottom figure of Fig. 2 clearly show that, under the strongly connected topology, the proposed adaptive control is effective, multiple agents keep up with the leader with a bounded tracking error, i.e., $\forall\, i = 1, 2, 3, x_i - x_0$ are UUB such that the overall system is cooperatively UUB.

Fig. 3 depicts that by applying the distributed estimator (7) to each agent in the network, each agent is able to estimate the active leader′s velocity v_0 with a bounded error.

4.2 Fixed topology (spanning tree and the leader is connected to the root node)

In this subsection, we consider the followers under a topology with a spanning tree and the leader connected to the root node as shown in Fig. 4. The weights of edges are all set to 1. According to the definition in Section 2, the following matrices are deduced:

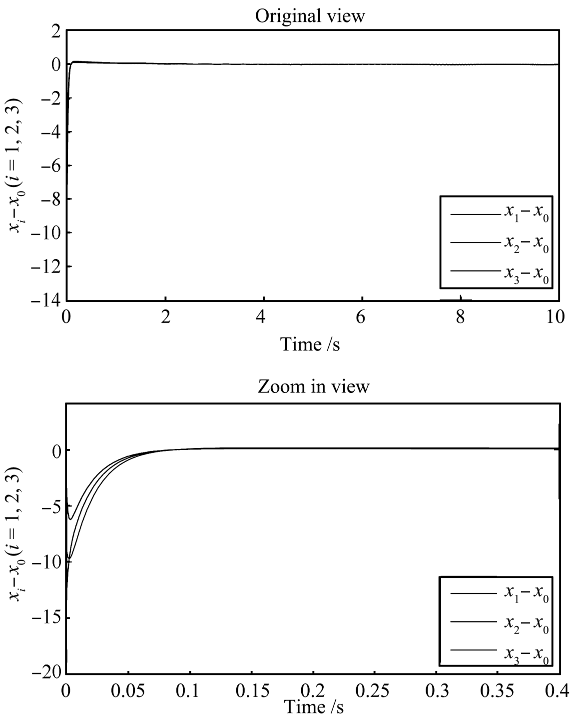

Fig. 2 Tracking error $x_i - x_0$ $(i = 1, 2, 3)$ under fixed topology in Fig. 1

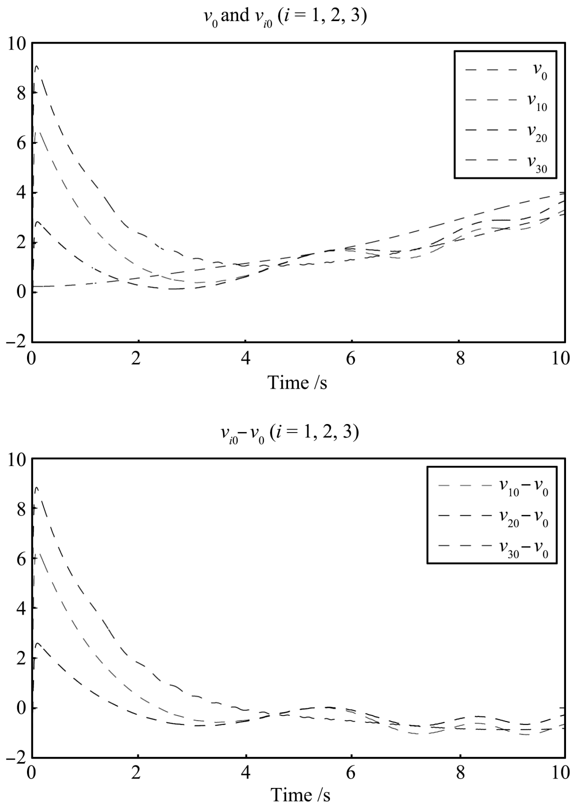

Fig. 3 Estimator of v_{i0} $(i = 1, 2, 3)$ and v_0 under fixed topology in Fig. 1

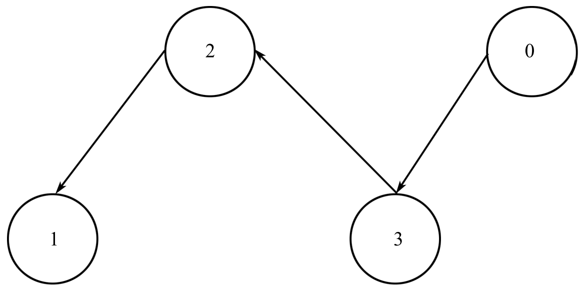

Fig. 4 Graph having a spanning tree and the leader connected to the root node

$$L=\begin{bmatrix}1 & -1 & 0\\ 0 & 1 & -1\\ 0 & 0 & 0\end{bmatrix},\quad B=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 1\end{bmatrix}\tag{20}$$

such that Lemma 3 can be verified. From Lemma 2, P and Q can be found as follows:

$$P=\begin{bmatrix}0.3333 & 0 & 0\\ 0 & 0.5 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$Q=\begin{bmatrix}0.6667 & -0.3333 & 0\\ -0.3333 & 1 & -0.5\\ 0 & -0.5 & 2\end{bmatrix}$$

The disturbances are $n_1 = 0.18, n_2 = 0.1, n_3 = 0.14$, respectively. The initial states of leader and followers are $[x_0(0), x_1(0), x_2(0), x_3(0)] = [7, 4.5, -0.5, -2.5]$ and $v_0(0) = 0.3$. Based on (8), the control gains are chosen as $r_v = 0.75$, $k_v = 80 > 78.0579$, $r_n = 4 > 3.5$, and $r_w = 4 > 3.5$.

The error state in the top figure of Fig. 5 and zoom-in view of bottom figure of Fig. 5 clearly show that, under the topology with a spanning tree and the leader connected to the root node, the proposed adaptive control is able to drive multiple agents to track the leader with a bounded tracking error, i.e., $\forall\, i = 1, 2, 3, x_i - x_0$ are UUB such that the overall system is cooperatively UUB.

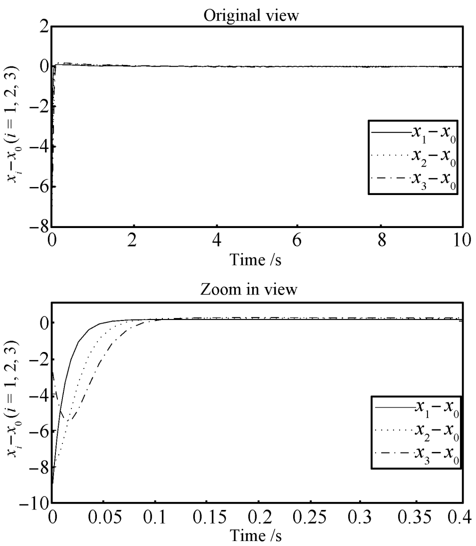

Fig. 5 Tracking error $x_i - x_0$ $(i = 1, 2, 3)$ under fixed topology in Fig. 4

Fig. 6 depicts that by applying the distributed estimator (7) to each agent in the network, each agent is able to estimate the active leader′s velocity v_0 with a bounded error.

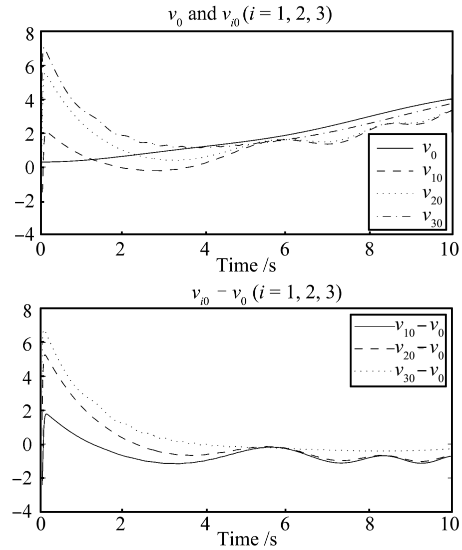

Fig. 6 Estimator of v_{i0} $(i = 1, 2, 3)$ and v_0 under fixed topology in Fig. 4

5 Conclusion and future direction

This paper studied the cooperative tracking problem of networked unknown nonlinear systems to an active leader, whose states can only be partly measured. To solve the problem, NN technique was applied in modeling the unknown followers′ dynamics. A distributed adaptive control along with distributed estimator and adaptive updaters were proposed for each agent in the network. Disturbance compensators were also designed for the disturbance in the input channel. Lyapunov-based convergence analysis was given for both cases, the graph was strongly connected with the leader connected to any agent in the network or there was a spanning tree in the graph with the leader connected to the root node. Simulation results illustrated the effectiveness of the proposed adaptive cooperative tracking control.

The next step is to extend the current problem to the switching topologies case as well as the experimental validation.

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