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干扰解耦问题

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摘 要

本文在文献[1]的基础上，讨论了一般系统的干扰解耦问题，给出了系统干扰解耦问题有解的充分必要条件及实现干扰解耦所需的反馈阵的解法．

设有系统

$$
\begin{cases} \dot{x}' = A'x' + D'f + B'u \qquad x' \in \mathbb{R}^{n'} \\ y' = C'x'. \end{cases} \tag{1}
$$

文献[1]中讨论了(A', B')完全能控时的干扰解耦问题，给出了检验算法 L．现在，当(A', B')不完全能控时，仍采用 Yokoyama 控制结构相伴标准形[2,3]．设

$$
A' = \begin{bmatrix} A_{00} & 0 & \cdots\cdots\cdots & & 0 \\ 0 & 0 & (I_\nu 0) & 0 & 0 \\ \vdots & \vdots & 0 & (I_{\nu-1} 0) & \vdots \\ 0 & \vdots & \vdots & & (I_2 0) \\ -A_0 & -A_\nu & -A_{\nu-1} \cdots & & -A_1 \end{bmatrix},
$$

$$
D' = \begin{bmatrix} D_0 \\ D_\nu \\ D_{\nu-1} \\ \vdots \\ D_2 \\ D_1 \end{bmatrix}, \qquad B' = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ B_1 \end{bmatrix}, \tag{2}
$$

$$
C' = [C_0 \quad C_\nu \quad C_{\nu-1} \cdots C_1].
$$

其中$\det B_1 \neq 0$，I_i为$n_i \times n_i$阶单位阵，A_i为$m \times n_i$阶阵，C_i为$p \times n_i$阶阵，D_i为$n_i \times q$阶阵，$i = 0, 1, 2, \cdots \nu$．A_{00}代表系统(1)的不能控振型，是$n_0 \times n_0$阶方阵．$m = n_1 \geqslant n_2 \geqslant \cdots \geqslant n_\nu$，且记$n = n_1 + n_2 + \cdots + n_\nu$．这时，系统(1)通过状态反馈

$$
u = -K'x' \tag{3}
$$

能改变的是(2)中的小矩阵A_i，$i = 0, 1, 2, \cdots, \nu$．

记

$$
x' = \begin{pmatrix} x_0 \\ x_1 \end{pmatrix}, \qquad x_0 \in \mathbb{R}^{n_0},\ x \in \mathbb{R}^n,\ n_0 + n = n'.
$$

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这时,(3)式中 $K' = (K_0 \quad K)$, 即

$$u = -(K_0 \quad K)\begin{pmatrix} x_0 \\ x \end{pmatrix},$$

则在文献[1]的符号 J 及 $\tilde{I}_1$ 下,有

$$\overline{A}' = A' - B'K' = \begin{bmatrix} A_{00} & 0 \\ -\tilde{I}_1\overline{A}_0 & J - \tilde{I}_1\overline{A} \end{bmatrix},$$

其中

$$\begin{aligned} \overline{A}_0 &= A_0 + B_1K_0, \\ \overline{A} &= [\overline{A}_\nu\overline{A}_{\nu-1}\cdots\overline{A}_1], \\ \overline{A}_i &= A_i + B_1K_i, \quad i = 1, 2, \cdots, \nu, \\ K &= [K_\nu K_{\nu-1}\cdots K_1]. \end{aligned}$$

记

$$\begin{aligned} C &= [C_\nu C_{\nu-1}\cdots C_1], \\ D^\tau &= [D_\nu^\tau D_{\nu-1}^\tau\cdots D_1^\tau]. \end{aligned}$$

则系统的干扰解耦问题有解的充分必要条件是：是否存在 K_0 与 K,使

$$\begin{cases} (C_0 \quad C)\begin{pmatrix} D_0 \\ D \end{pmatrix} = 0 \\ (C_0 \quad C)\overline{A}'\begin{pmatrix} D_0 \\ D \end{pmatrix} = 0 \\ \cdots \\ (C_0 \quad C)\overline{A}'^{n'-1}\begin{pmatrix} D_0 \\ D \end{pmatrix} = 0 \end{cases}$$

成立,即

$$\begin{cases} C_0D_0 + CD = 0 \\ C_0A_{00}D_0 + CJD - C\tilde{I}_1(\overline{A}_0 \quad \overline{A})\begin{pmatrix} D_0 \\ D \end{pmatrix} = 0 \\ C_0A_{00}^2D_0 + CJ^2D - CJ\tilde{I}_1(\overline{A}_0 \quad \overline{A})\begin{pmatrix} D_0 \\ D \end{pmatrix} - C\tilde{I}_1(\overline{A}_0 \quad \overline{A})\overline{A}'\begin{pmatrix} D_0 \\ D \end{pmatrix} = 0 \\ C_0A_{00}^3D_0 + CJ^3D - CJ^2\tilde{I}_1(\overline{A}_0 \quad \overline{A})\begin{pmatrix} D_0 \\ D \end{pmatrix} - CJ\tilde{I}_1(\overline{A}_0 \quad \overline{A})\overline{A}'\begin{pmatrix} D_0 \\ D \end{pmatrix} \\ \qquad - C\tilde{I}_1(\overline{A}_0 \quad \overline{A})\overline{A}'^2\begin{pmatrix} D_0 \\ D \end{pmatrix} = 0 \\ \cdots\cdots \\ C_0A_{00}^{n'-1}D_0 + CJ^{n'-1}D - CJ^{n'-2}\tilde{I}_1(\overline{A}_0 \quad \overline{A})\begin{pmatrix} D_0 \\ D \end{pmatrix} - \cdots - CJ\tilde{I}_1(\overline{A}_0 \quad \overline{A})\overline{A}'^{n'-3}\begin{pmatrix} D_0 \\ D \end{pmatrix} \\ \qquad - C\tilde{I}_1(\overline{A}_0 \quad \overline{A})\overline{A}'^{n'-2}\begin{pmatrix} D_0 \\ D \end{pmatrix} = 0. \end{cases} \tag{4}$$

对方程(4),讨论如下:

1) 若 $\operatorname{rank} C\tilde{I}_1 = p$，则由(4)式中的第二式开始，可以依次解得

$$\begin{cases}(\bar{A}_0\ \bar{A})\begin{pmatrix}D_0\\D\end{pmatrix} = (C\tilde{I}_1)^+[C_0A_{00}D_0 + CJD] + [I-(C\tilde{I}_1)^+(C\tilde{I}_1)]X_0\\ (\bar{A}_0\ \bar{A})\bar{A}'\begin{pmatrix}D_0\\D\end{pmatrix} = (C\tilde{I}_1)^+\left[C_0A_{00}^2D_0 + CJ^2D - CJ\tilde{I}_1(\bar{A}_0\ \bar{A})\begin{pmatrix}D_0\\D\end{pmatrix}\right]\\ \qquad + [I-(C\tilde{I}_1)^+(C\tilde{I}_1)]X_1\\ \cdots\cdots\\ (\bar{A}_0\ \bar{A})\bar{A}'^{n'-2}\begin{pmatrix}D_0\\D\end{pmatrix} = (C\tilde{I}_1)^+\left[C_0A_{00}^{n'-1}D_0 + CJ^{n'-1}D - CJ^{n'-2}\tilde{I}_1(\bar{A}_0\ \bar{A})\begin{pmatrix}D_0\\D\end{pmatrix}\right.\\ \left. -\cdots - CJ\tilde{I}_1(\bar{A}_0\ \bar{A})\bar{A}'^{n'-3}\begin{pmatrix}D_0\\D\end{pmatrix}\right] + [I-(C\tilde{I}_1)^+(C\tilde{I}_1)]X_{n'-2}.\end{cases} \tag{5}$$

这里，$(C\tilde{I}_1)^+$ 为 $(C\tilde{I}_1)$ 的伪逆. $X_0, X_1, \cdots, X_{n'-2}$ 为 $m\times q$ 阶任意阵.

当选 $X_i = 0, \forall i$，则将(5)式中各式的解代入其它式，经整理可得

$$\begin{cases}(\bar{A}_0\ \bar{A})\begin{pmatrix}D_0\\D\end{pmatrix} = (\bar{C}_0\ \bar{C})\begin{pmatrix}D_0\\D\end{pmatrix}\\ (\bar{A}_0\ \bar{A})\bar{A}'\begin{pmatrix}D_0\\D\end{pmatrix} = (\bar{C}_0\ \bar{C})\bar{C}'\begin{pmatrix}D_0\\D\end{pmatrix}\\ \cdots\cdots\\ (\bar{A}_0\ \bar{A})\bar{A}'^{n'-2}\begin{pmatrix}D_0\\D\end{pmatrix} = (\bar{C}_0\ \bar{C})\bar{C}^{n'-2}\begin{pmatrix}D_0\\D\end{pmatrix},\end{cases} \tag{6}$$

其中

$$\bar{C}_0 = (C\tilde{I}_1)^+C_0A_{00},\ \bar{C} = (C\tilde{I}_1)^+CJ,\ \bar{C}' = \begin{bmatrix}A_{00} & 0\\ -\tilde{I}_1\bar{C}_0 & J-\tilde{I}_1\bar{C}\end{bmatrix}. \tag{7}$$

于是

$$\bar{A}'^i\begin{pmatrix}D_0\\D\end{pmatrix} = \bar{C}'^i\begin{pmatrix}D_0\\D\end{pmatrix},\qquad i = 0, 1, 2, \cdots, n'-2. \tag{8}$$

把(8)式代入(6)式，可得

$$\begin{cases}(\bar{A}_0\ \bar{A})\begin{pmatrix}D_0\\D\end{pmatrix} = (\bar{C}_0\ \bar{C})\begin{pmatrix}D_0\\D\end{pmatrix}\\ (\bar{A}_0\ \bar{A})\bar{C}'\begin{pmatrix}D_0\\D\end{pmatrix} = (\bar{C}_0\ \bar{C})\bar{C}'\begin{pmatrix}D_0\\D\end{pmatrix}\\ \cdots\cdots\\ (\bar{A}_0\ \bar{A})\bar{C}'^{n'-2}\begin{pmatrix}D_0\\D\end{pmatrix} = (\bar{C}_0\ \bar{C})\bar{C}'^{n'-2}\begin{pmatrix}D_0\\D\end{pmatrix},\end{cases}$$

即

$$[(\bar{A}_0\ \bar{A}) - (\bar{C}_0\ \bar{C})]\left[\begin{pmatrix}D_0\\D\end{pmatrix}\bar{C}'\begin{pmatrix}D_0\\D\end{pmatrix}\cdots\bar{C}'^{n'-2}\begin{pmatrix}D_0\\D\end{pmatrix}\right] = 0. \tag{9}$$

(9)式为一线性方程组. 设其解为 $(\psi_0\ \psi)$，则有

$$(\bar{A}_0\ A) = (\psi_0\ \psi) + (\bar{C}_0\ \bar{C}),$$

于是

$$K_0 = B_1^{-1}[\phi_0 + \bar{C}_0 - A_0]$$
$$K = B_1^{-1}[\phi + \bar{C} - A]. \tag{10}$$

其中

$$A = [A_\nu A_{\nu-1} \cdots A_1].$$

2) 若 $\mathrm{rank}\ C\tilde{I}_1 = p_1 < p$. 设 $C^1 = C$,则存在 $p \times p$ 阶非异阵 T_1,使

$$T_1 C^1 = \begin{pmatrix} C_1^1 \\ C_2^1 \end{pmatrix} \begin{matrix} \} \ p_1 \text{ 行} \\ \} \ (p - p_1) \text{ 行} \end{matrix}$$

满足

$$\mathrm{rank} C_1^1 \tilde{I}_1 = p_1, \qquad C_2^1 \tilde{I}_1 = 0.$$

设

$$T_1 C_0 = \begin{pmatrix} C_{01} \\ C_{02} \end{pmatrix} \begin{matrix} \} \ p_1 \text{ 行} \\ \} \ (p - p_1) \text{ 行,} \end{matrix}$$

则式(4)中从第二式起,变为

$$C_{01} A_{00} D_0 + C_1^1 J D - C_1^1 \tilde{I}_1 (\bar{A}_0\ \ \bar{A}) \begin{pmatrix} D_0 \\ D \end{pmatrix} = 0,$$

$$C_{02} A_{00} D_0 + C_2^1 J D = 0,$$

$$C_{01} A_{00}^2 D_0 + C_1^1 J^2 D - C_1^1 J \tilde{I}_1 (\bar{A}_0\ \ \bar{A}) \begin{pmatrix} D_0 \\ D \end{pmatrix}$$
$$- C_1^1 \tilde{I}_1 (\bar{A}_0\ \ \bar{A}) \bar{A}' \begin{pmatrix} D_0 \\ D \end{pmatrix} = 0,$$

$$C_{02} A_{00}^2 D_0 + C_2^1 J^2 D - C_2^1 J \tilde{I}_1 (\bar{A}_0\ \ \bar{A}) \begin{pmatrix} D_0 \\ D \end{pmatrix} = 0,$$

$$\cdots\cdots$$

$$C_{01} A_{00}^{n'-1} D_0 + C_1^1 J^{n'-1} D - C_1^1 J^{n'-2} \tilde{I}_1 (\bar{A}_0\ \ \bar{A}) \begin{pmatrix} D_0 \\ D \end{pmatrix}$$
$$- \cdots - C_1^1 J \tilde{I}_1 (\bar{A}_0\ \ \bar{A}) \bar{A}'^{n'-3} \begin{pmatrix} D_0 \\ D \end{pmatrix} - C_1^1 \tilde{I}_1 (\bar{A}_0\ \ \bar{A}) \bar{A}'^{n'-2} \begin{pmatrix} D_0 \\ D \end{pmatrix} = 0,$$

$$C_{02} A_{00}^{n'-1} D_0 + C_2^1 J^{n'-1} D - C_2^1 J^{n'-2} \tilde{I}_1 (\bar{A}_0\ \ \bar{A}) \begin{pmatrix} D_0 \\ D \end{pmatrix}$$
$$- \cdots - C_2^1 J \tilde{I}_1 (\bar{A}_0\ \ \bar{A}) \bar{A}'^{n'-3} \begin{pmatrix} D_0 \\ D \end{pmatrix} = 0.$$

再由 Hamilton-cayley 定理,增加

$$C_{02} A_{00}^{n'} D_0 + C_2^1 J^{n'} D - C_2^1 J^{n'-1} \tilde{I}_1 (\bar{A}_0\ \ \bar{A}) \begin{pmatrix} D_0 \\ D \end{pmatrix} - \cdots - C_2^1 J^2 \tilde{I}_1 (\bar{A}_0\ \ \bar{A}) \bar{A}'^{n'-3} \begin{pmatrix} D_0 \\ D \end{pmatrix}$$
$$- C_2^1 J \tilde{I}_1 (\bar{A}_0\ \ \bar{A}) \bar{A}'^{n'-2} \begin{pmatrix} D_0 \\ D \end{pmatrix} = 0,$$

可得

$$\left\{\begin{array}{l}C_{02}A_{00}D_0+C_2^1JD=0\\ \begin{pmatrix}C_{01}\\ C_{02}A_{00}\end{pmatrix}A_{00}D_0+\begin{pmatrix}C_1^1\\ C_2^1J\end{pmatrix}JD-\begin{pmatrix}C_1^1\\ C_2^1J\end{pmatrix}\tilde{I}_1(\bar{A}_0\ \ \bar{A})\begin{pmatrix}D_0\\ D\end{pmatrix}=0\\ \begin{pmatrix}C_{01}\\ C_{02}A_{00}\end{pmatrix}A_{00}^2D_0+\begin{pmatrix}C_1^1\\ C_2^1J\end{pmatrix}J^2D-\begin{pmatrix}C_1^1\\ C_2^1J\end{pmatrix}J\tilde{I}_1(\bar{A}_0\ \ \bar{A})\begin{pmatrix}D_0\\ D\end{pmatrix}\\ \qquad\qquad-\begin{pmatrix}C_1^1\\ C_2^1J\end{pmatrix}\tilde{I}_1(\bar{A}_0\ \ \bar{A})\bar{A}'\begin{pmatrix}D_0\\ D\end{pmatrix}=0\\ \cdots\cdots\\ \begin{pmatrix}C_{01}\\ C_{02}A_{00}\end{pmatrix}A_{00}^{n'-1}D_0+\begin{pmatrix}C_1^1\\ C_2^1J\end{pmatrix}J^{n'-1}D-\begin{pmatrix}C_1^1\\ C_2^1J\end{pmatrix}J^{n'-2}\tilde{I}_1(\bar{A}_0\ \ \bar{A})\begin{pmatrix}D_0\\ D\end{pmatrix}\\ \qquad\qquad-\cdots-\begin{pmatrix}C_1^1\\ C_2^1J\end{pmatrix}\tilde{I}_1(\bar{A}_0\ \ \bar{A})\bar{A}'^{n'-2}\begin{pmatrix}D_0\\ D\end{pmatrix}=0.\end{array}\right. \tag{11}$$

如果① $\mathrm{rank}\begin{pmatrix}C_1^1\\ C_2^1J\end{pmatrix}\tilde{I}_1=p$，则回到情形 1)；②若 $C_2^1J=0$ 且 $C_{02}A_{00}D_0=0, C_{02}A_{00}^2D_0=0, \cdots, C_{02}A_{00}^{n_0}D_0=0$，则式(11)从第二式起就等价于

$$\left\{\begin{array}{l}C_{01}A_{00}D_0+C_1^1JD-C_1^1\tilde{I}(\bar{A}_0\ \ \bar{A})\begin{pmatrix}D_0\\ D\end{pmatrix}=0\\ C_{01}A_{00}^2D_0+C_1^1J^2D-C_1^1J\tilde{I}_1(\bar{A}_0\ \ \bar{A})\begin{pmatrix}D_0\\ D\end{pmatrix}-C_1^1\tilde{I}_1(\bar{A}_0\ \ \bar{A})\bar{A}'\begin{pmatrix}D_0\\ D\end{pmatrix}=0\\ \cdots\cdots\\ C_{01}A_{00}^{n'-1}D_0+C_1^1J^{n'-1}D-C_1^1J^{n'-2}\tilde{I}_1(\bar{A}_0\ \ \bar{A})\begin{pmatrix}D_0\\ D\end{pmatrix}\\ \qquad\qquad-\cdots-C_1^1\tilde{I}_1(\bar{A}_0\ \ \bar{A})\bar{A}'^{n'-2}\begin{pmatrix}D_0\\ D\end{pmatrix}=0.\end{array}\right.$$

它可类似于情形 1) 中的方式处理；③ 除①，② 外，则以 $\begin{pmatrix}C_{01}\\ C_{02}A_{00}\end{pmatrix}$ 代替 C_0，以 $\begin{pmatrix}C_1^1\\ C_2^1J\end{pmatrix}$ 代替 C，重复上述讨论过程.

于是，可得检验算法 LNC：

i) 置 $C_0^1=C_0$，$C^1=C$，$i=1$ 及 $p_0=0$.

ii) 计算 $(C_0^iC^i)\begin{pmatrix}D_0\\ D\end{pmatrix}$. 若 $(C_0^iC^i)\begin{pmatrix}D_0\\ D\end{pmatrix}\neq 0$，则转 viii).

iii) 计算 $\mathrm{rank}\,C^i\tilde{I}_1=p_i$. 若 $p_i=p$，则转 x).

iv) 引入 $p\times p$ 阶非异阵

$$T_{i1}=\begin{pmatrix}I_{p_{i-1}} & 0\\ Z_i & T'_{i1}\end{pmatrix}.$$

T'_{i1} 也是非异阵，使

$$T_{i1}C^i=\begin{pmatrix}C_1^{i1}\\ C_2^{i1}\end{pmatrix}\begin{matrix}\}\ p_i\ 行\\ \}\ (p-p_i)\ 行\end{matrix}$$

满足

$$\mathrm{rank}\, C_1^{i1}\tilde{I}_1 = p_i, \qquad C_2^{i1}\tilde{I}_1 = 0.$$

这时，记

$$T_{i1}C_0^i = \begin{pmatrix} C_{01}^{i1} \\ C_{02}^{i1} \end{pmatrix} \begin{matrix} \} \ p_i \ 行 \\ \} \ (p-p_i) \ 行. \end{matrix}$$

v) 计算 $C_2^{i1}J$. 若 $C_2^{i1}J = 0$，则转 ix).

vi) 引入 $p \times p$ 阶非异阵

$$T_{i2} = \begin{pmatrix} I_{p_i} & Y_i \\ 0 & T'_{i2} \end{pmatrix}.$$

T'_{i2} 也是非异阵，设

$$T_iC^i = T_{i2}T_{i1}C^i = \begin{pmatrix} C_1^i \\ C_2^i \end{pmatrix} \} \ p_i \ 行,$$

取 T_{i2}，使 C_1^i 的前面尽可能多地增加全零列，记

$$T_iC_0^i = T_{i2} \cdot T_{i1}C_0^i = \begin{pmatrix} C_{01}^i \\ C_{02}^i \end{pmatrix} \} \ p_i \ 行.$$

vii) 计算 $(C_{02}^iA_{00} \ \ C_2^iJ)\begin{pmatrix} D_0 \\ D \end{pmatrix}$. 若$(C_{02}^iA_{00} \ \ C_2^iJ)\begin{pmatrix} D_0 \\ D \end{pmatrix} = 0$，则取

$$C_0^{i+1} = \begin{pmatrix} C_{01}^i \\ C_{02}^iA_{00} \end{pmatrix}, \quad C^{i+1} = \begin{pmatrix} C_1^i \\ C_2^iJ \end{pmatrix},$$

并用 $i+1$ 代替 i，转 iii).

viii) 干扰解耦问题无解，停.

ix) 计算 $C_{02}^{i1}A_{00}D_0$，$C_{02}^{i1}A_{00}^2D_0, \cdots$，$C_{02}^{i}A_0^{n_0}D_0$，直到出现有某一个 j_0，$C_{02}^{i1}A_{00}^{j_0}D_0 \neq 0$，则转 viii).

x) 干扰解耦问题有解.

这里，与文献[1]中检验算法 L 相同，第 vi) 步是为了使计算过程不出现死循环所必须的. 显然，检验算法 L 是该算法的特殊情况.

为此，有如下结论：

定理. 对形如(2)式的系统(1)，存在全状态反馈$u = -(K_0K)\begin{pmatrix} X_0 \\ X \end{pmatrix}$，使闭环系统是干扰解耦的充分必要条件为：对检验算法 LNC，存在正整数N，或者 $\mathrm{rank}\, C^N\tilde{I}_1 = p$；或者 $C_2^{N1}J = 0$，且 $C_{02}^{N1}A_{00}^kD_0 = 0$，$k = 1, 2, \cdots, \min(n_0, q)$. 这时，实现干扰解耦的反馈阵 K_0 及K 由下式确定：

$$K_0 = B_1^{-1}(\psi_0 + \bar{C}_0 - A_0), \qquad K = B_1^{-1}(\psi + \bar{C} - A). \tag{12}$$

而$(\psi_0\psi)$为线性方程组

$$(\psi_0\psi)(D'\tilde{C}'D'\tilde{C}'^2D'\cdots\tilde{C}'^{n'-2}D') = 0 \tag{13}$$

的解，其中

$$\left\{\tilde{C}' = \begin{bmatrix} A_{00} & 0 \\ -\tilde{I}_1\bar{C}_0 & J - \tilde{I}_1\bar{C} \end{bmatrix},\right.$$

$$\begin{cases}\bar{C}_0 = (C^N\tilde{I}_1)^+ C_0^N A_{00} \\ \bar{C} = (C^N\tilde{I}_1)^+ C^N J,\end{cases} \tag{14}$$

而$(C^N\tilde{I}_1)^+$为$(C^N\tilde{I}_1)$的伪逆.

例.

$$A = \begin{bmatrix} a & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ \times & \times & \times & \times & \times \\ \times & \times & \times & \times & \times \end{bmatrix}, \quad B' = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad D' = \begin{bmatrix} d_0 \\ d_3 \\ d_2 \\ d_{12} \\ d_{11} \end{bmatrix},$$

$$C' = \begin{bmatrix} b & 1 & 0 & 0 & 0 \\ c & 0 & 0 & 0 & 1 \end{bmatrix}.$$

即 $n = 4$, $m = n_1 = 2$, $n_2 = n_3 = 1$, $n_0 = 1$, $p = 2$, $A_{00} = a$. 按检验算法 LNC 的步骤如下:

i) $C_0^1 = \begin{pmatrix} b \\ c \end{pmatrix}$, $C^1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$, $i = 1$, $p_0 = 0$.

ii) 要求 $bd_0 + d_3 = 0$, $cd_0 + d_{11} = 0$.

iii) $p_1 = 1$.

iv) $T_{11} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\begin{pmatrix} C_1^{11} \\ C_2^{11} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}$, $\begin{pmatrix} C_{01}^{11} \\ C_{02}^{11} \end{pmatrix} = \begin{pmatrix} C \\ b \end{pmatrix}$.

v) $C_2^{11} J \neq 0$.

vi) $T_{12} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $\begin{pmatrix} C_1^1 \\ C_2^1 \end{pmatrix} = \begin{pmatrix} C_1^{11} \\ C_2^{11} \end{pmatrix}$, $\begin{pmatrix} C_{01}^1 \\ C_{02}^1 \end{pmatrix} = \begin{pmatrix} C_{01}^{11} \\ C_{02}^{11} \end{pmatrix}$.

vii) 要求 $bad_0 + d_2 = 0$,

$$C_0^2 = \begin{pmatrix} C \\ ba \end{pmatrix}, \quad C^2 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \quad i = 2.$$

iii) $p_2 = 1$.

iv) $T_{21} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $\begin{pmatrix} C_1^{21} \\ C_2^{21} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix}$, $\begin{pmatrix} C_{01}^{21} \\ C_{02}^{21} \end{pmatrix} = \begin{pmatrix} C \\ ba \end{pmatrix}$.

v) $C_2^{21} J \neq 0$.

vi) $T_{22} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $\begin{pmatrix} C_1^2 \\ C_2^2 \end{pmatrix} = \begin{pmatrix} C_1^{21} \\ C_2^{21} \end{pmatrix}$, $\begin{pmatrix} C_{01}^2 \\ C_{02}^2 \end{pmatrix} = \begin{pmatrix} C_{01}^{21} \\ C_{02}^{21} \end{pmatrix}$.

vii) 要求 $ba^2 d_0 + d_{12} = 0$.

$$C_0^3 = \begin{pmatrix} C \\ ba^2 \end{pmatrix}, \quad C^3 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \quad i = 3.$$

iii) $p_3 = 2$.

x) 问题有解.

故问题有解的充要条件是

$$\begin{cases} cd_0 + d_{11} = 0 \\ bd_0 + d_3 = 0 \\ bad_0 + d_2 = 0 \\ ba^2 d_0 + d_{12} = 0. \end{cases} \tag{15}$$

这时,存在正整数 $N = 3$,而

$$\bar{C}_0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} C \\ ba^2 \end{pmatrix} a = \begin{pmatrix} ba^3 \\ ca \end{pmatrix},$$

$$\bar{C}=\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

$$\tilde{C}'=\begin{bmatrix} a & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ -ba^3 & 0 & 0 & 0 & 0 \\ -ca & 0 & 0 & 0 & 0 \end{bmatrix}.$$

方程(13)为

$$(\psi_0\ \ \psi)\begin{bmatrix} d_0 & ad_0 & a^2d_0 \\ d_3 & d_2 & d_2 \\ d_2 & d_{12} & -ba^3d_0 \\ d_{12} & -ba^3d_0 & -ba^4d_0 \\ d_{11} & -cad_0 & -ca^2d_0 \end{bmatrix}=0$$

由式(15),它实际上为

$$(\psi_0\ \ \psi)\begin{pmatrix} d_0 \\ d_3 \\ d_2 \\ d_{12} \\ d_{11} \end{pmatrix}=0$$

由(15)式知, $d_3=-bd_0$, $d_2=-abd_0$, $d_{12}=-a^2bd_0$, $d_{11}=-cd_0$. 由此可解得 $(\psi_0\psi)$, 进而由式(12)确定所需的反馈阵.

由此可见,若一个线性控制系统,是干扰解耦的,则实现干扰解耦的状态反馈阵由线性方程组所确定. 本文给出的检验算法 LNC,是易于在计算机上实现的.

参 考 文 献

[1] Xu Kekang, Wang Shilin and Han Kyengcheng, On Output Invariance for Distubance, Proceeding of the 8th. Triennial World Congress of IFAC. Vol. I, Session 1—4, 84—89.

[2] R. Yokoyama, E. Kinnen, Phase-varible Canonical Forms for Multi-input, Multi-output System, Int. J. Control, **17**(1973), No. 6, 1297—1312.

[3] 韩京清, 线性控制系统结构与反馈系统计算, 全国控制理论及其应用学术交流会论文集, 科学出版社, pp. 43—55 (1981).

DISTURBANCE DECOUPLING PROBLEM

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Abstract

This paper, on the basis of [1], discusses disturbance decoupling problem in general dynamic system. We obtain the necessary and sufficient conditions for solvablity of the disturbance decoupling problem as well as the desirable state feelback matrix to realize disturbance decoupling.