On Optimal Fault Detection for Discrete-time Markovian Jump Linear Systems

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Abstract This paper deals with the problem of fault detection for discrete-time Markovian jump linear systems (MJLS). Using an observer-based fault detection filter (FDF) as a residual generator, the design of the FDF is formulated as an optimization problem for maximizing stochastic H_-/H_∞ or H_∞/H_∞ performance index. With the aid of an operator optimization method, it is shown that a unified optimal solution can be derived by solving a coupled Riccati equation. Numerical examples are given to show the effectiveness of the proposed method.

Key words Fault detection filter, Markovian jump linear system (MJLS), observer, coupled Riccati equation

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During the past three decades, research on observer-based robust fault detection and isolation (FDI) has received much attention[1−7]. In reviewing of the development of FDI techniques, there are two main approaches which are widely used for linear time-invariant (LTI) systems with $\mathcal{L}_2$-norm bounded unknown inputs and faults. One is the H_∞ filtering scheme which ensures a prescribed bound on the $\mathcal{L}_2$-induced gain from the disturbance to the error between the fault and the residual[8−12]. The other one is the H_∞ optimization scheme which involves solving a two-objective optimization problem. In [13], a unified H_∞ optimization solution is given in the framework of maximizing H_-/H_∞ and H_∞/H_∞ performance indices by coprime factorization approach, and in [14−15] this unified approach has been extended to linear continuous time-varying (LCTV) systems and linear discrete time-varying (LDTV) systems, respectively. In [16], a finite horizon H_-/H_∞ and H_∞/H_∞ FDI formulation is proposed for LDTV systems and an optimal solution is derived by solving a Riccati equation.

On the other hand, Markovian jump systems are appropriate to model different plants subject to component failures, sudden environment disturbances, abrupt changes in subsystems interconnections, incomplete information in communication channel and random delays. The problems of control and filtering for Markovian jump linear systems (MJLS) have been deeply investigated[17−22]. With the increasing demands for system safety and reliability, it is of significance to study the problem of FDI for MJLS. In [23], the problem of fault detection for MJLS is formulated into a two-object optimization problem and a numerical solution is given via iterative linear matrix inequality (LMI) algorithms. Later, the H_∞ filtering scheme together with LMI technique is applied in [24], and the same idea has also been extended to the fault detection problem for various systems with Markovian jump characteristic. For example, for networked control systems, [25−27] model the systems with packet dropouts and time-delay as MJLS and observer-based H_∞-FDFs are designed, while in [28] fault detection for discrete-time MJLS with partially known transition probabilities is concerned. Recently, in [29−30], an H_∞ fault isolation algorithm is addressed based on the geometric approach for both continuous-time and discrete-time MJLS, respectively. Notice that although there exist some results on fault detection of discrete-time systems such as the Krein space based H_∞ filtering method, the coprime factorization based optimization approach or the matrix norm optimization based approach[10, 15−16], the problem of optimal fault detection for MJLS could not be handled by direct application of the existing results due to the fact that MJLS is intrinsically stochastic and mode-dependent. To authors′ best knowledge, the problem of optimal fault detection for discrete-time MJLS in the H_∞ optimization scheme has not been published in the open literature and the research remains significant and challenging, which motivates our present study.

In this paper, the problem of optimal fault detection for discrete-time MJLS will be investigated. By constructing an observer-based FDF and defining input-output operators that map from fault and unknown input to residual, the problem of designing optimal FDF for discrete-time MJLS is formulated in the framework of optimizing H_-/H_∞ or H_∞/H_∞ performance index. A new adjoint operator based optimization scheme is proposed for solving the aforementioned optimization problem and an analytical unified optimal solution is obtained by solving a coupled Riccati equation. Numerical examples are given to show the effectiveness of the proposed method.

Notations. $\mathbf{R}^n$ means the n-dimensional Euclidean space. I and 0 denote identity matrix and zero matrix with appropriate dimensions, respectively. $X > 0$ ($X < 0$) denotes X is positive (negative) definite. $\mathrm{E}\{\boldsymbol{\vartheta}(k)\}$ means the mathematical expectation of $\boldsymbol{\vartheta}(k)$. $\|\boldsymbol{\alpha}(k)\|_2$ stands for the deterministic l_2-norm of $\boldsymbol{\alpha}(k)$ with $\|\boldsymbol{\alpha}(k)\|_2^2 = \sum_{k=0}^{\infty}\boldsymbol{\alpha}^{\mathrm{T}}(k)\boldsymbol{\alpha}(k)$, while $\|\boldsymbol{\zeta}(k)\|_{2,\mathrm{E}}$ for the stochastic case with $\|\boldsymbol{\zeta}(k)\|_{2,\mathrm{E}}^2 = \mathrm{E}\{\sum_{k=0}^{\infty}\boldsymbol{\zeta}^{\mathrm{T}}(k)\boldsymbol{\zeta}(k)\}$. $\langle\boldsymbol{\mu}(k),\boldsymbol{\varsigma}(k)\rangle = \mathrm{E}\{\Sigma_{k=0}^{\infty}\boldsymbol{\mu}^{\mathrm{T}}(k)\boldsymbol{\varsigma}(k)\}$ gives the definition of the inner product for vector $\boldsymbol{\mu}(k)$ and $\boldsymbol{\varsigma}(k)$ with appropriate dimensions.

1 Problem formulation

Consider the following discrete-time MJLS:

$$\begin{cases} \boldsymbol{x}(k+1) = A(\theta(k))\boldsymbol{x}(k) + B(\theta(k))\boldsymbol{u}(k) + \\ \quad B_d(\theta(k))\boldsymbol{d}(k) + B_f(\theta(k))\boldsymbol{f}(k) \\ \boldsymbol{y}(k) = C(\theta(k))\boldsymbol{x}(k) + D_d(\theta(k))\boldsymbol{d}(k) + \\ \quad D_f(\theta(k))\boldsymbol{f}(k) \\ \boldsymbol{x}(0) = 0,\ \theta(0) = i_0 \end{cases} \tag{1}$$

where $\boldsymbol{x}(k) \in \mathbf{R}^n$, $\boldsymbol{u}(k) \in \mathbf{R}^{n_u}$, $\boldsymbol{y}(k) \in \mathbf{R}^{n_y}$, $\boldsymbol{d}(k) \in \mathbf{R}^{n_d}$ and $\boldsymbol{f}(k) \in \mathbf{R}^{n_f}$ denote the state, control input, measurement output, unknown input and fault to be detected, respectively; $\boldsymbol{f}(k)$ and $\boldsymbol{d}(k)$ are l_2-norm bounded. $\{\theta(k)\}$ is a discrete-time homogeneous Markov chain taking values in a finite set $\Omega = \{1, 2, \cdots, N\}$ with transition probability matrix $\Lambda = [\lambda_{ij}]_{i,j\in\Omega}$, where λ_{ij} is defined as

$$\lambda_{ij} = \Pr\{\theta(k+1) = j|\theta(k) = i\}$$

with $\sum_{j=1}^{N}\lambda_{ij} = 1$. Denote by A_i, B_i, B_{di}, B_{fi}, C_i, D_{di} and D_{fi} the values of $A(\theta(k))$, $B(\theta(k))$, $B_d(\theta(k))$, $B_f(\theta(k))$, $C(\theta(k))$, $D_d(\theta(k))$ and $D_f(\theta(k))$, respectively, for $\theta(k) = i \in \Omega$. A_i, B_i, B_{di}, B_{fi}, C_i, D_{di} and D_{fi} are known constant matrices with appropriate dimensions.

For system (1), the following definition is first introduced.

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Definition 1[17]**.** System (1) with $\boldsymbol{u}(k)=0$, $\boldsymbol{d}(k)=0$ and $\boldsymbol{f}(k)=0$ is mean square stable if

$$\mathrm{E}\{\|\boldsymbol{x}(k)\|^2\}\rightarrow 0 \quad \text{as} \quad k\rightarrow\infty$$

for any initial condition $\boldsymbol{x}_0$ and initial distribution $i_0\in\Omega$.

The core of fault detection is to generate a residual which is robust to disturbance and sensitive to fault. For this purpose, an observer-based FDF can be considered as a residual generator. In order to guarantee the mean square stability of the proposed FDF, the following assumptions are used throughout this paper[19].

A1. $(\mathcal{A}^{\mathrm{T}},\mathcal{C}^{\mathrm{T}})$ is mean square stabilizable;

A2. $(\mathcal{B}_R,\mathcal{A}_R)$ is mean square detectable, where

$$\mathcal{A}=(A_1,\cdots,A_N),\ \mathcal{C}=(C_1,\cdots,C_N)$$
$$A_{Ri}:=A_i^{\mathrm{T}}-C_i^{\mathrm{T}}(D_{di}D_{di}^{\mathrm{T}})^{-1}D_{di}B_{di}^{\mathrm{T}}$$
$$B_{Ri}:=(I-D_{di}^{\mathrm{T}}(D_{di}D_{di}^{\mathrm{T}})^{-1}D_{di})B_{di}^{\mathrm{T}}$$
$$\mathcal{A}_R=(A_{R1},\cdots,A_{RN}),\ \mathcal{B}_R=(B_{R1},\cdots,B_{RN})$$

A3. The instantaneous value of mode $\theta(k)$ is available in real time.

Remark 1. When $B_{di}D_{di}^{\mathrm{T}}=0$, the assumption that $(\mathcal{B}_R,\mathcal{A}_R)$ is mean square detectable can be simplified into $(\mathcal{B}_d,\mathcal{A})$ is mean square detectable, where $\mathcal{B}_d=(B_{d1},\cdots,B_{dN})$. It should be noticed that the duality of stabilizable and detectable in the deterministic case can not be directly extended to MJLS. We refer to [19] for the definitions and duality of stabilizable and detectable for MJLS.

In this paper, the following observer-based FDF is considered for MJLS (1)

$$\begin{cases}\hat{\boldsymbol{x}}(k+1)=A(\theta(k))\hat{\boldsymbol{x}}(k)+B(\theta(k))\boldsymbol{u}(k)+\\ \qquad L(\theta(k))(\boldsymbol{y}(k)-C(\theta(k))\hat{\boldsymbol{x}}(k))\\ \boldsymbol{r}(k)=V(\theta(k))(\boldsymbol{y}(k)-C(\theta(k))\hat{\boldsymbol{x}}(k))\end{cases}\tag{2}$$

where $\hat{\boldsymbol{x}}(k)\in\mathbf{R}^n$ is an estimate of $\boldsymbol{x}(k)$, $\boldsymbol{r}(k)\in\mathbf{R}^r$ is the generated residual, $L(\theta(k))$ is the observer gain matrix and $V(\theta(k))$ is the (regular) post-filter to be determined.

By defining $\boldsymbol{e}(k)=\boldsymbol{x}(k)-\hat{\boldsymbol{x}}(k)$, it follows from (1) and (2) that the filtering error dynamics can be obtained as below

$$\begin{cases}\boldsymbol{e}(k+1)=(A(\theta(k))-L(\theta(k))C(\theta(k)))\boldsymbol{e}(k)+\\ \qquad (B_{\boldsymbol{d}}(\theta(k))-L(\theta(k))D_d(\theta(k)))\boldsymbol{d}(k)+\\ \qquad (B_{\boldsymbol{f}}(\theta(k))-L(\theta(k))D_f(\theta(k)))\boldsymbol{f}(k)\\ \boldsymbol{r}(k)=V(\theta(k))(C(\theta(k))\boldsymbol{e}(k)+D_{\boldsymbol{d}}(\theta(k))\boldsymbol{d}(k)+\\ \qquad D_{\boldsymbol{f}}(\theta(k))\boldsymbol{f}(k))\end{cases}\tag{3}$$

Recalling that system (1) is linear and the input signals and output signals of system (1) are defined in the same field[31], an operator that maps $\boldsymbol{f}\mapsto\boldsymbol{r}$ and an operator that maps $\boldsymbol{d}\mapsto\boldsymbol{r}$ can be defined, respectively, which means that

$$\boldsymbol{r}_{\boldsymbol{f}}(k)=\mathcal{G}_{\boldsymbol{rf}}\boldsymbol{f}(k),\ \boldsymbol{r}_{\boldsymbol{d}}(k)=\mathcal{G}_{\boldsymbol{rd}}\boldsymbol{d}(k)$$

where $\boldsymbol{r}_{\boldsymbol{f}}(k)=\boldsymbol{r}(k)|_{\boldsymbol{d}(k)=0}$, $\boldsymbol{r}_{\boldsymbol{d}}(k)=\boldsymbol{r}(k)|_{\boldsymbol{f}(k)=0}$.

Define

$$\|\mathcal{G}_{rf}\|_\infty=\sup_{\boldsymbol{f}\in l_2,\ \|\boldsymbol{f}\|_2\neq0}\frac{\|\boldsymbol{r}_f(k)\|_{2,\mathrm{E}}^2}{\|\boldsymbol{f}(k)\|_2^2}$$
$$\|\mathcal{G}_{rd}\|_\infty=\sup_{\boldsymbol{d}\in l_2,\ \|\boldsymbol{d}\|_2\neq0}\frac{\|\boldsymbol{r}_{\boldsymbol{d}}(k)\|_{2,\mathrm{E}}^2}{\|\boldsymbol{d}(k)\|_2^2}$$
$$\|\mathcal{G}_{rf}\|_-=\inf_{\boldsymbol{f}\in l_2,\ \|\boldsymbol{f}\|_2\neq0}\frac{\|\boldsymbol{r}_{\boldsymbol{f}}(k)\|_{2,\mathrm{E}}^2}{\|\boldsymbol{f}(k)\|_2^2}$$

Similar to the deterministic cases in [14−16], the sensitivity of residual to fault can be evaluated by $\|\mathcal{G}_{\boldsymbol{rf}}\|_\infty$ or $\|\mathcal{G}_{\boldsymbol{rf}}\|_-$, while the robustness of residual to unknown input can be evaluated by $\|\mathcal{G}_{\boldsymbol{rd}}\|_\infty$. Furthermore, $\|\mathcal{G}_{\boldsymbol{rf}}\|_\infty$ and $\|\mathcal{G}_{\boldsymbol{rf}}\|_-$ represents the best and the worst sensitivity criteria of fault detection, respectively.

Based on the definitions above, the FDF design problem can be formulated as: find a suitable observer gain matrix $L(\theta(k))$ and a regular post-filter $V(\theta(k))$ such that system (3) is mean square stable and satisfies the following performance:

$$\max_{L(\theta(k)),V(\theta(k))}\frac{\|\mathcal{G}_{\boldsymbol{rf}}\|_\infty}{\|\mathcal{G}_{\boldsymbol{rd}}\|_\infty}\quad\text{or}\quad\max_{L(\theta(k)),V(\theta(k))}\frac{\|\mathcal{G}_{\boldsymbol{rf}}\|_-}{\|\mathcal{G}_{\boldsymbol{rd}}\|_\infty}\tag{4}$$

Remark 2. The proposed performance index $\|\mathcal{G}_{\boldsymbol{rf}}\|_\infty/\|\mathcal{G}_{\boldsymbol{rd}}\|_\infty$ or $\|\mathcal{G}_{\boldsymbol{rf}}\|_-/\|\mathcal{G}_{\boldsymbol{rd}}\|_\infty$ which is slightly different from the LTI case can be seen as a stochastic version of H_∞/H_∞ or H_-/H_∞ performance for LTV systems. However, there exists no explicit coprime factorization realization for MJLS and the equivalence of the norm between generalized transfer function matrix and input-output operator does not hold for MJLS, which indicates that the existing technique in [14] or [16] cannot be applied to MJLS directly. To solve the aforementioned problem, an adjoint operator based optimization method will be proposed and a mode-dependent optimal FDF will be derived.

2 Main results

Before deriving the main results of this paper, the following definitions and lemmas which play the key role should be given.

Definition 2[32]**.** Let G_s denotes an operator or a system mapping from l_2-norm bounded space S_1 to l_2-norm bounded space S_2. An operator $G_s^\sim$ is said to be the adjoint operator of G_s from space S_2 to S_1 if $\langle G_s\boldsymbol{\mu},\boldsymbol{\varsigma}\rangle=\langle\boldsymbol{\mu},G_s^\sim\boldsymbol{\varsigma}\rangle$ for all $\boldsymbol{\mu}\in S_1$ and $\boldsymbol{\varsigma}\in S_2$.

Definition 3[32]**.** Let G_s denotes an operator or a system mapping from l_2-norm bounded input space S_1 to l_2-norm bounded output space S_2, then G_s is co-isometric if $\|G_s^\sim\boldsymbol{\varphi}(k)\|_{S_2}=\|\boldsymbol{\varphi}(k)\|_{S_1}$. Here, $\|\cdot\|_S$ denotes the l_2-norm of a signal defined in space S.

Lemma 1. Consider the following residual generators:

$$\begin{cases}\hat{\boldsymbol{x}}^m(k+1)=A(\theta(k))\hat{\boldsymbol{x}}^m(k)+B(\theta(k))\boldsymbol{u}(k)+\\ \qquad L^m(\theta(k))(\boldsymbol{y}(k)-C(\theta(k))\hat{\boldsymbol{x}}^m(k))\\ \boldsymbol{r}^m(k)=V^m(\theta(k))(\boldsymbol{y}(k)-C(\theta(k)\hat{\boldsymbol{x}}^m(k)),\ m=1,2\end{cases}$$

where $L^m(\theta(k))$ is the observer gain matrix such that $A(\theta(k))-L^m(\theta(k))C(\theta(k))$ is mean square stable and $V^m(\theta(k))$ is the post-filter. Then

$$\boldsymbol{r}^2(k)=Q\boldsymbol{r}^1(k)\tag{5}$$

where Q is an operator that maps $\boldsymbol{r}^1(k)\mapsto\boldsymbol{r}^2(k)$.

Proof. The proof can be readily derived by applying Lemma 1 in [16] to MJLS. In fact, for the following residual generators

$$\begin{cases}\hat{\boldsymbol{x}}^m(k+1)=A(\theta(k))\hat{\boldsymbol{x}}^m(k)+B(\theta(k))\boldsymbol{u}(k)+\\ \qquad L^m(\theta(k))(\boldsymbol{y}(k)-C(\theta(k))\hat{\boldsymbol{x}}^m(k))\\ \boldsymbol{\varepsilon}^m(k)=\boldsymbol{y}(k)-C(\theta(k)\hat{\boldsymbol{x}}^m(k),\ m=1,2\end{cases}$$

where $L^m(\theta(k))$ is the observer gain matrix that ensures the mean square stability of $A(\theta(k))-L^m(\theta(k))C(\theta(k))$, an operator Q_ε that guarantees $\boldsymbol{\varepsilon}^2(k)=Q_\varepsilon\varepsilon^1(k)$ exists, which can be realized by the following MJLS:

$$\begin{cases} \boldsymbol{\eta}(k+1) = (A(\theta(k)) - L^2(\theta(k))C(\theta(k))\boldsymbol{\eta}(k) + \\ \quad (L^1(\theta(k)) - L^2(\theta(k)))\boldsymbol{\nu}(k) \\ \boldsymbol{\varepsilon}^{Q_\varepsilon}(k) = C(\theta(k)\boldsymbol{\eta}(k) + \boldsymbol{\nu}(k), \quad \boldsymbol{\eta}(0) = 0 \end{cases} \tag{6}$$

Since $V(\theta(k))$ is regular, i.e., there exists $V(\theta(k))$ such that

$$V^+(\theta(k))V(\theta(k)) = I$$

where $V^+(\theta(k))$ denotes the left-inverse of $V(\theta(k))$, then we have:

$$\boldsymbol{r}^2(k) = V^2(\theta(k))\boldsymbol{\varepsilon}^2(k) = V^2(\theta(k))Q_\varepsilon\boldsymbol{\varepsilon}^1(k) = \\ V^2(\theta(k))Q_\varepsilon(V^1(\theta(k))^+\boldsymbol{r}^1(k)$$

Thus, Q can be represented as

$$\begin{cases} \boldsymbol{\eta}(k+1) = (A(\theta(k)) - L^2(\theta(k))C(\theta(k))\boldsymbol{\eta}(k) + \\ \quad L^1((\theta(k)) - L^2(\theta(k)))(V^1(\theta(k))^+\boldsymbol{r}^1(k) \\ \boldsymbol{r}^2(k) = V^2(\theta(k))(C(\theta(k)\boldsymbol{\eta}(k) + (V^1(\theta(k))^+\boldsymbol{r}^1(k)) \\ \boldsymbol{\eta}(0) = 0 \end{cases} \tag{7}$$

□

Lemma 2. For system (3), consider the operator $\mathcal{G}_{rd}$ that maps $\boldsymbol{d}(k) \mapsto \boldsymbol{r}_d(k)$ which is realized by the following discrete-time MJLS:

$$\begin{cases} \boldsymbol{e}(k+1) = A_e(\theta(k))\boldsymbol{e}(k) + B_e(\theta(k))\boldsymbol{d}(k) \\ \boldsymbol{r}_d(k) = C_e(\theta(k))\boldsymbol{e}(k) + D_e(\theta(k))\boldsymbol{d}(k) \\ \boldsymbol{e}(0) = 0, \ \theta(0) = i_0 \end{cases} \tag{8}$$

where $A_e(\theta(k)) = A(\theta(k)) - L(\theta(k))C(\theta(k))$, $B_e(\theta(k)) = B(\theta(k)) - L(\theta(k))D_d(\theta(k))$, $C_e(\theta(k)) = V(\theta(k))C(\theta(k))$, $D_e(\theta(k)) = V(\theta(k))D_d(\theta(k))$. Let $\mathcal{G}^{\sim}_{\boldsymbol{rd}}$ be the adjoint operator of $\mathcal{G}_{\boldsymbol{rd}}$. If there exists a semi-positive definite matrix $P_i \geq 0$ satisfying the following equations:

$$\begin{cases} B_{ei}B_{ei}^{\rm T} + A_{ei}\bar{P}_{ei}A_i^{\rm T} = P_i \\ B_{ei}D_{ei}^{\rm T} + A_{ei}\bar{P}_iC_{ei}^{\rm T} = 0 \\ D_{ei}D_{ei}^{\rm T} + C_{ei}\bar{P}_iC_{ei}^{\rm T} = I \end{cases} \tag{9}$$

where

$$P_i = \sum_{j=1}^{N} \lambda_{ij}P_j$$

then $\mathcal{G}_{\boldsymbol{r}d}$ is co-isometric.

Proof. From (8), we know that

$$\boldsymbol{r}_d(k) = \begin{cases} C_e(\theta(k)) \sum\limits_{l=0}^{k-1} \Phi(k,l+1)B_e(\theta(l))\boldsymbol{d}(l) + \\ \quad D_e(\theta(k))\boldsymbol{d}(k), \ 0 < k \leq \infty \\ D_e(\theta(0))\boldsymbol{d}(0), \ k = 0 \end{cases} \tag{10}$$

where $\Phi(k,l)$ is the transition matrix defined by

$$\Phi(k,l) = \begin{cases} A_e(\theta(k-1))A_e(\theta(k-2))\cdots A_e(\theta(l)), \ 0 < l < k \\ I, \quad k = l \end{cases}$$

Let $\mathcal{G}^{\sim}_{\boldsymbol{rd}}\boldsymbol{d}(k) = \boldsymbol{d}_a(k)$. Applying the same idea in [33] based on Definition 2, we have:

$$\langle \mathcal{G}_{\boldsymbol{rd}}\boldsymbol{d}(k), \boldsymbol{r_d}(k)\rangle \quad = \langle \boldsymbol{d}(k), \mathcal{G}^{\sim}_{\boldsymbol{rd}}\boldsymbol{r_d}(k)\rangle = {\rm E}\left\{\sum_{k=0}^{\infty} \boldsymbol{d}^{\rm T}(k)\boldsymbol{d}_a(k)\right\}$$

i.e.

$$\sum_{k=0}^{\infty} {\rm E}\Big\{\boldsymbol{r_d}^{\rm T}(k)[C_e(\theta(k)) \sum_{l=0}^{k-1} \Phi(k,l+1)B_e(\theta(l))\boldsymbol{d}(l) + \\ D_e(\theta(k))\boldsymbol{d}(k)]\Big\} = \\ \sum_{k=0}^{\infty}{\rm E}\Big\{\sum_{l=0}^{k-1}[(\Phi^{\rm T}(k,l+1)C_e^{\rm T}(\theta(k))\boldsymbol{r_d}(k))^{\rm T}B_e(\theta(l))\boldsymbol{d}(l)] + \\ \boldsymbol{r_d}^{\rm T}(k)D_e(\theta(k))\boldsymbol{d}(k)\Big\} = \\ \sum_{k=0}^{\infty}{\rm E}\Big\{\boldsymbol{d}^{\rm T}(k)B_e^{\rm T}(\theta(k)) \sum_{l=k+1}^{\infty} \Phi^{\rm T}(l,k+1)C_e^{\rm T}(\theta(l))\boldsymbol{r_d}(l) + \\ \boldsymbol{d}^{\rm T}(k)D_e^{\rm T}(\theta(k))\boldsymbol{r_d}(k)\Big\} = \\ {\rm E}\left\{\sum_{k=0}^{\infty} \boldsymbol{d}_a{}^{\rm T}(k)\boldsymbol{d}(k)\right\} = {\rm E}\left\{\sum_{k=0}^{\infty} \boldsymbol{d}^{\rm T}(k)\boldsymbol{d}_a(k)\right\}$$

then, $\boldsymbol{d}_a(k)$ can be chosen as

$$\boldsymbol{d}_a(k) = B_e^{\rm T}(\theta(k)) \sum_{l=k+1}^{\infty} \Phi^{\rm T}(l,k+1)C_e^{\rm T}(\theta(l))\boldsymbol{r}_d(l) + \\ D_e^{\rm T}(\theta(k))\boldsymbol{r}_d(k)$$

In the following, let

$$\boldsymbol{x}_a(k) = \sum_{l=k+1}^{\infty} \Phi^{\rm T}(l,k+1)C_e^{\rm T}(\theta(l))\boldsymbol{r}_d(l)$$

then the state-space representation of $\mathcal{G}^{\sim}_{rd}$ can be obtained as

$$\begin{cases} \boldsymbol{x}_a(k-1) = A_e^{\rm T}(\theta(k))\boldsymbol{x}_a(k) + C_e^{\rm T}(\theta(k))\boldsymbol{r}_d(k) \\ \boldsymbol{d}_a(k) = B_e^{\rm T}(\theta(k))\boldsymbol{x}_a(k) + D_e^{\rm T}(\theta(k))\boldsymbol{r}_d(k) \\ \boldsymbol{x}_a(\infty) = 0 \end{cases} \tag{11}$$

For system (11), when $\theta(k) = p$ and $\theta(k-1) = q$, define

$$\mathbb{V}(\boldsymbol{x}_a(k), \theta(k)) = \boldsymbol{x}_a^{\rm T}(k)P_p\boldsymbol{x}_a(k), \ P_p \geq 0$$

We have:

$${\rm E}\left\{\sum_{k=0}^{\infty} \boldsymbol{d}_a^{\rm T}(k)\boldsymbol{d}_a(k)\right\} = {\rm E}\Big\{\sum_{k=0}^{\infty} \boldsymbol{d}_a^{\rm T}(k)\boldsymbol{d}_a(k) + \\ \mathbb{V}(\boldsymbol{x}_a(k-1), \theta(k-1)) - \mathbb{V}(\boldsymbol{x}_a(k), \theta(k))\Big\} + \\ \boldsymbol{x}_a(\infty)^{\rm T}P_p\boldsymbol{x}_a(\infty) - \boldsymbol{x}_a(-1)^{\rm T}P_{\theta(-1)}\boldsymbol{x}_a(-1) = \\ {\rm E}\Big\{\sum_{k=0}^{\infty} \boldsymbol{x}_a^{\rm T}(k)(B_{ep}B_{ep}^{\rm T} + A_{ep}\bar{P}_pA_{ep}^{\rm T} - P_p)\boldsymbol{x}_a(k) + \\ 2\boldsymbol{x}_a{}^{\rm T}(k)(B_{ep}D_{ep}^{\rm T} + A_{ep}\bar{P}_pC_{ep}^{\rm T})\boldsymbol{r_d}(k) + \\ \boldsymbol{r_d}^{\rm T}(D_{ep}D_{ep}^{\rm T} + C_{ep}\bar{P}_pC_{ep}^{\rm T})\boldsymbol{r_d}(k)\Big\} \tag{12}$$

where $P_{\theta(-1)} = 0$ and $\bar{P}_p = \sum_{q=1}^{N} \lambda_{pq}P_q$.

From (12) and Definition 3, if $\|\boldsymbol{d}_a\|_{2,{\rm E}}^2 = \|\mathcal{G}^{\sim}_{\boldsymbol{rd}}\boldsymbol{r_d}\|_{2,{\rm E}}^2 = \|\boldsymbol{r_d}\|_{2,{\rm E}}^2$, i.e., the following equations hold:

$$\begin{cases} B_{ep}B_{ep}^{\rm T} + A_{ep}\bar{P}_pA_{ep}^{\rm T} = P_p \\ B_{ep}D_{ep}^{\rm T} + A_{ep}\bar{P}_pC_{ep}^{\rm T} = 0 \\ D_{ep}D_{ep}^{\rm T} + C_{ep}\bar{P}_pC_{ep}^{\rm T} = I \end{cases} \tag{13}$$

then $\mathcal{G}_{\boldsymbol{rd}}$ is co-isometric. Relabel $p = i$ and $q = j$, and then (13) turns to (9). □

Remark 3. The representation of $\mathcal{G}^{\sim}_{\boldsymbol{rd}}$ is not unique. In arriving $\boldsymbol{d}_a(k)$ above, we have utilized the fact that for $\forall\, \boldsymbol{\mu}$ and $\boldsymbol{\varsigma}$ with appropriate dimensions, a sufficient condition that ${\rm E}\{\boldsymbol{\mu}\} = {\rm E}\{\boldsymbol{\varsigma}\}$ holds, if $\boldsymbol{\mu} = \boldsymbol{\varsigma}$, which is different from the result by the Dirac function approach in [19].

Based on Lemmas 1 and 2, we are now in position to give the main result of this paper.

Theorem 1. Under Assumptions A1 and A2 are fulfilled, the following matrix pair:

$$L_{o,i} = (B_{\boldsymbol{d}i}D_{\boldsymbol{d}i}^{\mathrm{T}} + A_i\bar{P}_{o,i}C_i^{\mathrm{T}})(D_{\boldsymbol{d}i}D_{\boldsymbol{d}i}^{\mathrm{T}} + C_i\bar{P}_{o,i}C_i^{\mathrm{T}})^{-1} \quad (14)$$

$$V_{o,i} = (D_{\boldsymbol{d}i}D_{\boldsymbol{d}i}^{\mathrm{T}} + C_i\bar{P}_{o,i}C_i^{\mathrm{T}})^{-\frac{1}{2}} \quad (15)$$

with

$$R_{\boldsymbol{d},i} = D_{\boldsymbol{d}i}D_{\boldsymbol{d}i}^{\mathrm{T}} + C_i\bar{P}_{o,i}C_i^{\mathrm{T}} > 0$$

gives an optimal solution to the FDF design problem, where

$$P_{o,i} = \sum_{j=1}^{N} \lambda_{ij}P_{o,j}$$

is the solution of the following coupled Riccati equation:

$$\begin{aligned}P_{o,i} = {} & A_i\bar{P}_{o,i}A_i^{\mathrm{T}} + B_{\boldsymbol{d}i}B_{\boldsymbol{d}i}^{\mathrm{T}} - (B_{\boldsymbol{d}i}D_{\boldsymbol{d}i}^{\mathrm{T}} + A_i\bar{P}_{o,i}C_i^{\mathrm{T}})\times \\ & (D_{\boldsymbol{d}i}D_{\boldsymbol{d}i}^{\mathrm{T}} + C_i\bar{P}_{o,i}C_i^{\mathrm{T}})^{-1}(B_{\boldsymbol{d}i}D_{\boldsymbol{d}i}^{\mathrm{T}} + A_i\bar{P}_{o,i}C_i^{\mathrm{T}})^{\mathrm{T}}\end{aligned} \quad (16)$$

Proof. Let $\boldsymbol{r}_o(k)$ be the optimal generated residual for the FDF design problem. Since system (3) is linear, by applying Lemma 1, we know that there exists an operator Q_r such that

$$\begin{aligned}\boldsymbol{r}(k) = \boldsymbol{r}_f(k) + \boldsymbol{r}_{\boldsymbol{d}}(k) = \mathcal{G}_{\boldsymbol{r}f}\boldsymbol{f}(k) + \mathcal{G}_{\boldsymbol{r}\boldsymbol{d}}\boldsymbol{d}(k) = \\ Q_{\boldsymbol{r}}\boldsymbol{r}_o(k) = Q_{\boldsymbol{r}}(\boldsymbol{r}_{o,f}(k) + \boldsymbol{r}_{o,\boldsymbol{d}}(k))\end{aligned}$$

where $\boldsymbol{r}_{o,f}(k) = \boldsymbol{r}_o(k)|_{\boldsymbol{d}(k)=0}$ and $\boldsymbol{r}_{o,\boldsymbol{d}}(k) = \boldsymbol{r}_o(k)|_{\boldsymbol{f}(k)=0}$, which shows that

$$\boldsymbol{r}_{\boldsymbol{d}}(k) = Q_{\boldsymbol{r}}\boldsymbol{r}_{o,\boldsymbol{d}}(k)$$

On the other hand, consider the operator $\mathcal{G}_{rd}$ that maps $\boldsymbol{d} \mapsto \boldsymbol{r}$ in system (3) , we have:

$$\boldsymbol{r}_{o,\boldsymbol{d}}(k) = \mathcal{G}_{\boldsymbol{r}\boldsymbol{d},o}\boldsymbol{d}(k)$$

where $\mathcal{G}_{\boldsymbol{r}\boldsymbol{d},o} = \mathcal{G}_{\boldsymbol{r}\boldsymbol{d}}|_{L_i=L_{o,i},V_i=V_{o,i}}$, which concludes that

$$\mathcal{G}_{\boldsymbol{r}\boldsymbol{d}} = Q_r\mathcal{G}_{\boldsymbol{r}\boldsymbol{d},o}$$

Similarly, we have:

$$\mathcal{G}_{\boldsymbol{r}f} = Q_{\boldsymbol{r}}\mathcal{G}_{\boldsymbol{r}f,o}$$

where $\mathcal{G}_{\boldsymbol{r}f,o} = \mathcal{G}_{\boldsymbol{r}f}|_{L_i=L_{o,i},V_i=V_{o,i}}$.

Hence,

$$\frac{\|\mathcal{G}_{\boldsymbol{r}f}\|_\infty}{\|\mathcal{G}_{\boldsymbol{r}\boldsymbol{d}}\|_\infty} = \frac{\|Q_{\boldsymbol{r}}\mathcal{G}_{\boldsymbol{r}f,o}\|_\infty}{\|Q_{\boldsymbol{r}}\mathcal{G}_{\boldsymbol{r}\boldsymbol{d},o}\|_\infty}$$

According to Lemma 2, we know that if the following equations hold

$$\begin{aligned}P_{o,i} = {} & (B_{\boldsymbol{d}i} - L_{o,i}D_{\boldsymbol{d}i})(B_{\boldsymbol{d}i} - L_{o,i}D_{\boldsymbol{d}i})^{\mathrm{T}} + \\ & (A_i - L_{o,i}C_i)\bar{P}_{o,i}(A_i - L_{o,i}C_i)^{\mathrm{T}}\end{aligned} \quad (17)$$

$$(B_{\boldsymbol{d}i} - L_{o,i}D_{\boldsymbol{d}i})D_{\boldsymbol{d}i}^{\mathrm{T}}V_{o,i}^{\mathrm{T}} + (A_i - L_{o,i}C_i)\bar{P}_{o,i}C_i^{\mathrm{T}}V_{o,i}^{\mathrm{T}} = 0 \quad (18)$$

$$V_{o,i}(D_{\boldsymbol{d}i}D_{\boldsymbol{d}i}^{\mathrm{T}} + C_i\bar{P}_{o,i}C_i^{\mathrm{T}})V_{o,i}^{\mathrm{T}} = I \quad (19)$$

then, $\mathcal{G}_{rd}$ is co-isometric.

Thus, according to Theorem 4.5-2 in [31] and Definition 2, we have that

$$\begin{aligned}\|\mathcal{G}_{\boldsymbol{r}\boldsymbol{d}}\|_\infty = \|\mathcal{G}_{\boldsymbol{r}\boldsymbol{d}}^{\sim}\|_\infty = \sup\frac{\langle \boldsymbol{d}_a(k), \boldsymbol{d}_a(k)\rangle}{\|\boldsymbol{r}_{\boldsymbol{d}}(k)\|_{2,\mathrm{E}}^2} = \\ \sup\frac{\langle (Q_{\boldsymbol{r}}\mathcal{G}_{\boldsymbol{r}\boldsymbol{d},o})^{\sim}\boldsymbol{r}_{\boldsymbol{d}}(k), (Q_{\boldsymbol{r}}\mathcal{G}_{\boldsymbol{r}\boldsymbol{d},o})^{\sim}\boldsymbol{r}_{\boldsymbol{d}}(k)\rangle}{\|\boldsymbol{r}_{\boldsymbol{d}}(k)\|_{2,\mathrm{E}}^2} = \\ \sup\frac{\langle \boldsymbol{r}_{\boldsymbol{d}}(k), Q_r\mathcal{G}_{\boldsymbol{r}\boldsymbol{d},o}\mathcal{G}_{\boldsymbol{r}\boldsymbol{d},o}^{\sim}Q_r^{\sim}\boldsymbol{r}_{\boldsymbol{d}}(k)\rangle}{\|\boldsymbol{r}_{\boldsymbol{d}}(k)\|_{2,\mathrm{E}}^2} = \|Q_r\|_\infty\end{aligned}$$

and the following inequality immediately establishes

$$\frac{\|\mathcal{G}_{\boldsymbol{r}f}\|_\infty}{\|\mathcal{G}_{\boldsymbol{r}\boldsymbol{d}}\|_\infty} = \frac{\|Q_{\boldsymbol{r}}\mathcal{G}_{\boldsymbol{r}f,o}\|_\infty}{\|Q_{\boldsymbol{r}}\mathcal{G}_{\boldsymbol{r}\boldsymbol{d},o}\|_\infty} \le \frac{\|Q_{\boldsymbol{r}}\|_\infty \cdot \|\mathcal{G}_{\boldsymbol{r}f,o}\|_\infty}{\|Q_{\boldsymbol{r}}\|_\infty} = \|\mathcal{G}_{\boldsymbol{r}f,o}\|_\infty$$

which gives the optimal value of maximizing the performance index $\|\mathcal{G}_{\boldsymbol{r}f}\|_\infty/\|\mathcal{G}_{\boldsymbol{r}\boldsymbol{d}}\|_\infty$.

Furthermore, by solving (19), $V_{o,i}$ can be derived as

$$V_{o,i} = (D_{\boldsymbol{d}i}D_{\boldsymbol{d}i}^{\mathrm{T}} + C_i\bar{P}_{o,i}C_i^{\mathrm{T}})^{-\frac{1}{2}}$$

Substitute $V_{o,i}$ into (18), $L_{o,i}$ can be obtained as

$$L_{o,i} = (B_{\boldsymbol{d}i}D_{\boldsymbol{d}i}^{\mathrm{T}} + A_i\bar{P}_{o,i}C_i^{\mathrm{T}})(D_{\boldsymbol{d}i}D_{\boldsymbol{d}i}^{\mathrm{T}} + C_i\bar{P}_{o,i}C_i^{\mathrm{T}})^{-1}$$

Finally, with the aid of $L_{o,i}$ and $V_{o,i}$, (17) converts to the coupled Riccati equation (16). From [34], we know that if the assumptions A1 and A2 are satisfied, (16) has a positive semi-definite stabilizing solution $\mathcal{P}_o = (P_{o,1}, \cdots, P_{o,N})$. Moreover, due to Theorem 3.1 in [35], one can conclude that when the stabilizing solution exists, system (3) is mean square stable.

Following the same idea, we can prove that

$$\frac{\|\mathcal{G}_{\boldsymbol{r}f}\|_-}{\|\mathcal{G}_{\boldsymbol{r}\boldsymbol{d}}\|_\infty} = \frac{\|Q_{\boldsymbol{r}}\mathcal{G}_{\boldsymbol{r}f,o}\|_-}{\|Q_{\boldsymbol{r}}\mathcal{G}_{\boldsymbol{r}\boldsymbol{d},o}\|_\infty} \le \frac{\|Q_{\boldsymbol{r}}\|_\infty\|\mathcal{G}_{\boldsymbol{r}f,o}\|_-}{\|Q_{\boldsymbol{r}}\|_\infty} = \|\mathcal{G}_{\boldsymbol{r}f,o}\|_-$$

i.e., the matrix pair of $(L_{o,i}, V_{o,i})$ is also an optimal solution to maximizing the performance index $\|\mathcal{G}_{\boldsymbol{r}f}\|_-/\|\mathcal{G}_{\boldsymbol{r}\boldsymbol{d}}\|_\infty$, which completes the proof. □

Remark 4. It is worth mentioning that the optimal solution to problem (4) is not unique. For any real constant β, we have:

$$\frac{\|\beta\mathcal{G}_{\boldsymbol{r}f}\|_\infty}{\|\beta\mathcal{G}_{\boldsymbol{r}\boldsymbol{d}}\|_\infty} = \frac{\|\mathcal{G}_{\boldsymbol{r}f}\|_\infty}{\|\mathcal{G}_{\boldsymbol{r}\boldsymbol{d}}\|_\infty} \quad \text{or} \quad \frac{\|\beta\mathcal{G}_{\boldsymbol{r}f}\|_-}{\|\beta\mathcal{G}_{\boldsymbol{r}\boldsymbol{d}}\|_\infty} = \frac{\|\mathcal{G}_{\boldsymbol{r}f}\|_-}{\|\mathcal{G}_{\boldsymbol{r}\boldsymbol{d}}\|_\infty}$$

which implies that the matrix pair $(L_{o,i}, \beta V_{o,i})$ is also an optimal solution to (4). For solving the coupled Riccati equation (16), there exist many computational algorithms, see for example[36−37] and references therein.

Remark 5. Note that if the set Ω contains only one mode, the results in this paper will coincide with the one given in [15−16, 38] for the infinite horizon case, while if A_i, B_i, $B_{\boldsymbol{d}i}$, B_{fi}, C_i, $D_{\boldsymbol{d}i}$ and D_{fi} are time-varying deterministic matrices, our results will be identical with the one in [15−16] by choosing $\mathbb{V}(k) = \boldsymbol{x}_a^{\mathrm{T}}(k)P(k)\boldsymbol{x}_a(k)$.

Remark 6. With different problem formulations, the existing results in [23, 25] can solve a two-objective H_∞/H_∞ FD problem which should generally be realized by solving two H_∞-type Riccati inequalities while the proposed theorem gives a unified solution to both H_-/H_∞ and H_∞/H_∞ FD problems and only needs to solve one H_2-type Riccati equation. Meanwhile, our solution is analytical and independent from the fault distribution matrices $B_f(\theta(k))$ and $D_f(\theta(k))$, which is computationally simple.

3 Numerical examples

Example 1. To illustrate the effectiveness of the proposed method, a classical economic system proposed in [39] will be considered in the following difference equation form (For more details, please refer to [28, 40−41] and the reference therein)

$$\begin{cases} \boldsymbol{C}_t^E = c\boldsymbol{Y}_{t-1} \\ \boldsymbol{J}_t = w(\boldsymbol{Y}_{t-1} - \boldsymbol{Y}_{t-2}) \\ \boldsymbol{Y}_t = \boldsymbol{C}_t^E + \boldsymbol{J}_t + \boldsymbol{G}_t^E \end{cases} \quad (20)$$

where $\boldsymbol{C}^E$ is the consumption expenditure, $\boldsymbol{Y}$ is national income, $\boldsymbol{J}$ is induced private investment, $\boldsymbol{G}^E$ is government expenditure, c is marginal propensity to consume, w is the accelerator coefficient, and t is the subscript for time with $t = kT = k\,(T = 1)$. From (20), by defining $\boldsymbol{x}(k+1) = [\boldsymbol{x}_1(k+1)\ \boldsymbol{x}_2(k+1)]^{\mathrm{T}}$ and letting $\boldsymbol{x}_1(k+1) = \boldsymbol{x}_2(k)\ \boldsymbol{x}_2(k+1) = Y_k$, we have:

$$\boldsymbol{x}(k+1) = A\boldsymbol{x}(k) + B\boldsymbol{G}^E(k)$$

where

$$A = \begin{bmatrix} 0 & 1 \\ -w & 1-s+w \end{bmatrix},\ B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

s is marginal propensity to save which is related to c by $c = (1-s)$. There exist three different economic scenarios, i.e., "Norm", "Boom" and "Slump", which changes from one mode to another in a Markovian jump sense. The abrupt economic events or emergent political factors can be modelled as the fault occurence in the system. We consider the above system with the following stable system matrices calculated in [28] with

$$A_1 = \begin{bmatrix} 0 & 1 \\ -0.5 & -0.2 \end{bmatrix},\ A_2 = \begin{bmatrix} 0 & 1 \\ -0.7 & 0.4 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 0 & 1 \\ 0.3 & -0.2 \end{bmatrix}$$

$$B_{f1} = B_{f2} = B_{f3} = \begin{bmatrix} 0 \\ -0.5 \end{bmatrix}$$

$$B_{d1} = B_{d2} = B_{d3} = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix}$$

$$C_1 = C_2 = C_3 = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

$$D_{f1} = D_{f2} = D_{f3} = 0.9,\ D_{\boldsymbol{d}1} = D_{\boldsymbol{d}2} = D_{\boldsymbol{d}3} = 0.4$$

$$p_{11} = 0.67,\ p_{12} = 0.17,\ p_{13} = 0.16$$

$$p_{21} = 0.30,\ p_{22} = 0.47,\ p_{23} = 0.23$$

$$p_{31} = 0.26,\ p_{32} = 0.10,\ p_{33} = 0.64$$

We choose

$$J(k) = \sqrt{\frac{1}{k}\sum_{k=0}^{k=k_T} \boldsymbol{r}^{\mathrm{T}}(k)\boldsymbol{r}(k)}$$

as the residual evaluation function, where k_T denotes the length of the evaluation time window. The corresponding threshold is $J_{th} = \sup_{f(k)=0} \mathrm{E}\{J(k)\}$. The unknown input $\boldsymbol{d}(k)$ is shown as in Fig. 1 and $\theta(k)$ changes as shown in Fig. 2. The fault signal is simulated as

$$\boldsymbol{f}(k) = \begin{cases} 1, & k \in [20, 40] \\ -1, & k \in [60, 80] \\ 0, & \text{else} \end{cases}$$

Applying Theorem 1, we have:

$$L_1 = \begin{bmatrix} 0.0059 & 0.2493 \end{bmatrix}^{\mathrm{T}},\ V_1 = 2.4911$$

$$L_2 = \begin{bmatrix} 0.0122 & 0.2511 \end{bmatrix}^{\mathrm{T}},\ V_2 = 2.4888$$

$$L_3 = \begin{bmatrix} 0.0063 & 0.2451 \end{bmatrix}^{\mathrm{T}},\ V_3 = 2.4915$$

Fault $\boldsymbol{f}(k)$ and the corresponding residual $\boldsymbol{r}(k)$ are displayed in Fig. 3. Fig. 4 shows the residual evaluation function for both fault-free and faulty cases. It can be seen from the simulation results that the generated residual can deliver fault alarms soon after the fault occurs.

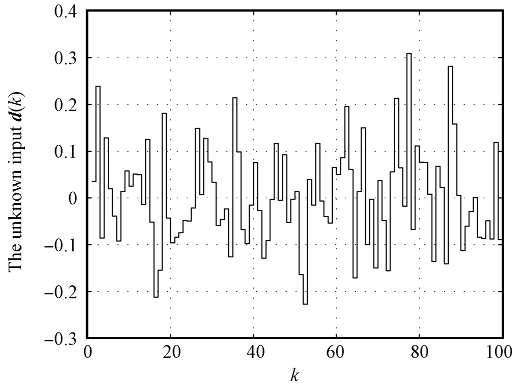

Fig. 1 Unknown input $\boldsymbol{d}(k)$

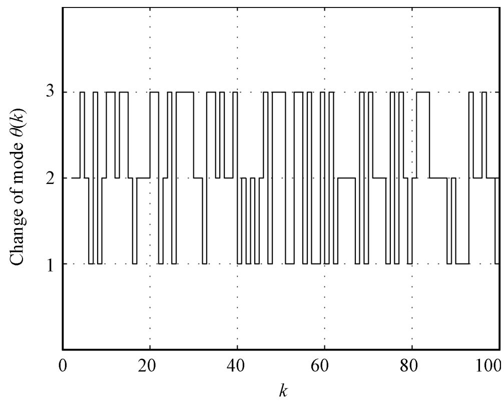

Fig. 2 The operation mode $\theta(k)$

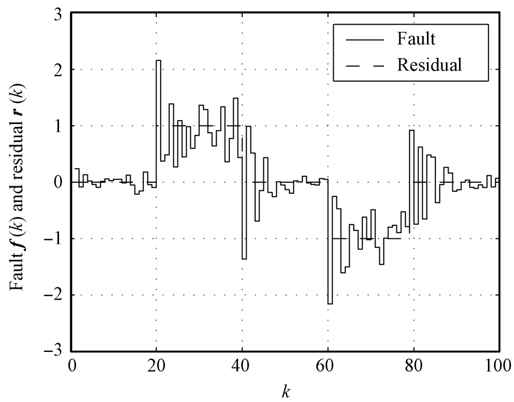

Fig. 3 Fault $\boldsymbol{f}(k)$ and residual $\boldsymbol{r}(k)$

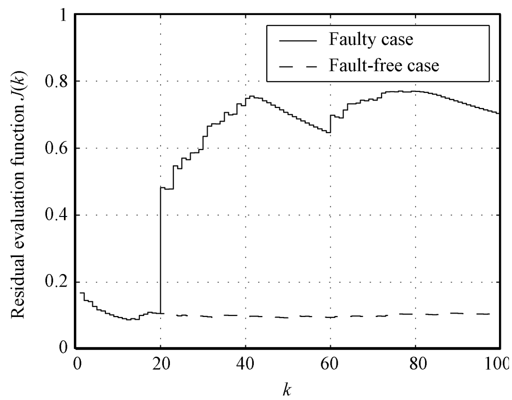

Fig. 4 Residual evaluation function $J(k)$

Example 2. For the purpose of illustrating the advantage of the proposed method in this paper, we compare it with the two-objective method addressed in [23]. Consider the two-mode MJLS given in the following form taken from [23]

$$\begin{aligned}
A_1 &= \begin{bmatrix} 0.1 & 0 & 1 & 0 \\ 0 & 0.1 & 0 & 0.5 \\ 0 & 0 & 0.2 & 0 \\ 0 & 0 & 0 & 0.1 \end{bmatrix} \\
A_2 &= \begin{bmatrix} 0.3 & 0 & -1 & 0 \\ -0.1 & 0.2 & 0 & -0.5 \\ 0 & 0 & -0.2 & 0 \\ 0 & 0 & 0 & -0.5 \end{bmatrix} \\
B_{f1} &= B_{f2} = \begin{bmatrix} 1 & 1 & 2 & -2 \end{bmatrix}^{\mathrm{T}} \\
B_{\boldsymbol{d}1} &= B_{\boldsymbol{d}2} = \begin{bmatrix} 0.8 & -2.4 & 1.6 & 0.8 \end{bmatrix}^{\mathrm{T}} \\
C_1 &= C_2 = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \\
D_{f1} &= D_{f2} = \begin{bmatrix} 2 & -1 \end{bmatrix}^{\mathrm{T}} \\
D_{\boldsymbol{d}1} &= D_{\boldsymbol{d}2} = \begin{bmatrix} 0.2 & 0.4 \end{bmatrix}^{\mathrm{T}} \\
p_{11} &= 0.2,\ p_{12} = 0.8,\ p_{21} = 0.6,\ p_{22} = 0.4
\end{aligned}$$

The unknown input $\boldsymbol{d}(k)$ is still shown as in Fig. 1 and the fault signal is simulated as

$$\boldsymbol{f}(k) = \begin{cases} 0.3, & k \in [20, 40] \\ -0.3, & k \in [60, 80] \\ 0, & \text{else} \end{cases}$$

Applying Theorem 1, we have:

$$\begin{aligned}
L_1 &= \begin{bmatrix} 1.1169 & 1.3664 \\ -5.1180 & -3.3317 \\ 3.2727 & 2.2831 \\ 1.6652 & 1.1293 \end{bmatrix} \\
L_2 &= \begin{bmatrix} 2.0747 & 0.8907 \\ -4.7665 & -3.5017 \\ 3.3262 & 2.2515 \\ 1.7529 & 1.0716 \end{bmatrix} \\
V_1 &= \begin{bmatrix} 3.2952 & -0.2294 \\ -0.2294 & 1.8917 \end{bmatrix} \\
V_2 &= \begin{bmatrix} 3.3260 & -0.4484 \\ -0.4484 & 1.9852 \end{bmatrix}
\end{aligned}$$

Figs. 5 and 6 display the generated residuals using different methods. It can be seen from the simulation results that our proposed algorithm can generate much more sensitive residual signals than the two-objective method in [23] when an incipient fault occurs.

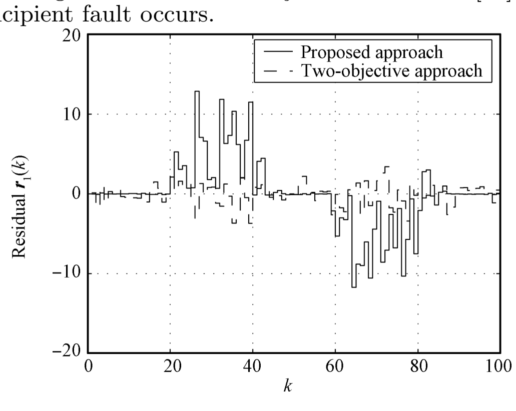

Fig. 5 Generated residual $\boldsymbol{r}_1(k)$

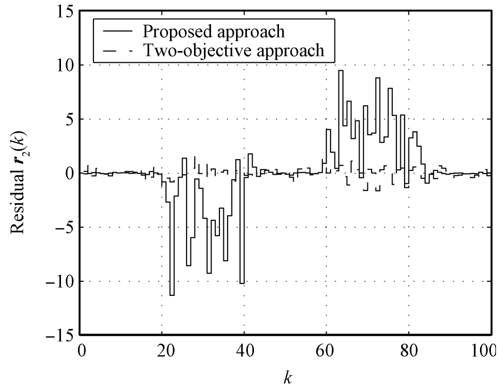

Fig. 6 Generated residual $\boldsymbol{r}_2(k)$

4 Conclusion

In this paper, the problem of optimal fault detection for discrete-time MJLS has been investigated. An observer-based FDF has been considered as a residual generator and the design of the FDF has been formulated in the framework of maximizing stochastic H_-/H_∞ or H_∞/H_∞ performance index. A unified solution has been obtained by solving a coupled Riccati equation in terms of a generalized operator-aided optimization method. The achieved result has been illustrated by numerical examples.

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